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A MULTI-PERIOD FACILITY LOCATION MODEL FOR SERVICE
ORGANIZATIONS OPERATING IN A COMPETITIVE ENVIRONMENT

By

Sven W. Brose

Diplom-Wirtschaftsingenieur, Technische Universitat Dortmund, Germany 2008

A Dissertation
Submitted to the J.B. Speed School of Engineering
of the University of Louisville
in Partial Fulfillment of the Requirements
for the Degree of

Doctor of Philosophy in Industrial Engineering

Department of Industrial Engineering
Speed School of Engineering
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May 2017

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ABSTRACT

A MULTI-PERIOD FACILITY LOCATION MODEL FOR SERVICE ORGANIZATIONS OPERATING IN A COMPETITIVE ENVIRONMENT

Sven W. Brose

March 27th, 2017

Facility location is an important problem faced by companies in many industries. Finding an optimal location of facilities and determining their size involves the consideration of many factors including: proximity to customers and suppliers, availability of skilled employees and support services, and cost-related factors, for example, construction or leasing costs, utility costs, taxes, availability of support services, and others.

In addition, the demand of the surrounding region plays an important role in location decisions. A high population density may not necessarily cause a proportional demand for products or services. The demography of a region could dictate the demand for products and this, in turn affects a facility's size and location.

The location of a company's competitors also affects the location of that company's facilities. Just because a competitor has a facility in a particular location does not necessarily mean the company must also locate its facilities there. Local restrictions such as laws or subsidies and taxes play an important role in a facility location decision. Infrastructural aspects also play a role in the location decision. A facility that has good connections to highways, air transportation, and railroads is more attractive to those with infrastructural bottlenecks.

Developed countries may not necessarily have higher demand for products, compared to emerging markets. In fact, the opposite could be true. Massive declines in sales have caused a deep impact on the economic survival of traditional industries in the last two decades, both in the United States and Europe. In some industries, the focus has now shifted towards the closure of facilities because markets are saturated and new business models have caused changes in business practices. How long to operate a facility in a market that is declining is thus a strategic question. Reports on closures of drugstores or building centers are part of the daily news in Germany. Furthermore, the growth of e-commerce has changed businesses completely and will dominate the development of business process automation.

Developed countries are facing demographic changes, including declining populations, and increase in immigration – this causes significant changes in markets and demands and in the product mix companies must offer.

Demand often is seasonal. Renting outdoor equipment is of interest for companies during the summer, but not in other seasons.

Location decisions are sometimes not based on a careful, rational and systematic analysis, rather a political decision meant to satisfy multiple parties.

Another important aspect in facility location modeling is that many models focus on the current demand, and do not adequately consider future demand. However, while making location decisions in an industry in decline, carefully and accurately considering future demand is especially important and the question in focus is to shrink or close down certain facilities with the objective of keeping a certain market share or maximizing profit, especially in a competitive environment.

Finally, strategic aspects of a company have significant influence on a facility and its location. For example, the types and level of service offered impacts customers' patronage.

Although numerous factors influence the location of a facility, existing models do not take all of them into consideration.

This thesis develops a multi-period, mathematical programming location model for facilities operating in a competitive environment. The objective is to maximize the profit in a variable demand environment. The solution of the problem can either be done in a robust or in a dynamic manner. A robust solution methodology forecasts the demand for

all the periods in the planning horizon and provides a robust solution that may be suboptimal for any given period, but optimal when all the periods are taken into consideration. A problem with this approach is that it requires accurate forecast of demand for multiple periods into the future, which is not easy especially when the number of periods is large or the length of each period is long. The dynamic solution methodology circumvents this problem by only considering the demand for the next period in making location decisions. The drawback of this approach is that it may call for frequent closing of existing facilities and opening of new or previously closed facilities and the cost of doing so may be rather high in many environments. However, the cost of opening and closing facilities may not be as high as the benefits of doing so for many service organizations.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iii
ABSTRACT	v
LIST OF TABLES	xv
LIST OF FIGURES	xix
CHAPTER 1 INTRODUCTION	1
1.1 Motivation	1
1.2 A multi-period facility location model for service organizations operating in a competitive environment	4
1.3 Scope of this dissertation	5
CHAPTER 2 LITERATURE REVIEW	7
2.1 Papers on classification schemes for facility location	7
2.2 A classification scheme for facility location models focusing on opening, closing, and capacity decisions incorporating competition.....	12
2.3 Literature on shrinking and facility closing.....	17

2.4 Literature on expanding or opening new facilities and on variable demand.....	23
2.5 Literature analysis.....	24
CHAPTER 3 MODELS FOR MULTI-PERIOD FACILITY LOCATION	26
3.1 Introduction	26
3.2 Model 1: Single product model	27
3.2.1 Mathematical formulation	27
3.2.2 Computational results.....	32
3.2.3 Performed checks	36
3.2.4 Extension of initial model	38
3.2.5 Retrieving the relaxed solution from the model.....	41
3.2.6 Solving the model in dynamic manner.....	42
3.3 Model 2: Allowing multiple products and additional constraints	59
3.3.1 Mathematical formulation	59
3.3.2 Computational results.....	65
3.3.3 Additional modifications.....	66
3.3.4 Some thoughts on the attraction parameter	67

3.3.5 Computational results.....	67
3.3.6 Complexity in real world applications	68
3.3.7 Technical implementation and corresponding challenges	69
3.3.8 Retrieving the model's bounds.....	71
3.3.9 Modification of parameters	72
3.4 Summary.....	74
CHAPTER 4 A DECOMPOSITION APPROACH	75
4.1 Introduction	75
4.2 Decomposition and database realization	76
4.3 Computational results	79
4.4 Combining different solutions	80
4.4.1 Selection strategy 1: Select if open for all products.....	83
4.4.2 Selection where at least three facilities are open / closed	90
4.4.3 Selection where at least five facilities are open / closed	94
4.4.4 Selection where at least eight facilities are open /closed	95
4.5 Summary.....	100
CHAPTER 5 A SIMULATED ANNEALING ALGORITHM.....	101

5.1 Introduction	101
5.2 Modification of algorithm	102
5.3 Technical challenges.....	104
5.4 Computational results	105
5.4.1 Solution times.....	105
5.4.2 The choice of temperature and cooling rate	108
5.5 Pre-processing	112
5.6 Conclusion	115
CHAPTER 6 CASE STUDY	116
6.1 Introduction	116
6.2 About Dortmunder Volksbank	117
6.3 The model	119
6.3.1 Gathering model data and database implementation.....	119
6.3.2 Mathematical formulation	122
6.3.3 Computational results.....	131
6.3.4 Applying simulated annealing.....	141
6.4 Conclusion	144

CHAPTER 7 CONCLUSIONS	146
7.1 Contributions	146
7.2 Further Works	147
7.2.1 Extension of the attraction function	147
7.2.2 Incorporating additional constraints	147
7.2.3 Combination of solutions	147
REFERENCES	149
Appendix A The Lingo File of model 1	159
Appendix B The Lingo file of model 1 using a dynamic approach	163
Appendix C Data used for comparison of robust and dynamic solution approach	167
Appendix D The Lingo File of model 2	178
Appendix E The Lingo File of model 2 including extensions	183
Appendix F The JAVA source code for the simulated annealing algorithm	191
Appendix G Ruby script for gathering distance information	199
Appendix H Data for case study	201
Appendix I LINGO file for case study model	420
Appendix J JAVA Source Code of annealing algorithm for case study	428

CURRICULUM VITA	437
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LIST OF TABLES

Table 1 Test data for model 1 with 3 facilities, 3 demand points, and 3 time periods.....	32
Table 2 Variables and runtimes for different datasets.....	39
Table 3 Objective function values for different model sizes both for mixed integer model and relaxed model	41
Table 4 Results for constant data for dynamic and robust approach	47
Table 5 Results for alternating demand for dynamic and robust approach.....	51
Table 6 Results for alternating prices for dynamic and robust approach.....	55
Table 7 Variables and runtimes for different datasets for extended model	65
Table 8 Database view "modeldata_mp_10_view"	69
Table 9 Database table "modeldata_mp_10_initials_cap"	70
Table 10 Database table "modeldata_mp_10_initials"	70
Table 11 Database table "modeldata_mp_10_facilities_npr"	70
Table 12 The model's bounds for the extended model.....	71

