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In the context of a theoretical model of expected profit maximization, this paper shows how historic institutional data can be used to assist enrollment managers in determining the level of financial aid for students with varying demographic and quality characteristics. Optimal tuition pricing in conjunction with empirical estimation of matriculation probability functions illustrates how financial aid can be used to maximize net tuition revenue given institutionally determined objectives. The model provides insight to the level of price sensitivity of prospective matriculants at a medium-sized comprehensive private college.

The recent economic decline and financial market collapse have negatively impacted family, state government, and college budgets. Shrinking endowments and uncertain enrollments have prompted institutions of higher education to focus on revenue enhancement and cost containment. In this environment, the decisions made by financial aid administrators have become more important in achieving financial stability given other institutional objectives such as student quality and diversity. This paper uses rules of optimal tuition pricing developed in a theoretical model, in conjunction with empirical estimation of matriculation probability functions, to illustrate how financial aid can be used to maximize net tuition revenue given institutionally determined quality objectives. The results may be particularly useful to colleges and universities that find themselves more heavily dependent on tuition revenue as a result of shrinking endowments and the current economic recession.

Following Somers and St. John (1997), we focus on institution-specific data from a comprehensive, predominantly undergraduate private college. First, this paper develops a theoretical model of optimal tuition pricing that specifies the optimal level of financial aid for each applicant, based on individual demographic and academic quality characteristics. Second we use historic data from this institution to estimate matriculation probability functions associated with applicants that possess varied demographic backgrounds. Finally, using these estimated probability functions, we simulate how the institution should allocate financial aid among various applicants in order to attain institutional objectives.

Studies of Enrollment and Student Aid

Early matriculation probability models, including Cabrera (1994), Dembowski (1980), Fuller, Manski and Wise (1982), Tierney (1980, 1982) and Weiler (1987), focused on student enrollment probabilities using historical data based on tuition and the demographic characteristics of the applicants. These papers forecast the matriculation probability of an individual applicant, but generally do not identify a net tuition level (stated tuition less financial aid) designed to promote specific institutional objectives.

Ehrenberg and Sherman (1984) present a model of optimal tuition pricing for highly selective institutions where the objective is to maximize utility, which is a function of prestige, student quality, diversity of the student body, and other attributes. While utility maximization models may be appropriate for the most prestigious institutions, where there are long waiting lists of acceptable students and large endowments that act as a cushion in turbulent times, the same cannot be said for the majority of colleges and universities in the United States. According to Peterson's (2009), approximately 6 percent of all colleges accept 30 percent or less of their applicant pool and only 11 percent accept less than 60%. This group of highly selective institutions includes public universities, many of which have deep applicant pools due to taxpayer subsidized tuition rates.

Epple, Romano, and Sieg (2006) have recently presented a price discrimination model for higher education that gives rise to a continuum of institutions of varying quality. They conclude that while top-tier institutions face minimum competition from alternatives that are perceived to be of higher quality, institutions outside of the top tier have many close substitutes and thus limited market power. At these institutions, the relationship between incremental costs and revenues guide admissions policies. Without large endowments, they operate in an environment where tuition-paying students must produce revenue large enough to cover the costs of operation. Private institutions that are not in the highly selective top tier generally operate without long waiting lists. For these institutions, a net tuition revenue maximization strategy may be the key to the maintenance of financial stability. This has become increasingly important to enrollment managers at public institutions as well, since the impact of the recent economic downturn has had negative effects on state expenditures for postsecondary education.

Spaulding (2003) and Spaulding and Olswang (2005) provide the framework for the analysis of financial aid on enrollment yield at a specific public institutions. The focus of these studies is on maximizing yield to meet institutional enrollment goals. Our analysis reorients these objectives to the use of financial aid to generate sufficient tuition revenue to ensure financial stability. While of growing importance to public institutions, this objective has been at the heart of most private colleges and universities in America for a number of years.

The Theoretical Model

Bosshardt, Lichtenstein, and Zaporowski (2008, 2009) present optimal tuition pricing models for academic institutions with enough market power to use price-discriminating techniques that tailor financial aid to each applicant. Rules of optimal tuition pricing are developed where the enrollment decisions of applicants are stochastic in nature and the probability of matriculation is a function of both the demographic characteristics of the applicant and the effective tuition that the applicant must pay. Following Bosshardt, Lichtenstein, and Zaporowski, this paper presents a theoretical model of optimal tuition pricing that specifies the optimal level of financial aid for each applicant, based on individual demographic and academic quality characteristics.

Consider an institution that is composed of both commuter and resident students. The physical plant is sufficiently large such that there is no binding capacity constraint. Over the relevant range of tuition pricing, the institution can accommodate more students than will actually matriculate. The cost of delivering education to the student body has a fixed cost component, f, which reflects expenses such as interest cost on debt, insurance, utilities, and the wage bill for tenured faculty. The marginal cost of delivering education to an additional student is denoted as v. Marginal cost is assumed to be greater than or equal to zero indicating that some institutional costs such as cleaning expenses, laboratory supplies, and the cost of hiring adjunct faculty may be related to the size of the student body.