Table 13 Database view MP_5_DEMANDS_P01_VIEW	77
Table 14 Database view MP_5_FACILITIES_P02_VIEW.....	78
Table 15 Database view MP_5_INITIALS_CAP_P03_VIEW	78
Table 16 Solving times for 5x5x5x5 sub problems in seconds.....	79
Table 17 Results tableau for opening status of facilities depending on the sub problem	80
Table 18 Result tableau for sub problems with five products.....	83
Table 19 Selection of all lines with open status equals 0.....	84
Table 20 Objective function value for 5x5x5x5 model using different selection strategies	85
Table 21 Results tableau for 10x10x10x10 model and its sub problems.....	86
Table 22 Results tableau with at least three facilities open for 10x10x10x10 model	90
Table 23 Results tableau with at least eight facilities open for 10x10x10x10 model.....	95
Table 24 Objective function values depending on selection strategy	99
Table 25 Computational times for each step of the SA algorithm with respect to problem size for the mixed-integer model	106
Table 26 Computational times for each step of the SA algorithm with respect to problem size for the relaxed model	106

Table 27 Potential solving times for all configurations	108
Table 28 Annealing steps depending on the choice of cooling rate and temperature	109
Table 29 Means and standard deviations for objective function values using different initial configurations	113
Table 30 p-values for <i>t</i> -test on means for 5x5x5x5.....	114
Table 31 p-values for <i>t</i> -test on means for 10x10x10x10.....	114
Table 32 Database view " CS_COMPETITORS_VIEW"	121
Table 33 Database table " CS_ATTRACTIVITY_VIEW"	121
Table 34 Database table "CASE_STUDY_DATA_INITIALS_CAP"	121
Table 35 Database table "CASE_STUDY_INITIALS"	121
Table 36 Database table "CS_PRODUCT_VIEW"	121
Table 37 Database table "CASE_STUDY_DATA_FACILITIES_NPR"	121
Table 38 Database table "CS_DEMAND_VIEW"	122
Table 39 Open status for sub problems.....	134
Table 40 Open status for sub-problems.....	137
Table 41 Open status for potential new locations	141
Table 42 Data used for first test - keeping demand and prices constant over time.....	167

Table 43 Data used for second test - Alternating demand and keeping prices constant over time	175
Table 44 Data used for third test - Alternating prices and keeping demand constant over time	176
Table 45 Data used for forth test - Contantly sinking demand	176
Table 46 Initial capacity of facilities.....	201
Table 47 prices and variable cost	202
Table 48 expansion, shrinkage, opening, closing and fixed cost	206
Table 49 Demand data for case study for all 13 products	207
Table 50 Competitors with highest attraction with respect to demand point.....	245
Table 51 Attraction value for Dortmunder Volksbank's facilities with respect to demand points.....	283

LIST OF FIGURES

Figure 1: Solver status for model with 100 locations, 100 demand points and 100 time periods.....	40
Figure 2 LINGO error message for 150x150x150 model.....	41
Figure 3 Dynamic solution approach	43
Figure 4 LINGO solver status for extended model 2.....	68
Figure 5 Procedure of decomposition algorithm.....	82
Figure 6 Solution times for 10x10x10x10 sub problems (in seconds)	86
Figure 7 Average runtime per annealing step for the relaxed model.....	107
Figure 8 Simulated annealing for 10x10x10x10 model temperature 10^6 and cooling rate 0.05.....	109
Figure 9 Simulated annealing for 10x10x10x10 model temperature 100 and cooling rate 0.001.....	110
Figure 10 Simulated annealing for 10x10x10x10 model temperature 10^7 and cooling rate 0.01.....	111

Figure 11 Simulated annealing for 10x10x10x10 model temperature 10,000 and cooling rate 0.01.....	111
Figure 12 City of Dortmund and its districts (Stadt Dortmund, 3/Dez - Stabsstelle Dortmunder Statistik, 2015).....	118
Figure 13 Procedure for solving sub-problems in case study	133
Figure 14 Scenario 1: Facility configuration and assigned demands in period 1	135
Figure 15 Scenario 1: Facility configuration and assigned demands in period 10	136
Figure 16 Scenario 2: Facility configuration and assigned demands in period 1	138
Figure 17 Scenario 2: Facility configuration and assigned demands in period 5	139
Figure 18 Scenario 2: Facility configuration and assigned demands in period 10	140

CHAPTER 1

INTRODUCTION

Location problems deal with the allocation of resources in space. According to Brandeau & Chiu, 1989 the general location paradigm deals with one or more facilities, called servers, serving a set of customers that are distributed in a specific manner in a region. Location decisions are critical and strategic and must be made thoroughly because very often these require high investments (Klose and Drexl, 2005).

1.1 Motivation

Empty parking lots? Useless supermarkets? A McKinsey study states that online-shopping for commodities will become more and more prevalent and that it will force the closure of commodity markets (n-tv Nachrichtenfernsehen GmbH, 2013). This study provided an outlook on a possible future - but it also dealt with the past which has shown that facility closure or shrinking has always been a concern for planners and needs to be conducted thoroughly.

There are numerous examples illustrating the necessity of facility shrinking or closure. Schlecker (Alter, 2012), General Motors (in Germany especially Adam Opel AG)

(Süddeutsche Zeitung, 2013), airlines (particularly the regional market) (Döring, 2014) or building centers like Praktiker AG (Dierig and Seidel, 2013) are just a few of them.

Another topical example is Deutsche Bank. Customers use the bank's branches increasingly less often. Deutsche Bank is now planning a massive restructuring of its branches network (Spiegel Online, 2016).

The closing-down of facilities is an operational problem that occurs frequently and not something that occurs rarely (<http://plantclosings.com>, 2012).

Apart from political decisions, industrial and society trends also influence location decisions and will impact discussions and decisions in location theory.

Numerous new business models have been created during the last years. The influence of e-commerce is omnipresent. Old business models are constantly challenged. For example there are some general stores in Germany, such as Karstadt (Karstadt, 2006), that failed to change their business model recently and had to close many locations during the last few years. Extremely short periods for bringing products and services to markets and a highly competitive environment make companies permanently rethink their location strategies and associated decisions. In some cases, electronic commerce has led to shop closings because more and more products are sold via the Internet.

In contrast to Europe and the United States, previously depressed economic regions are beginning to prosper and generate high demand for consumer products. The effect is that

companies are interested in investing in those regions such as India, China and even parts of Africa. A majority of the demand for many products is from those regions/areas and not necessarily from the saturated markets in developed countries.

The employment cost in Europe and the US are much higher than they are in countries such as India, China or Eastern Europe. Thus it has been and still is attractive for companies to move production capacities into regions with lower labor cost. Another societal trend that must be mentioned is that there is a movement towards possessing things for a shorter period of time instead of owning them for lifetime. Rental business is becoming more and more prevalent. In Germany “car sharing” has become more and more prevalent in cities. Renting is especially interesting for people who do not want to invest money into things they hardly ever use. As rental companies must keep their offering up to date, facility location is an important topic for them.

All over the world, large cities tend to grow even larger. An urban concentration takes place and causes rural exodus. A widespread distribution network with rather small facilities is very often replaced by large facilities that serve certain demand. Even financial service companies such as banks or insurances concentrate their businesses. For Example Hypo Vereinsbank is currently undergoing such a strategic change by rationalization of its branches and moving to online banking.

Companies face an enormous pressure in bringing products to market. Life cycles for products have become shorter, thus the time to return on investment has reduced. A

strong competitive environment meets very low distinction in the range of products and services offered.

1.2 A multi-period facility location model for service organizations operating in a competitive environment

Very often, decisions on facility location are made without considering the competitors' behavior or by taking the time horizon into account. Subsidies or political interests are sometimes the factors considered in making location decisions, instead of a sound analysis involving the numerous factors that impact location decisions. In many cases companies realize very late that their choice of location is not optimal. This dissertation develops a model which enables companies to select sites for their businesses according to their strategy. The model analyzes the strategic position of the company and forms a guideline for the decision. It investigates which facilities should be closed, (re) opened, shrunk or expanded. If facilities are to shrink or to expand, the model also determines their new capacities. It shows the impact on the market share and takes the costs for closure and opening into consideration.

A number of papers deal with location theory and its applications, but a few have been written for modeling a competitive environment in the case of declining demand. Existing papers in this area of research are mostly static in nature, do not offer multiperiod approaches, nor do they incorporate the behavior of competitors in the market.

Problems addressed in the model are:

- 1) The set of facilities that must be reduced in size, and the corresponding size
- 2) The set of facilities that must be expanded in size, and the corresponding size
- 3) The set of facilities that must be closed
- 4) The set of facilities that must be opened
- 5) The set of demand locations that will be assigned to a company's own facilities

Two possible settings are discussed.

- 1) Robust solution: The demand for all periods is known at the beginning. The model then is solved only once in a way that the solution is robust for all periods in the planning horizon. This requires more accuracy in demand forecast.
- 2) Dynamic solution: Beginning with an initial solution, the model is resolved whenever there is a significant change in demand or other parameters. With such an approach, the cost of closing or opening facilities and moving from one solution in one period to another in the next, must be considered.

To solve the problem mathematically, mixed-integer programming models will be developed and heuristic procedures will be applied.

1.3 Scope of this dissertation

This dissertation contains a review of the literature on location theory. It begins with an overview on the basis of locating theory, explains the classification of location models

and takes parameters that influence the choice of locations into consideration. Models for the closing-down and shrinking of facilities are presented as well as models that deal with a high variation of demands

The dissertation also encapsulates the development of the mathematical model for this problem and extends the model in a separate sub chapter. Alternate solution approaches are presented and a decomposition approach is developed. Additionally, a modified simulated annealing procedure is applied to the model.

A case study is carried out and the results are discussed. Finally steps for future research are presented.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews the literature on facility location. Location optimization has been studied since the beginning of the 19th century. The early papers focused on a more economic level in the theory of land usage see (von Thünen, 1910) or in the theory of central locations (Christaller, 1933). Location theory on a business level was formally introduced by Weber (1909). Isard (1956) extended Weber's work. Others also started working on location problems by considering the problem of locating two competing vendors along a straight line e.g. Hotelling (1929). A number of different researchers have studied the location problem from multiple perspectives and have published various papers (Aboolian et al. 2006), (Akinc & Khumuwala, 1977), (Ashtiani et al. 2013), (Chandrasekaran & Daughety, 1981), (Cooper, 1972), (Drezner et al. 1985), (Drezner T. , 1998), (Drezner T. , 1994), (Efroymson & Ray, 1966), (Elzinga & Hearn, 1972), (Geoffrion, 1975), (Goldman, 1971), (Hotelling, 1929), (Plastria F. , 2005), (ReVelle et al. 2008), (van Roy & Erlenkotter, 1982), (Weber, 1909).

2.1 Papers on classification schemes for facility location

In order to get a more organized view of the scope of the literature, a set of relevant papers are classified in this chapter. First existing classification schemes are introduced.

Second, these schemes are used to set up a taxonomy to classify the models on facility location and to get a deeper insight into the topic especially on competitive location models and models in the field of expansion, shrinking, opening and closing. Multiple researchers have set up classification schemes for facility location models.

Papers on location theory have been classified in Francis et al. (1983). They distinguish between four classes of problems, continuous planar, discrete planar, mixed planar, and discrete network problems.

Daskin (Daskin et al. 2008) splits location models into analytical models, continuous models, network models and discrete models. According to Daskin, (2008), analytical models are not applicable for certain problems. The main differences according to this classification are to be found in the decision space (market) (Haase & Hoppe, 2008). (Klose & Drexl, 2005) do the same clustering as they distinguish between continuous location models, network models and mixed-integer programming models. The latter two types of models actually are discrete optimization models, so this classification has much in common with the classifications mentioned above. They narrow down the mixed-integer programming models into (1) single- vs. multi-stage models, (2) uncapacitated vs. capacitated models, (3) single- vs. multi-product models, (4) static vs. dynamic models, (5) deterministic vs. stochastic models, (6) models with and without routing options included, and (6) single- vs. multi-objective models.

A further categorization splits location models into static and deterministic, continuous and stochastic problems, (Owen & Daskin, 1998). Static and deterministic problems can

be broken down into median-based-models, covering based-models, center models and other models. As the covering models focus on designing public services they might not be practical for business applications, but because those models play an important part in location theory, they will be at least sketched.

The p -center problem locates p facilities having the goal of minimizing the maximum distance between a demand point and its nearest facility. Thus the p -center problem is also called the minimax problem. It is probably one of the most known problems in this category. For business applications the median-problems are much more applicable, among these is the p -median model. The p -median problem, introduced by Hakimi (1965), minimizes the total demand-weighted travel distance between the demand points and the facilities. In contrast to the p -median model, the covering problem addresses a certain service level. A demand is covered, if it can be served in a certain time or distance. This measure describes the desired service level. The objective of the covering problem is to minimize the total cost for reaching that service level.

Uncertainty of the future is part of any model as it is the strategic nature of facility location problems, (Owen & Daskin, 1998). Dynamic location models consider an extended planning horizon and generate robust location decisions. Longer planning horizons and uncertainties in facility location are addressed by dynamic location models. Although the dynamic location models consider an extended planning horizon, they are static in nature, because all parameters are defined in advance and possible parameter changes in later time periods are not considered.

Stochastic models assume that the input parameters are mostly unknown in real-world applications. Owen and Daskin (1998) split the models into two different approaches, which are the probabilistic approach and the scenario planning approach. The former uses stochastic tools, the latter considers scenarios in a what-if setting.

Brandeau and Chiu (1989) presented an overview of representative problems and established a classification scheme, which appears to be more general. Their work contains a survey of more than 50 papers with problems in location theory. Their classification is divided into three characteristics which are

- The objective of the model
- The decision variables
- The system parameters.

Hamacher and Nickel (1998) introduced a 5-parameter classification using a formalization based on queuing theory. It takes five parameters into account and was built on the research of Eiselt et al. (1993), (Eiselt & Laporte, 1989), who focused on competitive location models. They indicate that their taxonomy has been a useful tool in designing and structuring lectures and research papers. The benefit of their classification is that not only classes of specific location problems are described, (Hamacher & Nickel, 1998). The taxonomy uses five parameters to describe the underlying problem. The structure reminds one of Kendall's notation used in queuing theory, (Kendall, 1953). These parameters are

- P1: Information about the number and type of new facilities
- P2: Type of the location model with respect to the decision space
- P3: Description of particulars of the specific location model
- P4: The relation between new and existing facilities
- P5: Description of the objective function,

A subset of symbols for each position is introduced with respect to continuous, network and discrete location models. Although the classification can be applied to a large variety of location models, this approach does not take a facility expansion or shrinkage into consideration.

Haase and Hoppe (2008) expanded the scheme by including additional parameters for competitive location models. The resulting taxonomy consists of six parameters using a similar notation as described above, which are

- Characteristics of competition
- Characteristics of decision space
- Modeling of demand or market share
- Pricing strategy
- Objective
- Additional parameters.

Haase and Hoppe (2008) mention that only a few authors have discussed the shrinking or closing of facilities.

Some more specialized classification schemes can be found by Handler and Mirchandani (1973) in a 4-position scheme, applicable to network location models with objective functions of the center type. Carrizosa et al. (1995) developed a 6-position scheme for the classification of planar models, referring mainly to the Weber Problem.

Very little attention has been paid in the literature to closing or relocation of facilities. Reville et al. (2005) give a first overview on papers both for a competitive and a non-competitive setting, but do not set up a classification scheme. Finally they end up with two static models for the shrinkage problem.

2.2 A classification scheme for facility location models focusing on opening, closing, and capacity decisions incorporating competition

According to the different taxonomies described above, it is necessary to set up a scheme that addresses the objective of the model in this dissertation in order to get an expanded and more diversified view on the body of literature. For a classification of the papers, the following parameters will be used for the literature review.

1) Decision space

The decision space is essential for the location problem and its formulation. It can be either continuous or discrete. In the case of a continuous decision space the variables used in the model are continuous. Continuous models are most likely used for site generation. The underlying decision space is of geographic nature. The feasible solution can be found inside the given the polygon, (Plastria F. , 2002). The

underlying network is known in discrete network location models. Thus the question is whether or not to locate a facility on a certain node on a network. Typical examples are the set covering location model and the maximal covering location problem.

2) Objective

The objective of the model plays an important part. Objectives can be profit or market share maximization, cost-minimization, achieving a certain service level or coverage, maximizing the market share or be based on a certain covering level. Very often several objectives are tried to be combined. The underlying decision space is essential for the objective. Current et al. mention that multiple shareholders with very often contradictory objectives are involved in location decisions, (Plastria F. , 2002).