The institution makes an offer in the form of an effective tuition, t, where t is defined as stipulated tuition less scholarships and grants. The applicant accepts this offer with probability p, which is assumed to be a decreasing function of effective tuition. The probability function is allowed to vary by the demographic and quality characteristics of the applicant. The probability function is expressed as

$$(1) \qquad p = p(t, \beta), \frac{\partial p}{\partial t} < 0,$$

where β is a vector of demographic and quality characteristics of the applicant. The partial derivative of t with respect to ß can be either positive or negative. For expositional ease, we assume that this function is linear.

Assuming that there are N applicants, where N_c is the number of applicants who will not live in the institution's residential housing and N_r is the number of applicants who will live in on-campus housing, then $N = N_c + N_r$. The decision of each applicant to matriculate is expressed as a Bernoulli random variable x_i, which takes on the value one if the applicant enrolls with probability p and takes on a value of zero with probability (1-p) if the applicant does not enroll.

N_c and N_r are partitioned into k mutually exclusive sub-groups such that $N_{1,c} + N_{2,c} + ... + N_{k,c} = N_c$ and $N_{1,r} + N_{2,r} + ... + N_{k,r} = N_r$. There are 2k distinct student quality-demographic profiles for the combined commuter and resident populations. For example, one demographic profile could be applicants from high-income families who planned to major in accounting, had high school averages above 90, and standardized test (SAT) scores above 1200. Since the institution acts as a price discriminator, there can be a different effective tuition charged to each of the 2k quality-demographic sub-groups. $t_{j,c}$ is defined as the effective tuition charged to commuter applicants in sub-group j as j varies from 1 to k. Similarly, $t_{j,r}$ denotes effective tuition charged to resident applicants in sub-group j.

For commuter students, the profit function can be expressed as

$$(2) \quad \pi = \sum_{i=1}^{N_{1,c}} (t_{1,c} - v)x_i + \sum_{i=1}^{N_{2,c}} (t_{2,c} - v)x_i + ... + \sum_{i=1}^{N_{k,c}} (t_{k,c} - v)x_i - f.$$

For each commuter student's quality-demographic profile, the probability of matriculating is:

$$(3) \; p_{i,c} = a_{i,c} - b_{i,c}\, t_{i,c} \qquad \text{where } i = 1, 2, \dots k .$$

The intercept term ($a_{i,c}$) represents the probability that the student will matriculate at an effective tuition of zero. The slope coefficient ($b_{i,c}$) reflects the change in the probability of matriculating per dollar increase in effective tuition. From (2) and (3), expected profit can be expressed as

$$(4) \; E(\pi) = N_{1,c}\,(a_{1,c} - b_{1,c}\, t_{1,c})(t_{1,c} - v) + ... + N_{k,c}(a_{k,c} - b_{k,c}\, t_{k,c})(t_{k,c} - v) - f.$$

Maximizing expected profit yields the following optimal effective tuition for commuter students

$$(5)\quad t^{\star}_{i,c} = \frac{a_{i,c}}{2b_{i,c}} + \frac{v}{2} \qquad \text{where } i = 1, 2, \ldots k.$$

For residential students, the above analysis is altered since room and board fees, d, are an additional source of revenue if the student matriculates. The profit function for residential students is

$$(6)\quad \pi = \sum_{i=1}^{N_{1,r}} (t_{1,r} + d - v)x_i + \sum_{i=1}^{N_{2,r}} (t_{2,r} + d - v)x_i + \ldots + \sum_{i=1}^{N_{k,r}} (t_{k,r} + d - v)x_i - f.$$

For each resident student quality-demographic profile, the probability of matriculating is

$$(7)\ p_{i,r} = a_{i,r} - b_{i,r}\, t_{i,r} \qquad \text{where } i = 1, 2, \ldots k.$$

Expected profit can be expressed as

$$(8)\ E(\pi) = N_{1,r}\,(a_{1,r} - b_{1,r}\, t_{1,r})(t_{1,r} + d - v) + \ldots + N_{k,r}\,(a_{k,r} - b_{k,r}\, t_{k,r})(t_{k,r} + d - v) - f.$$

Maximizing expected profit yields the following optimal effective tuition for resident students

$$(9)\quad t^{\star}_{i,r} = \frac{a_{i,r}}{2b_{i,r}} + \frac{v-d}{2} \qquad \text{where } i = 1, 2, \ldots k.$$

Thus, tuition guidelines for the maximization of expected profit can be applied once the probability function for each student quality-demographic profile is estimated.

The Data

The data are composed of a pooled cross-sectional time series covering freshman class applicants from the 1993-94 through 1995-96 academic years at a medium-sized comprehensive college enrolling approximately 4,500 students, approximately 3,000 of which are undergraduates. The college had major programs in arts and sciences, business, and education. The data contained approximately 5,000 observations and averaged 1,666 applicants over the three-year period.