3) Distance measuring

Most of the models use distances as parameters that influence customer choices. The region is very often assumed to be a plane. The willingness of a customer to travel to a facility is increasing as the distance decreases¹. Though customers patronize facilities that are closer instead of using those far away. The distance between a customer i (demand point) and a facility j is denoted by d_{ij} . The probability of patronizing a facility with higher distances is lower than that for a closer facility. This is why the attractiveness of a facility decreases while the distance increases.

¹ Some authors use travel time instead of distance (Huff, 1964)

Analyses have found out an inversely proportional relationship between the attraction of a facility and the distance of a customer and the facility, (Plastria F. , 2002), (Drezner & Eiselt, 2002). Distances can then be given as Euclidean, rectangular, max, hexagonal or minowski distance. In some cases distances are only part of a general parameter that describes the level of attractiveness of a certain facility.

4) Time incorporation

Some models are static in nature and only consider a single time period. Others use an extended time horizon and multiple periods, are considered dynamic, and dynamic location models. Manne (1967) researched dynamic location models and pointed out the importance of the time horizons in his studies. Ballou (1968) emphasizes that future time decisions cannot be neglected in location analyses. Though the model can be defined as dynamic, this must not necessarily mean that the solution approach is dynamic. Mostly, the solution approach for such models is robust.

5) Competition

Competitive location models are used to consider multiple players in a model. Most models in facility location do not consider competitors. This is the difference between location-choice models, which assume that customers decide which facility they use and location-allocation models. In location-allocation models, each facility is assigned to one or more demand points.

6) Consumers and demand

In a competitive setting, it is important to understand how the market share of a facility can be estimated and influenced. Drezner and Eiselt (2002) mention that it depends on three factors which are

- Customer characteristics
- Facility attributes
- The spatial separation of customers and facilities.

Thus it is important to consider all these factors and decide whether or not to incorporate competition in a model. Drezner and Eiselt (2002) set up a customer choice process and list the main factors for patronage of a certain facility. When considering a utility function, the assumption is that customers react rationally and the customers' decisions depend on logic. According to Drezner and Eiselt (2002), some authors have questioned the complete rationality. Customers do not always know the prices in each store for each product before shopping, but it is to be assumed that they might have a rough idea of the price for grocery products and this influences their behavior. Ho et al. (1998) mention some other aspects for store loyalty which is habitual store visit behavior and the concept of basket size thresholds. They explain that customers will patronize nearby stores for small quantities with a higher probability though the prices might be higher. For higher quantities it is assumed that customers will accept longer drives, (Drezner & Eiselt, 2002). A study by Drezner and Furihata (2000) also states that distance is not correlated with mall patronage in Japan and the U.S. Without having empirical evidence for correlation – the same observation can be made by airports. People tend

to patronize airports especially because of the offered destinations and the ticket prices. (www.airliners.de, 2014)

7) Single / Multiple product

This parameter indicates whether a single or a multiple product setting is described within the problem formulation. A multiple product setting makes the problem even more complex. Most of the existing models do not consider a multiple products scenario.

8) Homogenous / heterogeneous facilities

Some models are set up as a heterogeneous facility design / layout, to give an example no different sizes for the facilities are considered, the facilities do not differ in quality or in service like opening hours. Such factors can have significant influence on the customer patronage of a facility. Thus factors such as size, service quality and product quality may be considered. Some authors have developed a utilization function that takes all parameters together, (Bloomstein et al. 1980), (Burnett, 1973), (Drezner T. , 1994), (Drezner & Furihata, 2000).

9) Constraints

Each model has a unique set of relevant constraints. Such constraints can be capacity, subsidies, taxes and so on. The general assumption in a model is an unlimited capacity of the facilities which is quite distant from reality. When designing facilities, capacity restrictions play an important part – production capacities, land restrictions or even transportation restrictions might to be considered. Gourdin et al. (2002)

mention capacitation constraints for telecommunication networks. Capacity restrictions are usually modeled as constraints in the mathematical model. For shrinking or closing purposes further constraints can also be a certain service level that should be kept as well as budget constraints or an exclusive set of facilities that have to be kept open.

10) Opening and closing

This parameter indicates whether opening and / or closing of facilities is relevant for the model.

11) Solution methods applied

Solution methods can be either (linear) programming techniques or heuristics. As many models become very large, the use of heuristics is necessary.

2.3 Literature on shrinking and facility closing

Only few papers have been published on shrinking or closing of facilities, in contrast to expanding. A good starting point is to be found in a model proposed by Klincewicz et al. (1988). They propose a discrete network model with the objective of minimizing the total discounted costs over a given planning horizon which indicates that it is a dynamic approach. Instead of distances, costs play an important part in the model. The model allows the usage of concave operating costs. Competitors are not considered, though it is to be assumed that the model should be applied to business and not to public services. The demands are considered by arbitrary demand patterns. This allows one to model

different demands in different time periods. As it is possible to shrink or expand facilities in this model, this allows modelling facilities with different sizes. The constraints in the model focus primarily on the capacity of the facilities and the demand. To incorporate the arbitrary demand patterns Klincewicz et al. (1988) allow both contraction and expansion as well as closing and opening of locations. For solving the model, the authors start with a myopic, initial solution and then apply a heuristic. Overall this model is very powerful and already incorporates a couple of questions that should be addressed in this dissertation, but it does not take competitors into consideration.

Melachrinoudis and Min (2000) published a paper focusing on supply chain management objectives. They consider a phase-out and relocation situation of a hybrid, two-echelon plant/warehousing facility with respect to changes in business environments. The decision space is discrete as the possible locations are known in advance. The objective of the model is to maximize the total profit in a given time horizon with respect to location incentives. Distances are measured in time (minutes). Competitors are not considered. The demands of the customers in certain periods are known as well and are not influenced by the location decision. Melachrinoudis and Min (2000) do not consider special product types. Instead they use units to model both capacity and demand. Because the goal is to relocate the facility, there is no difference made between homogenous and heterogeneous facilities. The major constraints of the model pertain to production capacity and demand satisfaction. The model either closes or opens exactly one facility. The model then computes where and when to open and close. The model is solved by using LINGO.

Another paper published by Wang et al. (2003) considers a budget constrained location problem with opening and closing of facilities. The model is formulated in a discrete decision space. A set of nodes of existing and possible locations is given. The objective of the model is to minimize the total weighted travel distance for customers. Distance is measured in Euclidean space, but the distances are also weighted such that a certain demand node could become more important than another. The model is a single planning period model. Because the objective is to minimize the total weighted travel distance, the customers' competitors are not considered in the model. The consumers are modeled using demand nodes. A consumer can be ranked higher by assigning a larger weight. As capacity does not play a part in the model, the demand is not considered in case of units. All facilities serve the same product and are equal. The model uses two major constraints, first a budget constraint for opening and closing of facilities and second the total number of open facilities desired. Facilities may be closed down or opened. As the problem is NP-hard, the authors developed three heuristics.

Zhang and Rushton (2008) developed a model that uses a discrete decision space to optimize the size and the location of facilities in a competitive environment with the objective of maximizing the spatial utility of users. Distances are measured in Euclidean space. The authors consider only one time period. In contrast to the models mentioned previously, the location of competitors is considered. The demand is also known. Zhang and Rushton (2008) consider a single product. Facilities can differ in size, while larger branches attract more customers. The model incorporates such constraints as budget and size. Locations can either be opened or closed. Heuristics are used to solve the model.

ReVelle et al. (2005) considered closing facilities in their model. They propose two models, one for firms with competition and another for firms without competition. As this dissertation will provide a model for firms with competition, only the characteristics of the former are described. The model uses a decision space in a network of discrete points. The assumption is to retain a given number of facilities losing as little market share to the competitors as possible. Thus, the model objective is to minimize the market share lost. Distances are Euclidean and the model does not consider multiple time periods, but takes competition into account. A demand point is lost to a competitor when the distance of one's own nearest facility is larger than the distance of the nearest competitor. The demand is also known in advance. The model uses both single products and homogenous facilities. The authors considered only closure of a fixed number of facilities. The model is solved using LINGO.

Bi-level models are of competitive nature and incorporate the reaction of the follower. A first model deals with the problem of locating new facilities in a market where competitors are already operating (Küçükaydin et al., 2012). The new facilities should be such that the profit is maximized. For this purpose, the authors introduce an attraction index. According to the bi-level approach, the competitor can react by adjusting both the attractiveness of existing facilities or by opening or closing others in order to maximize the profit. The distance measure is Euclidean. Multiple time periods are not considered explicitly, due to the bi-level approach. The consumers demand for a homogeneous product is modeled by the buying power at the demand points. Because it is possible to adjust the attractiveness of each facility, the model works with heterogeneous facilities. In this model it is not possible to adjust the attractiveness. For the existing company in

the market, it is possible to open or close facilities. The authors use a heuristic approach for solving the model.

Küçükaydin et al. (2011) proposed another, similar model, allowing the adjustment of the attractiveness of the follower, but not allowing the closing or opening of facilities. The assignment of demand points to certain facilities is determined by the attraction index.

A further bi-level approach is proposed by Drezner and Drezner, (1998). They considered a model for a company entering a market knowing that a future competitor is expected to enter the market. They state that the location for one's own facility should be chosen such that it is optimal after the market entry of the new competitor. The model objective is to maximize the market share. The model uses a continuous decision space with distance correction to ensure that a facility will not be located directly on a demand point. The distance measure is Euclidean. In this bi-level approach, only single time periods are considered. Demand is measured in buying power and is aggregated at pre-specified demand points. The model does not consider multiple products. As the model deals with several existing facilities having different attractiveness levels, the facilities are completely heterogeneous. It is not possible to influence the attractiveness level in the model directly. The model does not incorporate closing or opening decisions. The authors introduce three solution approaches: the brute force approach, the pseudo-mathematical programming approach, and the gradient search approach.

A paper for bank networks was published by Ruiz-Hermández et al. (2014). The paper contains closing down as well as long term operations' costs. It also assumes that it

sometimes might be necessary to resize branches. The model is set up for a competitive environment and takes a ceding market share into account. Distances are Euclidean. The model does not include multiple time periods. The customers are known, capacities may vary at the locations and facilities can have different sizes. The problem is solved using the CPLEX algorithm.

A paper by Bhaumik (2010) introduces the de-location problem with the objective of closing a pre-specified number of facilities and assigning demands to the remaining facilities. The objective of the model is to minimize the total cost of serving the demand nodes, assuming reassignment might cause a higher distribution cost. In the model distances are not explicitly considered, but costs are given for all assignment pairs. It is assumed that the cost is based on distances. In his paper, Bhaumik (2010) did not extend the model to multiple time periods, but the model could be computed period by period. Competition is not considered as the focus is on cost minimization. The demands of the nodes are not known. Facilities in the model can be assumed to be homogenous because Bhaumik (2010) did not consider any differences in terms of operational costs or attractiveness. The model postulates that demand is not lost though closure of facilities. The only impact is that there will be higher distribution cost due to the new assignments. A linear solver was used to run the model.

2.4 Literature on expanding or opening new facilities and on variable demand

Küçükaydin et al. (2011) proposed another model that focused on competitive facility location in a discrete environment. A new market entrant should locate facilities and to compete with a set of existing facilities. They aggregate demand at certain demand points in the plane. Küçükaydin et al.'s (2011) model takes both attractiveness and distance into account. Patronage depends on these two parameters. The attractiveness level can be influenced by each facility. The mixed-integer non-linear model's objective is to maximize the profit. The model is static in nature, as the attractiveness level is set only once and not changed over time. For solving a branch and bound algorithm is applied.

Dasci & Laporte (2005) analyzed a model dealing with demand uncertainty and the corresponding capacity acquisition. The model uses a market area approach which massively simplifies the mathematical model behind it. The approach should be considered as complementary to existing conventional solution procedures. The model points out that location and capacity decisions may differ substantially.

The challenges of variable demand are especially considered in a paper proposed by Arrondo et al. (2014). The model incorporates competition in a planar location design. The paper develops a heuristic that deals with the increasing complexity of variation in demand. Attraction is also incorporated in the model using a simple additive utility function. The objective of the model is to maximize the profit. The influence of variable

demand on the overall result was demonstrated in a paper published by Redondo et al. (2011).

2.5 Literature analysis

There is much literature on facility location. A variety of papers addresses many different location problems, both in a competitive and in a non-competitive environment. Starting with the Weber problem the basics for facility have been laid. Several papers have been published for classifying problems on facility location. Brandeau et al. (1989) propose a very general classification scheme. Hamacher and Nickel (1998) published a paper on a more detailed scheme. In this dissertation, a taxonomy was developed based on the work of Hamacher and Nickel (1998) and Haase and Hoppe (2008). Our taxonomy considers eleven parameters which are the decision space, the model's objective, distance measuring, time incorporation, the characteristics of competition, the modeling of demand or market share, single or multiple products, facility types, the incorporation of opening and closing and solution methods. Drezner and Hamacher (2002) have published a book on location analysis that addresses most aspects of location theory. It covers both continuous and discrete location models, addresses the public and the private sector and takes the modeling of demand into account. In the later chapters applications and further research topics are addressed. When dealing with the shrinking or closing of facility it becomes clear very quickly that only little attention has been paid to that area. Applying our taxonomy also shows that the objectives of the models in papers dealing with the shrinking or closing of facilities do mostly focus on special use cases (Melanchrinoudis & Min, 2000), (Ruiz-Hermández et al. 2014), (Bhaumik, 2010). Most of them do not

consider competition or multi periods in their models. A few bi-level models have been developed during the last years. Bi-level models are suitable for modeling competitive settings by using an approach from game theory, but they do not consider multiple periods. Finally it is very hard to solve them. For most of the models Heuristics need to be applied.

This dissertation focuses on a model both considering multiple time periods, competition, shrinking and closing as well as opening and expanding of facilities. The initial model is proposed in the following chapter and extended later.

CHAPTER 3

MODELS FOR MULTI-PERIOD FACILITY LOCATION

3.1 Introduction

Most location models in literature either do not consider multiple time periods or neglect competition in the model. The model introduced in this chapter attempts to overcome this limitation. It is based on a competitive, multi-period environment and not only takes downsizing or closing of certain facilities into consideration, but also assumes that a downsizing or closing of one facility might require the expansion of others. Contraction of an existing facility is also considered as an option. Furthermore, competitors are explicitly incorporated in our model. The demands are assumed to vary over a chosen time horizon.

First, the model with a single product is demonstrated. Second the model is then extended by allowing multiple products as well as the addition of several constraints, because the single product assumption limits the applicability. Both models are solved and computational results are provided.

3.2 Model 1: Single product model

The first proposed model considers an existing set of facilities operating in a competitive environment in a market with discrete demand points. The model considers two different scenarios. First the demand is known for all periods in advance. This is solved using a robust approach. Secondly an approach for solving the model period-by-period is applied, as demand can be predicted more accurately only for the next period. The two approaches are then compared. Operating a facility incurs a fixed cost. Closing and opening of facilities will incur a cost just as expansion or contraction. Finally, the model takes the competitive environment into consideration. This is done by using an attractiveness parameter which indicates the attractiveness of a certain facility. The model postulates that the facility with the highest attraction will be assigned to a demand point, if it is open. The highest attractiveness might also be at a competitor's site. The prices for the homogenous product can differ between the facilities as well as between time periods. A facility will be operating with a certain capacity in each period.

3.2.1 Mathematical formulation

Indexes

i	Set of facilities
j	Set of demands
t	Set of time periods

Model parameters

$initialCapacity_i$	The initial capacity of facility i in period I
$initialOpen_i$	The initial status of facility i in period I
$price_{it}$	The price for selling a single unit at facility i in period t
$variableCost_{it}$	The unit cost for selling a single unit at facility i in period t
$fixCost_{it}$	The cost for having open facility i in period t
$openingCost_{it}$	The cost for opening facility i in period t
$closingCost_{it}$	The cost for closing facility i in period t
$attraction_{ijt}$	The attraction value of a company's own facility i with respect to demand node j in period t
$competitorsAttraction_{jt}$	The highest attraction of competitors for demand node j in period t
$expansionCost_{it}$	Fixed cost for expansion of facility i in period t
$shrinkageCost_{it}$	Fixed cost for contraction of facility i in period t
$unitExpansionCost_{it}$	Unit (variable) cost for expansion of facility i in period t
$unitShrinkageCost_{it}$	Unit (variable) cost for contraction of facility i in period t
$demand_{jt}$	The demand at demand node j in period t
$assignment_{ijt}$	$= \begin{cases} 1, & \text{if } attraction_{ijt} \geq competitorsAttraction_{jt} \\ 0, & \text{else} \end{cases}$

This parameter can be set to 1 if the attractiveness of a company's own facility i relative to a demand node j is greater or equal than the highest attractiveness of the competitor for the same node j in period t . It is used for the assignment of facilities to a demand node.