The academic quality of the applicant, QINDEX, takes on a value of 1 through 5 and depends upon the applicant's high school average and SAT score. QINDEX is defined by the following minimum thresholds:

QINDEX = 1 if the applicant's high school average > 90 and SAT score > 1000
QINDEX = 2 if the applicant's high school average > 85 and SAT score > 900
QINDEX = 3 if the applicant's high school average > 80 and SAT score > 800
QINDEX = 4 if the applicant's high school average > 77 and SAT score > 800
QINDEX = 5 if the applicant's high school average > 74 and SAT score > 750

Two premium majors offered by the college are accounting and biology. Students from these two majors have historically enjoyed a high placement rate in graduate and professional schools and, for those not seeking a graduate education, have commanded a high entry level salary in the job market upon graduation. To capture the influence of planned major upon matriculation probability, we define the following dummy variables: BIO equals one for students listing either biology or bio-chemistry as their intended major and zero for all others. ACC equals one for students listing accounting as their intended major and zero for all others. RES equals one for prospective students wishing to live in college housing and zero for all others.

The variable INCOM is defined as the family income of the applicant measured in thousands of dollars. TUIT is defined as the per semester effective tuition offered to prospective students measured in thousands of dollars. This variable is the difference between stipulated tuition and the amount of financial aid offered.

An Econometric Model of Matriculation Probability

The matriculation probability of student i is assumed to be a linear function of the independent variables described above. For expositional clarity, separate regression equations were estimated for each of the ten possible residential-status, quality-index profiles. Each has the following form.

(10) $p_i = \beta_0 + \beta_1 \text{ TUIT}_i + \beta_2 \text{ ACC}_i + \beta_3 \text{ BIO}_i + \beta_4 \text{ INCOM}_i + \varepsilon_i$

The β's are parameters to be estimated and ε_i is an independently and identically distributed error term. The estimated β's can be interpreted as the change in matriculation probability that will occur with respect to a change in the specific independent variable of interest, holding all other independent variables constant. One does not actually observe matriculation probabilities, but only the realization of whether the student decides to matriculate or not. Consequently, we code the dependent variable as MATRIC, which takes on a value of one if the student matriculated and is equal to zero otherwise. An estimable version of equation (10) is as follows

(11) $\text{MATRIC}_i = \beta_0 + \beta_1 \text{ TUIT}_i + \beta_2 \text{ ACC}_i + \beta_3 \text{ BIO}_i + \beta_4 \text{ INCOM}_i + \varepsilon_i$

When one estimates the β parameters in the above model, the predicted value of MATRIC for a student with a specific set of demographic characteristics will yield the matriculation probability for this student. A variety of statistical techniques can be used to estimate the parameters of (11), including ordinary least squares regression (OLS), Probit and Logit analysis. Since it provides a reasonably intuitive representation of the change in matriculation probability resulting from a change in each of the independent variables, only the results of the OLS regressions are reported in Tables A1 and A2.

As suggested by economic theory, the sign of the coefficient on TUIT is negative and statistically significant at the 1 percent level in all ten cases. The absolute value of the estimated coefficients on TUIT ranges in value from .065 to .159. This implies that, *ceteris paribus,* (other things being equal) a $1,000 increase in effective tuition results in a decrease in matriculation probability in the range of 6.5 to 15.9 percent. This is higher than reported for a similar time period by Kane (1999) since it represents the probability of enrolling at a specific institution, rather than the probability of entering any two or four year institution. The absolute value of the coefficients on TUIT, controlling for QINDEX, are always larger for commuters than for resident students. This suggests that more price sensitive commuter students may be minimizing expenses by residing at home instead of more expensive on-campus housing.

The coefficient on the ACC dummy variable is always positive. The probability of matriculation for applicants planning to major in accounting is from .044 to .202 higher than the matriculation probability for other majors, holding the other explanatory variables constant. The coefficients on BIO are less consistent in that they are positive in only six of ten cases. The coefficient on INCOM is always positive and varies in the range from .0001 to .003. *Ceteris paribus,* a $10,000 increase in household income results in an increase in matriculation probability from .1 to 3 percent, consistent with Spaulding (2003)

Using the Forecasted Matriculation Probabilities to Optimize Financial Aid

Enrollment managers can use the results of the probability model to optimize their financial aid offers to students with varying demographic characteristics. In our model, there are two categories of residential status, three categories for student major and five categories for academic quality. Ignoring household income, there are thirty student profiles in total (2 x 3 x 5). The continuous nature of the INCOM variable in the model makes each applicant a unique case for the purposes of forecasting. Since we wish to illustrate how an enrollment management administrator can use the theoretical model in conjunction with the estimated probability functions to determine the optimal tuition for each student profile, we treat the INCOM variable as discrete. For our purposes, INCOM takes on only two values, high ($80,000) and low ($40,000). These values of high and low income were selected since the mean value of INCOM in our sample was approximately $60,000 with a standard deviation of approximately $20,000. Given the relatively small coefficient on the INCOM variable, little error is introduced into the analysis as a result of this simplification. Consequently, there are sixty student profiles that will be considered for purposes of forecasting.