Model variables

$$open_{it} = \begin{cases} 1, & \text{if facility } i \text{ remains open in } t \\ 0, & \text{if facility } i \text{ is closed in } t \end{cases}$$

$$opened_{it} = \begin{cases} 1, & \text{if facility } i \text{ is opened in } t \\ 0, & \text{else} \end{cases}$$

$$closed_{it} = \begin{cases} 1, & \text{if facility } i \text{ is closed in } t \\ 0, & \text{else} \end{cases}$$

$$expanded_{it} = \begin{cases} 1, & \text{if facility } i \text{ is expanded in } t \\ 0, & \text{else} \end{cases}$$

$$shrunk_{it} = \begin{cases} 1, & \text{if facility } i \text{ is shrunk in } t \\ 0, & \text{else} \end{cases}$$

$uPlus_{it}$ Expansion in units at facility i in t

$uMinus_{it}$ Contraction in units at facility i in t

$totalCapacity_{it}$ Maximum capacity in units at i in period t

$usedCapacity_{it}$ Capacity utilized in units at i in period t

Objective function

$$\begin{aligned} &Max \sum_i \sum_t (usedCapacity_{it} * (price_{it} - variableCost_{it}) - fixCost_{it} * open_{it} - \\ &closingCost_{it} * closed_{it} - openingCost_{it} * opened_{it}) - \sum_i \sum_t expansionCost_{it} * \\ &expanded_{it} - \sum_i \sum_t unitExpansionCost_{it} * uPlus_{it} - \sum_i \sum_t shrinkageCost_{it} * \\ &shrunk_{it} - \sum_i \sum_t unitShrinkageCost_{it} * uMinus_{it} \end{aligned}$$

Subject to

- 1) $open_{i1} = initialOpen_i, \quad \forall i$
- 2) $totalCapacity_{i1} = initialCapacity_i, \quad \forall i$
- 3) $usedCapacity_{it} \leq totalCapacity_{it}, \quad \forall i, t$
- 4) $usedCapacity_{it} \leq \sum_j assignment_{ijt} * demand_{jt}, \quad \forall i, t$
- 5) $totalCapacity_{it+1} = totalCapacity_{it} + uPlus_{it} - uMinus_{it}, \quad \forall i, t$
- 6) $open_{it+1} = open_{it} + opened_{it} - closed_{it}, \forall i, t$
- 7) $M * expanded_{it} \geq uPlus_{it}, \forall i, t$
- 8) $M * shrunk_{it} \geq uMinus_{it}, \forall i, t$
- 9) $totalCapacity_{it} \geq 0, \forall i, t$
- 10) $totalCapacity_{it} \geq uMinus_{it}, \forall i, t$
- 11) $expanded_{it} \leq open_{it}, \forall i, t$
- 12) $shrunk_{it} \leq open_{it}, \forall i, t$
- 13) $M * open_{it} \geq usedCapacity_{it}, \forall i, t$
- 14) $\sum_i assignment_{ijt} \leq 1, \forall j, t$
- 15) $assignment_{ijt}, expanded_{it}, shrunk_{it}, open_{it}, opened_{it}, closed_{it} \in \{0; 1\}, \forall i, j, t$

$$16) \quad uPlus_{it}, uMinus_{it}, totalCapacity_{it}, usedCapacity_{it} \in \mathbb{N}, \forall i, t$$

Explanation

The objective function maximizes the profit. It consists of six terms. The first term computes the net profit. The second term includes the fixed cost component when a facility is open. The third and fourth terms are for assigning a closing or opening cost that occurs when a facility is opened or closed, respectively. The last two terms consider the expansion or contraction costs. Constraints (1) and (2) are used to specify the open or closed status of each facility as well as the initial capacity. The usage must never be greater than the capacity. This is ensured by constraint (3). Constraint (4) addresses the considering that the usage cannot exceed the assigned demands. The capacity of the facilities might vary over time. Constraint (5) computes the capacity for the following period by adding the units of expansion or subtracting the units of shrinkage from the current capacity. Constraint (6) is a key constraint and is used for setting the status of a facility to open or close. Constraint (7) sets $expanded_{it}$ to 1 if the facility is expanded. Constraint (8) does the same for the contraction case. Constraint (9) ensures that the capacity will never become negative and constraint (10) states that the capacity will always be greater or equal than a potential contraction. Constraints (11) and (12) assert that only open facilities can be expanded or contracted, respectively. Finally, constraint (13) ensures that capacity will only be used if a facility is open and (14) postulates to assign a certain demand only to one of available facilities and only once.

3.2.2 Computational results

To demonstrate the validity of the model, it is first solved using a small problem set - three facilities, three demand locations and three periods – in LINGO solver. The corresponding LINGO model is in Appendix A. The following fictitious data was used and read from an Excel file.

Table 1 Test data for model 1 with 3 facilities, 3 demand points, and 3 time periods

Initial capacity	
initialCapacity	
Facility (<i>i</i>)	initialCapacity
1	500
2	500
3	500

Opensettings for period 1	
initialOpen	
Facility (<i>i</i>)	initalOpen
1	1
2	1
3	1

Price for selling one unit at <i>i</i> in <i>t</i>			
price	Period (<i>t</i>)		
Facility (<i>i</i>)	1	2	3

1	5,000	5,000	5,000
2	5,000	5,000	5,000
3	5,000	5,000	5,000

Variable cost for selling in period t at i			
variableCost	Period (t)		
Facility (i)	1	2	3
1	20	20	50,000
2	20	20	20
3	20	20	20

Unit cost for expanding i in t			
unitExpansionCost	Period (t)		
Facility (i)	1	2	3
1	5	5	5
2	5	5	5
3	5	5	5

Unit cost for shrinking i in t			
unitShrinkageCost	Period (t)		
Facility (i)	1	2	3
1	3	3	3
2	3	3	3
3	3	3	3

Expanding cost for i in t			
expansionCost	Period (t)		
Facility (i)	1	2	3
1	10	11	10
2	20	10	10
3	10	5	20

Contraction cost for i in t			
shrinkageCost	Period (t)		
Facility (i)	1	2	3
1	5	5	5
2	5	3	10
3	10	10	10

Demand at j in period t			
demand	Period (t)		
Demand location (j)	1	2	3
1	1,00000	0	50,0000
2	1,00000	0	50,0000
3	1,00000	0	50,0000

Attraction value for facility i for demand point j in period t			
attraction	Demand location (j)		
	1	2	3

	Period (t)			Period (t)			Period (t)		
Facility (i)	1	2	3	1	2	3	1	2	3
1	1	2	3	1	2	3	1	2	3
2	2	2	2	2	10	2	2	2	2
3	3	2	1	3	2	1	3	30	1

Highest attraction value of competitor				
competitorsAttraction		Period (t)		
Demand location (j)		1	2	3
1		1	1	1
2		1	50	1
3		1	1	1

Fix cost for running facility i in period t				
fixCost		Period (t)		
Facility (i)		1	2	3
1		10	50,000	50,000
2		10	50,000	50,000
3		10	50,000	50,000

Opening cost for i in t				
openingCost		Period (t)		
Facility (i)		1	2	3
1		1	1	1
2		1	1	1

3	1	1	1
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Closing cost for i in t			
closingCost	Period (t)		
Facility (i)	1	2	3
1	1	1	1
2	1	600,000	1
3	1	1	1

The model was solved in less than a second by running the LINGO solver on an Intel Core 2.7 GHz CPU with 4GB RAM on Windows 64Bit OS.

3.2.3 Performed checks

The model was tested with two additional data sets. Again, the solutions were obtained in less than one second using the same software and hardware. Several tests were applied to confirm the model is working correctly. All tests were conducted on all three different data sets and are listed below.