The historic period over which the data was collected generated an annual average number of applicants amounting to 1,666. We use this number of applicants in our simulation. The breakdown of applicants by student profile is based on the historic proportions of applicants from each of these groups. Table B3 shows the number of applicants for each student profile.

Using the rules of optimization shown in equations (5) and (9), the probability functions shown in Tables B1 and B2, and the applicant pool shown in Table B3, we solve for the optimal tuition for each student profile type in Tables B4, B5, B6 and B7. The model allows the cost of delivering education to have a variable component (v). In the simulations, we have considered the limiting case where variable cost is zero (v=0), indicating that the marginal cost of providing education to an additional student is equal to zero. In this scenario, the school has sufficient faculty and classroom space such that admitting an additional student does not require additional resources. Projections using this assumption appear in Tables B4 and B5. This contrasts with the projections in Tables B6 and B7 where the marginal cost of providing education is assumed to be $1,000 per semester per student. The tuition discount that appears in these tables is simply the difference between the published full semester tuition rate of $5,375 and the optimal tuition for that profile. The matriculation probability at the optimal tuition is solved for by inserting the optimal tuition into the appropriate probability function. The expected number of matriculants is found by multiplying the number of applicants in the appropriate pool by the matriculation probability. Expected net tuition revenue is the product of the expected number of matriculants and the optimal tuition.

Table B8 summarizes the data presented in Tables B4-B7. The following conlusions can be drawn. Residential students receive a deeper tuition discount than do commuter students. For example, when variable costs are zero, the average tuition discount for residential students is 56.6% compared to a 35.8% discount for commuter students. This result is intuitive since residential students bring an additional source of revenue in the form of housing payments to the institution. Of course, this result is dependent upon the assumption that the college has excess capacity in its dormitories. Similar results are obtained for the case where variable costs are positive. A second result is that as the magnitude of variable cost rises, the tuition discount diminishes. For example, for commuter students, as variable costs increase from $0 to $1,000 per student, the average tuition discount falls from $1,924 to

$1,416. These results are consistent with the behavior of a price discriminating producer, since *ceteris paribus,* as marginal costs increase, the optimal price increases.

In Table B9, we summarize how the tuition discount varies by academic quality. Generally, the size of the discount increases as academic quality improves. This occurs since many institutions compete for high-quality students resulting in a greater number of options for the applicant and a lower probability of matriculating.

The influence of the student's choice of academic major upon the optimal tuition discount is shown in Table B10. *Ceteris paribus,* students who plan on majoring in one of the premium subjects have a higher probability of matriculating and consequently require smaller tuition discounts. For example, in the case of commuter students where the institution faces zero variable costs, the average tuition discount for accounting majors is $1,654 and for biology majors is $1,890. The average tuition discount for all other majors aggregated is $2,168.

Conclusion

In the context of a theoretical model of expected profit maximization, this paper has shown how historic data can be used to assist enrollment administrators in determining the level of financial aid for students with varying demographic characteristics. The data needed to estimate the probability functions is readily available as long as the decision makers keep track of historic applicant characteristics, the financial aid offered to applicants and each applicant's matriculation decision. For the institution analyzed in this paper, the variables that had a significant effect on matriculation probability were: the choice of academic major, the academic quality of the student, the applicant's family income, and whether or not the applicant wished to live in on-campus housing. These variables by no means comprise an exhaustive set of the factors influencing a student's matriculation probability, but were important variables for the institution that we analyzed.

Our approach allows consideration of a cost structure comprised of both fixed and variable costs. In our simulation, we present a limiting case where the variable cost is zero, as well as a scenario where the marginal cost of delivering education at $1,000 per student. We have shown that greater variable costs lead to higher optimal effective tuition and therefore less financial aid. Since residential students provide an additional source of revenue to the institution, it is not surprising to find that they receive more financial aid than their commuting counterparts. It is interesting to observe that even though our model did not consider an academic quality constraint whereby the institution actively recruits high quality students to improve its reputation; higher quality students nonetheless generally received more financial aid than their academically inferior peers. Students pursuing marquis majors tend to receive less financial aid *ceteris paribus* when estimates of matriculation probability play a role in determining the size of the financial aid offer.

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Appendix A
Table A1: Ordinary Least Squares Estimates of Equation 11 for Commuter Students

QINDEX	VARIABLE	COEFFICIENT	*t*-Ratio	R^2	*N*
1	CONSTANT	.7096*	14.28	.223	448
	TUIT	-.1354	9.47		
	ACC	.2015*	3.10		
	BIO	.0881	1.58		
	INCOM	.0011	1.32		
2	CONSTANT	.8562*	15.73	.184	469
	TUIT	-.1586*	9.01		
	ACC	.1058	1.63		
	BIO	.0635	1.04		
	INCOM	.0029	.003		
3	CONSTANT	.8307*	16.47	.173	524
	TUIT	-.1410*	8.52		
	ACC	.1234*	2.23		
	BIO	.1318*	1.65		
	INCOM	.0028*	2.76		
4	CONSTANT	.8187*	6.59	.178	101
	TUIT	-.1183*	2.65		
	ACC	.1012	0.71		
	BIO	-.1560	0.83		
	INCOM	.0004	0.16		
5	CONSTANT	.8526*	14.01	.204	279
	TUIT	-.1341*	6.53		
	ACC	.1845*	2.41		
	BIO	.0619	0.58		
	INCOM	.0030*	2.40		

* indicates significance at the 5% level for a one tail test.
N indicates sample size
R^2 is the coefficient of determination
t-ratio is the absolute value of the estimated regression coefficient divided by its standard error.

Table A2: Ordinary Least Squares Estimates of Equation 11 for Resident Students

QINDEX	VARIABLE	COEFFICIENT	*t*-Ratio	R^2	*N*
1	CONSTANT	.3059*	14.28	.132	573
	TUIT	-.0871*	7.75		
	ACC	.0778	1.27		
	BIO	.0830*	1.99		
	INCOM	.0014*	2.57		
2	CONSTANT	.3881*	13.60	.147	763
	TUIT	-.0751*	8.73		
	ACC	.0442	0.85		
	BIO	-.0040	0.10		
	INCOM	.0006	1.42		
3	CONSTANT	.5167*	18.11	.176	996
	TUIT	-.0860*	9.83		
	ACC	.0953*	1.92		
	BIO	.0129	0.26		
	INCOM	.0001	0.30		
4	CONSTANT	.4068*	7.55	.131	331
	TUIT	-.0653*	4.13		
	ACC	.1163	1.18		
	BIO	-.0775	0.79		
	INCOM	.0011	1.15		
5	CONSTANT	.5067*	12.26	.125	518
	TUIT	-.0901*	7.40		
	ACC	.1256	1.62		
	BIO	-.0082	0.09		
	INCOM	.0015*	2.20		

* indicates significance at the 5% level for a one tail test.
N indicates sample size
R^2 is the coefficient of determination
t-ratio is the absolute value of the estimated regression coefficient divided by its standard error.

Appendix B

Table B1: Matriculation Probability Functions for Commuter Students by Income, Intended Major, and Student Quality

Income	Major	QINDEX	Probability Function
Low	Accounting	1	p = .9551 - .1354 TUIT
Low	Biology	1	p = .8417 - .1354 TUIT
Low	Other	1	p = .7536 - .1354 TUIT
High	Accounting	1	p = .9991 - .1354 TUIT
High	Biology	1	p = .8857 - .1354 TUIT
High	Other	1	p = .7976 - .1354 TUIT
Low	Accounting	2	p = 1.0780 - .1586 TUIT
Low	Biology	2	p = 1.0357 - .1586 TUIT
Low	Other	2	p = .9722 - .1586 TUIT
High	Accounting	2	p = 1.1940 - .1586 TUIT
High	Biology	2	p = 1.1517 - .1586 TUIT
High	Other	2	p = 1.0082 - .1586 TUIT
Low	Accounting	3	p = 1.0661 - .1410 TUIT
Low	Biology	3	p = 1.0745 - .1410 TUIT
Low	Other	3	p = .9427 - .1410 TUIT
High	Accounting	3	p = 1.1781 - .1410 TUIT
High	Biology	3	p = 1.1865 - .1410 TUIT
High	Other	3	p = 1.0547 - .1410 TUIT
Low	Accounting	4	p = .9359 - .1183 TUIT
Low	Biology	4	p = .6787 - .1183 TUIT
Low	Other	4	p = .8347 - .1183 TUIT
High	Accounting	4	p = .9519 - .1183 TUIT
High	Biology	4	p = .6947 - .1183 TUIT
High	Other	4	p = .8507 - .1183 TUIT
Low	Accounting	5	p = 1.1571 - .1341 TUIT
Low	Biology	5	p = 1.0345 - .1341 TUIT
Low	Other	5	p = .9726 - .1341 TUIT
High	Accounting	5	p = 1.2771 - .1341 TUIT
High	Biology	5	p = 1.1545 - .1341 TUIT
High	Other	5	p = 1.0926 - .1341 TUIT

Note: Low income variable = $40,000; High income variable = $80,000

Table B2: Matriculation Probability Functions for Resident Students by Income, Intended Major, and Student Quality