1. Test: If $assignment_{ijt} = 1$, is $usedCapacity_{it} > 0$?

This test was conducted to confirm that a facility only has used capacity if the corresponding assignment variable is set to 1.

2. If $assignment_{ijt} = 1$, is $attraction_{ijt} \geq competitorsAttraction_{jt}$?

This condition states that an assignment will only be made if attraction of own facility with respect to demand point is greater or equal than the attraction of all competitors in period t .

3. If $expanded_{it} = 1$, is $uPlus_{it} > 0$?

If a facility is expanded the variable $uPlus$ indicating the amount of expansion must be positive.

4. If $shrunk_{it} = 1$, is $uMinus > 0$?

If a facility is contracted, the variable $uMinus$ indicating the amount of contraction needs to be positive.

5. If $expanded_{it} = 1$, is $capacity_{it+1} = capacity_{it} + uPlus_{it}$?

If a facility is expanded in period t , the capacity of the facility in the preceding period must be expanded by $uPlus_{it}$ units.

6. If $shrunk_{it} = 1$, is $capacity_{it+1} = capacity_{it} - uMinus_{it}$?

If a facility is contracted in period t , the capacity of the facility in the preceding period must be reduced by $uMinus_{it}$ units.

7. If $closed_{it} = 1$, is $open_{it+1} = 0$?

If a facility is closed at the end of period t , it must be closed in the preceding period.

8. If $opened_{it} = 1$, is $open_{it+1} = 1$?

If a facility is opened at the end of period t , it must be open in the preceding period.

9. If $used_{it} > 0$, is $open_{it} = 1$?

This test was conducted to confirm that a facility only has used capacity if it is open.

All tests showed positive results and confirmed the model was working the expected way. The only remark that must be made here is that “M” is an arbitrarily large positive number, larger than the highest potential capacity. The highest potential capacity will never exceed the demand at all demand points in any period.

3.2.4 Extension of initial model

To get a better understanding of the model’s behavior, several additional scenarios were constructed. First, the number of demand locations was extended to 10. The problem was solved quickly. Second, the time periods were extended to 10. Again, a solution of the problem could be obtained in about 1 second. Third, the number of facilities was also extended to 10. For the first set of data, the model could not be solved even after x hours of computation time. For two subsequent data sets, the optimal solution could be determined in a few seconds.

Another larger data set with 100 facilities, 100 demand points, and 100 time periods was generated. The complexity of the data increased exponentially and working with Excel turned out to be cumbersome, but the model with 1,089,800 variables was solved in about 2.5 minutes.

The following table summarizes the solution times for the multiple datasets that were used to solve Model 1.

Table 2 Variables and runtimes for different datasets

Dataset	Number of variables	Elapsed runtime
3x3x3	102	< 1 sec.
5x5x5	340	< 1 sec
10x10x10*	1,880	30 – 250 sec.
100x100x100	1,089,800	n.a.

*Depending on the data

Figure 1: Solver status for model with 100 locations, 100 demand points and 100 time periods

In order to gauge the problem sizes that can be solved using standard software, several problem instances were generated. The 100x100x100 scenario could not be solved within a reasonable time. A 150x150x150 scenario with more than 3.5 million variables, resulted in an LINGO Buffer error.

Figure 2 LINGO error message for 150x150x150 model

3.2.5 Retrieving the relaxed solution from the model

To obtain an upper bound for the solution of the model, all binary and integer constraints were removed from the model. The following results were obtained.

Table 3 Objective function values for different model sizes both for mixed integer model and relaxed model

Data Set	Objective function value	
	Regular model	Relaxed model
5x5x5	5.50502E08	5.5116E8
10x10x10	4.18601E08	4.18671E8
100x100x100 ²	2.05094E11	1.10398E12

The results of the objective function values (OFVs) show that the relaxed model solutions

² Objective function value of a feasible but not of the optimal solution

provide an upper bound that is relatively close the OFV of the optimal solution for the first two (smaller) problem instances.

3.2.6 Solving the model in dynamic manner

It is relatively easier to forecast demands in the short term, than for the long term. Thus, the model was also solved using a dynamic approach; this means the problem is solved for each period considering only the demand for that period. A dynamic solution will result in more frequent opening and closing of facilities due to the myopic approach. An analysis was performed, whether a robust solution (solving the model once for the entire planning horizon, even though the demand forecasts, especially for the later periods in the planning horizon, may be inaccurate) or a dynamic approach (solving the model once in each period and using that solution and the next period's forecast demand to determine the next period's solution) will result in a better solution. With respect to the model and its mathematical formulation, a single period based approach was not implementable as the open status and current capacity is always known for the initial period. Thus, the consecutive period was included in the model and the all variables were determined for that period. After obtaining the variables values, the next run was begun using the just generated variable values as initial parameters. Figure 3 demonstrates the procedure.

Figure 3 Dynamic solution approach

The dynamic solution was implemented using LINGO scripting. The scripting made it possible to change the relevant variables in the model. The following script shows the implementation for the model with ten facilities, ten demand points and ten time periods. The corresponding model is shown in Appendix B.

```

TAKE dyn1.lng
width 200

GO
SOLU
PAUSE

ALTER ALL 'OPEN2' 'OPEN3'
ALTER ALL 'TOTALCAPACITY2' 'TOTALCAPACITY3'
ALTER ALL 'initialOpen1' 'initialOpen2'
ALTER ALL 'INITIALCPACITY1' 'INITIALCPACITY2'
ALTER ALL 'attraction1' 'attraction2'

```

```

ALTER ALL 'competitorsattraction1'competitorsattraction2'
ALTER ALL 'variableCost1'variableCost2'
ALTER ALL 'fixCost1'fixCost2'
ALTER ALL 'openingCost1'openingCost2'
ALTER ALL 'unitexpansionCostansionCost1'unitexpansionCostansionCost2'
ALTER ALL 'expansionCost1'expansionCost2'
ALTER ALL 'unitshrinkageCostCost1'unitshrinkageCostCost2'
ALTER ALL 'shrinkageCost1'shrinkageCost2'
ALTER ALL 'price1'price2'
ALTER ALL 'demand1'demand2'
GO
SOLU
PAUSE

```

```

ALTER ALL 'OPEN3'OPEN4'
ALTER ALL 'TOTALCAPACITY3'TOTALCAPACITY4'
ALTER ALL 'initialOpen2'initialOpen3'
ALTER ALL 'INITIALCPACITY2'INITIALCPACITY3'
ALTER ALL 'attraction2'attraction3'
ALTER ALL 'competitorsattraction2'competitorsattraction3'
ALTER ALL 'variableCost2'variableCost3'
ALTER ALL 'fixCost2'fixCost3'
ALTER ALL 'openingCost2'openingCost3'
ALTER ALL 'unitexpansionCostansionCost2'unitexpansionCostansionCost3'
ALTER ALL 'expansionCost2'expansionCost3'
ALTER ALL 'unitshrinkageCostCost2'unitshrinkageCostCost3'
ALTER ALL 'shrinkageCost2'shrinkageCost3'
ALTER ALL 'price2'price3'
ALTER ALL 'demand2'demand3'
GO
SOLU
PAUSE

```

```

ALTER ALL 'OPEN4'OPEN5'
ALTER ALL 'TOTALCAPACITY4'TOTALCAPACITY5'
ALTER ALL 'initialOpen3'initialOpen4'
ALTER ALL 'INITIALCPACITY3'INITIALCPACITY4'
ALTER ALL 'attraction3'attraction4'
ALTER ALL 'competitorsattraction3'competitorsattraction4'
ALTER ALL 'variableCost3'variableCost4'
ALTER ALL 'fixCost3'fixCost4'
ALTER ALL 'openingCost3'openingCost4'
ALTER ALL 'unitexpansionCostansionCost3'unitexpansionCostansionCost4'
ALTER ALL 'expansionCost3'expansionCost4'
ALTER ALL 'unitshrinkageCostCost3'unitshrinkageCostCost4'
ALTER ALL 'shrinkageCost3'shrinkageCost4'
ALTER ALL 'price3'price4'
ALTER ALL 'demand3'demand4'
GO
SOLU
PAUSE

```

```

ALTER ALL 'OPEN5'OPEN6'
ALTER ALL 'TOTALCAPACITY5'TOTALCAPACITY6'
ALTER ALL 'initialOpen4'initialOpen5'
ALTER ALL 'INITIALCPACITY4'INITIALCPACITY5'
ALTER ALL 'attraction4'attraction5'