Income	Major	QINDEX	Probability Function
Low	Accounting	1	p = .4397 - .0871 TUIT
Low	Biology	1	p = .4449 - .0871 TUIT
Low	Other	1	p = .3619 - .0871 TUIT
High	Accounting	1	p = .4957 - .0871 TUIT
High	Biology	1	p = .5009 - .0871 TUIT
High	Other	1	p = .4179 - .0871 TUIT
Low	Accounting	2	p = .4563 - .0751 TUIT
Low	Biology	2	p = .4081 - .0751 TUIT
Low	Other	2	p = .4121 - .0751 TUIT
High	Accounting	2	p = .4803 - .0751 TUIT
High	Biology	2	p = .4321 - .0751 TUIT
High	Other	2	p = .4361 - .0751 TUIT
Low	Accounting	3	p = .6160 - .0860 TUIT
Low	Biology	3	p = .5336 - .0860 TUIT
Low	Other	3	p = .5207 - .0860 TUIT
High	Accounting	3	p = .6200 - .0860 TUIT
High	Biology	3	p = .5076 - .0860 TUIT
High	Other	3	p = .5247 - .0860 TUIT
Low	Accounting	4	p = .5671 - .0653 TUIT
Low	Biology	4	p = .3733 - .0653 TUIT
Low	Other	4	p = .4508 - .0653 TUIT
High	Accounting	4	p = .6111 - .0653 TUIT
High	Biology	4	p = .4173 - .0653 TUIT
High	Other	4	p = .4948 - .0653 TUIT
Low	Accounting	5	p = .6923 - .0901 TUIT
Low	Biology	5	p = .5749 - .0901 TUIT
Low	Other	5	p = .5667 - .0901 TUIT
High	Accounting	5	p = .7523 - .0901 TUIT
High	Biology	5	p = .6349 - .0901 TUIT
High	Other	5	p = .6267 - .0901 TUIT

Note: Low income variable = $40,000; High income variable = $80,000

Table B3: Number of Student Applying for Admission by Income, Intended Major, Student Quality, and Residential Status

Income	Major	QINDEX	Number of Commuter Students	Number of Resident Students
Low	Accounting	1	4	5
Low	Biology	1	9	12
Low	Other	1	62	79
High	Accounting	1	4	5
High	Biology	1	9	12
High	Other	1	62	79
Low	Accounting	2	4	7
Low	Biology	2	10	16
Low	Other	2	64	105
High	Accounting	2	4	7
High	Biology	2	10	16
High	Other	2	64	105
Low	Accounting	3	4	9
Low	Biology	3	11	21
Low	Other	3	72	137
High	Accounting	3	4	9
High	Biology	3	11	21
High	Other	3	72	137
Low	Accounting	4	1	3
Low	Biology	4	2	7
Low	Other	4	14	45
High	Accounting	4	1	3
High	Biology	4	2	7
High	Other	4	14	45
Low	Accounting	5	2	4
Low	Biology	5	6	11
Low	Other	5	38	71
High	Accounting	5	2	4
High	Biology	5	6	11
High	Other	5	38	71

Note: Low income variable = $40,000; High income variable = $80,000

Table B4: Optimal Tuition, Expected Number of Matriculants and Net Tuition Revenue for Commuter Student Profiles (Variable Costs = 0)

Income	Major	Qindex	Optimal Tuition	Tuition Discount	Matriculation Probability at Optimal Tuition	Expected Number of Matriculants	Expected Net Tuition Revenue
Low	Accounting	1	$3,527	$1,848	0.478	2	$6,462
Low	Biology	1	$3,108	$2,267	0.421	4	$12,155
Low	Other	1	$2,783	$2,592	0.377	23	$64,496
High	Accounting	1	$3,689	$1,686	0.500	2	$7,071
High	Biology	1	$3,271	$2,104	0.443	4	$13,459
High	Other	1	$2,945	$2,430	0.399	25	$72,248
Low	Accounting	2	$3,398	$1,977	0.539	2	$7,357
Low	Biology	2	$3,265	$2,110	0.518	5	$16,448
Low	Other	2	$3,065	$2,310	0.486	31	$95,935
High	Accounting	2	$3,764	$1,611	0.597	2	$9,025
High	Biology	2	$3,631	$1,744	0.576	6	$20,339
High	Other	2	$3,431	$1,944	0.544	35	$120,194
Low	Accounting	3	$3,780	$1,595	0.533	2	$9,034
Low	Biology	3	$3,810	$1,565	0.537	6	$22,228
Low	Other	3	$3,343	$2,032	0.471	34	$113,250
High	Accounting	3	$4,178	$1,197	0.589	3	$11,032
High	Biology	3	$4,207	$1,168	0.593	6	$27,103
High	Other	3	$3,740	$1,635	0.527	38	$141,759
Low	Accounting	4	$3,956	$1,419	0.468	0	$1,601
Low	Biology	4	$2,869	$2,506	0.339	1	$2,039
Low	Other	4	$3,528	$1,847	0.417	6	$20,417
High	Accounting	4	$4,023	$1,352	0.476	0	$1,656
High	Biology	4	$2,936	$2,439	0.347	1	$2,137
High	Other	4	$3,596	$1,779	0.425	6	$21,207
Low	Accounting	5	$4,314	$1,061	0.579	1	$5,963
Low	Biology	5	$3,857	$1,518	0.517	3	$11,546
Low	Other	5	$3,626	$1,749	0.486	19	$67,552
High	Accounting	5	$4,762	$613	0.639	2	$7,264
High	Biology	5	$4,305	$1,070	0.577	3	$14,380
High	Other	5	$4,074	$1,301	0.546	21	$85,250

Note: Low income variable = $40,000; High income variable = $80,000

Table B5: Optimal Tuition, Expected Number of Matriculants and Net Tuition Revenue for Resident Student Profiles (Variable Costs = 0)

Income	Major	Qindex	Optimal Tuition	Tuition Discount	Matriculation Probability at Optimal Tuition	Expected Number of Matriculants	Expected Net Tuition Revenue
Low	Accounting	1	$1,837	$3,538	0.280	1	$2,521
Low	Biology	1	$1,866	$3,509	0.282	3	$6,263
Low	Other	1	$1,390	$3,985	0.241	19	$26,335
High	Accounting	1	$2,158	$3,217	0.308	2	$3,259
High	Biology	1	$2,188	$3,187	0.310	4	$8,070
High	Other	1	$1,711	$3,664	0.269	21	$36,196
Low	Accounting	2	$2,350	$3,025	0.280	2	$4,294
Low	Biology	2	$2,030	$3,345	0.256	4	$8,207
Low	Other	2	$2,056	$3,319	0.258	27	$55,467
High	Accounting	2	$2,510	$2,865	0.292	2	$4,782
High	Biology	2	$2,189	$3,186	0.268	4	$9,269
High	Other	2	$2,216	$3,159	0.270	28	$62,562
Low	Accounting	3	$2,894	$2,481	0.367	3	$9,057
Low	Biology	3	$2,415	$2,960	0.326	7	$16,251
Low	Other	3	$2,340	$3,035	0.319	44	$102,168
High	Accounting	3	$2,917	$2,458	0.369	3	$9,179
High	Biology	3	$2,438	$2,937	0.328	7	$16,509
High	Other	3	$2,363	$3,012	0.321	44	$103,830
Low	Accounting	4	$3,655	$1,720	0.328	1	$3,397
Low	Biology	4	$2,171	$3,204	0.232	2	$3,446
Low	Other	4	$2,764	$2,611	0.270	12	$33,903
High	Accounting	4	$3,992	$1,383	0.350	1	$3,959
High	Biology	4	$2,508	$2,867	0.254	2	$4,359
High	Other	4	$3,101	$2,274	0.292	13	$41,131
Low	Accounting	5	$3,154	$2,221	0.408	2	$5,710
Low	Biology	5	$2,503	$2,872	0.349	4	$9,396
Low	Other	5	$2,457	$2,918	0.345	25	$60,345
High	Accounting	5	$3,487	$1,888	0.438	2	$6,777
High	Biology	5	$2,836	$2,539	0.379	4	$11,560
High	Other	5	$2,790	$2,585	0.375	27	$74,474

Note: Low income variable = $40,000; High income variable = $80,000

Table B6: Optimal Tuition, Expected Number of Matriculants and Net Tuition Revenue for Commuter Student Profiles (Variable Costs > 0)

Income	Major	Qindex	Optimal Tuition	Tuition Discount	Matriculation Probability at Optimal Tuition	Expected Number of Matriculants	Expected Net Tuition Revenue
Low	Accounting	1	$4,027	$1,348	0.410	2	$6,332
Low	Biology	1	$3,608	$1,767	0.353	3	$11,841
Low	Other	1	$3,283	$2,092	0.309	19	$62,414
High	Accounting	1	$4,189	$1,186	0.432	2	$6,941
High	Biology	1	$3,771	$1,604	0.375	3	$13,145
High	Other	1	$3,445	$1,930	0.331	20	$70,166
Low	Accounting	2	$3,898	$1,477	0.460	2	$7,197
Low	Biology	2	$3,765	$1,610	0.439	4	$16,063
Low	Other	2	$3,565	$1,810	0.407	26	$93,382
High	Accounting	2	$4,264	$1,111	0.518	2	$8,866
High	Biology	2	$4,131	$1,244	0.497	5	$19,953
High	Other	2	$3,931	$1,444	0.465	30	$117,641
Low	Accounting	3	$4,280	$1,095	0.463	2	$8,876
Low	Biology	3	$4,310	$1,065	0.467	5	$21,845
Low	Other	3	$3,843	$1,532	0.401	29	$110,717
High	Accounting	3	$4,678	$697	0.519	2	$10,874
High	Biology	3	$4,707	$668	0.523	6	$26,720
High	Other	3	$4,240	$1,135	0.457	33	$139,225
Low	Accounting	4	$4,456	$919	0.409	0	$1,575
Low	Biology	4	$3,369	$2,006	0.280	1	$1,977
Low	Other	4	$4,028	$1,347	0.358	5	$20,007
High	Accounting	4	$4,523	$852	0.417	0	$1,631
High	Biology	4	$3,436	$1,939	0.288	1	$2,075
High	Other	4	$4,096	$1,279	0.366	5	$20,797
Low	Accounting	5	$4,814	$561	0.512	1	$5,883
Low	Biology	5	$4,357	$1,018	0.450	3	$11,352
Low	Other	5	$4,126	$1,249	0.419	16	$66,268
High	Accounting	5	$5,262	$113	0.572	1	$7,184
High	Biology	5	$4,805	$570	0.510	3	$14,186
High	Other	5	$4,574	$801	0.479	18	$83,965

Note: Low income variable = $40,000; High income variable = $80,000

Table B7: Optimal Tuition, Expected Number of Matriculants and Net Tuition Revenue for Resident Student Profiles (Variable Costs > 0)

Income	Major	Qindex	Optimal Tuition	Tuition Discount	Matriculation Probability at Optimal Tuition	Expected Number of Matriculants	Expected Net Tuition Revenue
Low	Accounting	1	$2,337	$3,038	0.236	1	$2,708
Low	Biology	1	$2,366	$3,009	0.239	3	$6,716
Low	Other	1	$1,890	$3,485	0.197	16	$29,333
High	Accounting	1	$2,658	$2,717	0.264	1	$3,446
High	Biology	1	$2,688	$2,687	0.267	3	$8,523
High	Other	1	$2,211	$3,164	0.225	18	$39,194
Low	Accounting	2	$2,850	$2,525	0.242	2	$4,508
Low	Biology	2	$2,530	$2,845	0.218	3	$8,727
Low	Other	2	$2,556	$2,819	0.220	23	$58,907
High	Accounting	2	$3,010	$2,365	0.254	2	$4,997
High	Biology	2	$2,689	$2,686	0.230	4	$9,788
High	Other	2	$2,716	$2,659	0.232	24	$66,001
Low	Accounting	3	$3,394	$1,981	0.324	3	$9,378
Low	Biology	3	$2,915	$2,460	0.283	6	$17,028
Low	Other	3	$2,840	$2,535	0.276	38	$107,311
High	Accounting	3	$3,417	$1,958	0.326	3	$9,500
High	Biology	3	$2,938	$2,437	0.285	6	$17,285
High	Other	3	$2,863	$2,512	0.278	38	$108,972
Low	Accounting	4	$4,155	$1,220	0.296	1	$3,478
Low	Biology	4	$2,671	$2,704	0.199	1	$3,642
Low	Other	4	$3,264	$2,111	0.238	11	$35,200
High	Accounting	4	$4,492	$883	0.318	1	$4,040
High	Biology	4	$3,008	$2,367	0.221	2	$4,555
High	Other	4	$3,601	$1,774	0.260	12	$42,428
Low	Accounting	5	$3,654	$1,721	0.363	2	$5,885
Low	Biology	5	$3,003	$2,372	0.304	3	$9,819
Low	Other	5	$2,957	$2,418	0.300	21	$63,148
High	Accounting	5	$3,987	$1,388	0.393	2	$6,952
High	Biology	5	$3,336	$2,039	0.334	4	$11,983
High	Other	5	$3,290	$2,085	0.330	23	$77,278

Note: Low income variable = $40,000; High income variable = $80,000

Table B8: Summary of Expected Number of Students, Tuition Revenue, and Tuition Disount for Commuter and Resident Students (Variable Costs > 0)

	Expected Number of Students	Expected Net Tuition Revenue	Expected Housing Revenue	Total Revenue	Average Tuition Discount	Discount as a percent of Stated Tuition
Variable Cost = 0						
Commuter Students	293	$1,010,606	$0	$1,010,606	$1,924	35.8%
Resident Students	318	$742,674	$437,560	$1,180,234	$3,041	56.6%
Variable Cost > 0						
Commuter Students	250	$989,097	$0	$989,097	$1,416	26.3%
Resident Students	275	$780,727	$377,762	$1,158,489	$2,533	47.1%

Table B9: Average Tuition Discount by Academic Quality Profiles, Residential Status, and Variable Cost Assumptions

	QINDEX	Average Tuition Discount	Discount as a Percent of Stated Tuition
Variable Cost = 0			
Commuter Students	1	$2,418	45.0%
	2	$2,072	38.6%
	3	$1,734	32.3%
	4	$1,856	34.5%
	5	$1,441	26.8%
Resident Students	1	$3,723	69.3%
	2	$3,224	60.0%
	3	$2,982	55.5%
	4	$2,444	45.5%
	5	$2,697	50.2%
Variable Cost > 0			
Commuter Students	1	$1,915	35.6%
	2	$1,570	29.2%
	3	$1,230	22.9%
	4	$1,353	25.2%
	5	$938	17.5%
Resident Students	1	$3,219	59.9%
	2	$2,724	50.7%
	3	$2,481	46.2%
	4	$1,941	36.1%
	5	$2,195	40.8%

Table B10: Average Tuition Discount by Academic Major, Residential Status, and Variable Cost Assumptions

	Major	Average Tuition Discount	Discount as a Percent of Stated Tuition
Variable Cost = 0			
Commuter Students	Accounting	$1,654	30.8%
	Biology	$1,890	35.2%
	Other	$2,168	40.3%
Resident Students	Accounting	$2,621	48.8%
	Biology	$3,138	58.4%
	Other	$3,174	59.1%
Variable Cost > 0			
Commuter Students	Accounting	$1,150	21.4%
	Biology	$1,385	25.8%
	Other	$1,663	30.9%
Resident Students	Accounting	$2,110	39.3%
	Biology	$2,635	49.0%
	Other	$2,666	49.6%