```

```

ALTER ALL 'competitorsattraction4'competitorsattraction5'
ALTER ALL 'variableCost4'variableCost5'
ALTER ALL 'fixCost4'fixCost5'
ALTER ALL 'openingCost4'openingCost5'
ALTER ALL 'unitexpansionCostansionCost4'unitexpansionCostansionCost5'
ALTER ALL 'expansionCost4'expansionCost5'
ALTER ALL 'unitshrinkageCostCost4'unitshrinkageCostCost5'
ALTER ALL 'shrinkageCost4'shrinkageCost5'
ALTER ALL 'price4'price5'
ALTER ALL 'demand4'demand5'
GO
SOLU
PAUSE

```

```

ALTER ALL 'OPEN6'OPEN7'
ALTER ALL 'TOTALCAPACITY6'TOTALCAPACITY7'
ALTER ALL 'initialOpen5'initialOpen6'
ALTER ALL 'INITIALCPACITY5'INITIALCPACITY6'
ALTER ALL 'attraction5'attraction6'
ALTER ALL 'competitorsattraction5'competitorsattraction6'
ALTER ALL 'variableCost5'variableCost6'
ALTER ALL 'fixCost5'fixCost6'
ALTER ALL 'openingCost5'openingCost6'
ALTER ALL 'unitexpansionCostansionCost5'unitexpansionCostansionCost6'
ALTER ALL 'expansionCost5'expansionCost6'
ALTER ALL 'unitshrinkageCostCost5'unitshrinkageCostCost6'
ALTER ALL 'shrinkageCost5'shrinkageCost6'
ALTER ALL 'price5'price6'
ALTER ALL 'demand5'demand6'
GO
SOLU
PAUSE

```

```

ALTER ALL 'OPEN7'OPEN8'
ALTER ALL 'TOTALCAPACITY7'TOTALCAPACITY8'
ALTER ALL 'initialOpen6'initialOpen7'
ALTER ALL 'INITIALCPACITY6'INITIALCPACITY7'
ALTER ALL 'attraction6'attraction7'
ALTER ALL 'competitorsattraction6'competitorsattraction7'
ALTER ALL 'variableCost6'variableCost7'
ALTER ALL 'fixCost6'fixCost7'
ALTER ALL 'openingCost6'openingCost7'
ALTER ALL 'unitexpansionCostansionCost6'unitexpansionCostansionCost7'
ALTER ALL 'expansionCost6'expansionCost7'
ALTER ALL 'unitshrinkageCostCost6'unitshrinkageCostCost7'
ALTER ALL 'shrinkageCost6'shrinkageCost7'
ALTER ALL 'price6'price7'
ALTER ALL 'demand6'demand7'
GO
SOLU
PAUSE

```

```

ALTER ALL 'OPEN8'OPEN9'
ALTER ALL 'TOTALCAPACITY8'TOTALCAPACITY9'
ALTER ALL 'initialOpen7'initialOpen8'
ALTER ALL 'INITIALCPACITY7'INITIALCPACITY8'
ALTER ALL 'attraction7'attraction8'

```

```

ALTER ALL 'competitorsattraction7'competitorsattraction8'
ALTER ALL 'variableCost7'variableCost8'
ALTER ALL 'fixCost7'fixCost8'
ALTER ALL 'openingCost7'openingCost8'
ALTER ALL 'unitexpansionCostansionCost7'unitexpansionCostansionCost8'
ALTER ALL 'expansionCost7'expansionCost8'
ALTER ALL 'unitshrinkageCostCost7'unitshrinkageCostCost8'
ALTER ALL 'shrinkageCost7'shrinkageCost8'
ALTER ALL 'price7'price8'
ALTER ALL 'demand7'demand8'
GO
SOLU
PAUSE

ALTER ALL 'OPEN9'OPEN10'
ALTER ALL 'TOTALCAPACITY9'TOTALCAPACITY10'
ALTER ALL 'initialOpen8'initialOpen9'
ALTER ALL 'INITIALCPACITY8'INITIALCPACITY9'
ALTER ALL 'attraction8'attraction9'
ALTER ALL 'competitorsattraction8'competitorsattraction9'
ALTER ALL 'variableCost8'variableCost9'
ALTER ALL 'fixCost8'fixCost9'
ALTER ALL 'openingCost8'openingCost9'
ALTER ALL 'unitexpansionCostansionCost8'unitexpansionCostansionCost9'
ALTER ALL 'expansionCost8'expansionCost9'
ALTER ALL 'unitshrinkageCostCost8'unitshrinkageCostCost9'
ALTER ALL 'shrinkageCost8'shrinkageCost9'
ALTER ALL 'price8'price9'
ALTER ALL 'demand8'demand9'
GO
SOLU
PAUSE

```

Several checks were performed to explore the models' behavior and to detect the differences of the two approaches (robust versus dynamic).

1. Demand data constant over time horizon

The two approaches show a difference both in terms of capacity and the opening status for facility 1. The robust solution contains an extension of the capacities; the dynamic one keeps the capacity on the initial level for all periods, due to the fact that the expansion cost cannot be offset by net profit from one period. The opening status is the same for both models. The following table shows the results.

Table 4 Results for constant data for dynamic and robust approach

Facility	Period	Open status		Capacity	
		Robust	Dynamic	Robust	Dynamic
1	1	1	1	1,000	1,000
2	1	1	1	1,000	1,000
3	1	1	1	1,000	1,000
4	1	1	1	1,000	1,000
5	1	1	1	1,000	1,000
6	1	1	1	1,000	1,000
7	1	1	1	1,000	1,000
8	1	1	1	1,000	1,000
9	1	1	1	1,000	1,000
10	1	1	1	1,000	1,000
1	2	0	1	1,000	1,000
2	2	1	1	20,000	1,000
3	2	1	1	20,000	1,000
4	2	1	1	20,000	1,000
5	2	1	1	20,000	1,000
6	2	1	1	20,000	1,000
7	2	1	1	20,000	1,000
8	2	1	1	20,000	1,000
9	2	1	1	20,000	1,000
10	2	1	1	20,000	1,000
1	3	0	1	1,000	1,000
2	3	1	1	10,000	1,000
3	3	1	1	10,000	1,000
4	3	1	1	10,000	1,000
5	3	1	1	10,000	1,000
6	3	1	1	10,000	1,000
7	3	1	1	10,000	1,000
8	3	1	1	10,000	1,000

9	3	1	1	10,000	1,000
10	3	1	1	10,000	1,000
1	4	0	1	1,000	1,000
2	4	1	1	10,000	1,000
3	4	1	1	10,000	1,000
4	4	1	1	10,000	1,000
5	4	1	1	10,000	1,000
6	4	1	1	10,000	1,000
7	4	1	1	10,000	1,000
8	4	1	1	10,000	1,000
9	4	1	1	10,000	1,000
10	4	1	1	10,000	1,000
1	5	0	1	1,000	1,000
2	5	1	1	10,000	1,000
3	5	1	1	10,000	1,000
4	5	1	1	10,000	1,000
5	5	1	1	10,000	1,000
6	5	1	1	10,000	1,000
7	5	1	1	10,000	1,000
8	5	1	1	10,000	1,000
9	5	1	1	10,000	1,000
10	5	1	1	10,000	1,000
1	6	0	1	1,000	1,000
2	6	1	1	10,000	1,000
3	6	1	1	10,000	1,000
4	6	1	1	10,000	1,000
5	6	1	1	10,000	1,000
6	6	1	1	10,000	1,000
7	6	1	1	10,000	1,000
8	6	1	1	10,000	1,000
9	6	1	1	10,000	1,000
10	6	1	1	10,000	1,000

1	7	0	1	1,000	1,000
2	7	1	1	10,000	1,000
3	7	1	1	10,000	1,000
4	7	1	1	10,000	1,000
5	7	1	1	10,000	1,000
6	7	1	1	10,000	1,000
7	7	1	1	10,000	1,000
8	7	1	1	10,000	1,000
9	7	1	1	10,000	1,000
10	7	1	1	10,000	1,000
1	8	0	1	1,000	1,000
2	8	1	1	10,000	1,000
3	8	1	1	10,000	1,000
4	8	1	1	10,000	1,000
5	8	1	1	10,000	1,000
6	8	1	1	10,000	1,000
7	8	1	1	10,000	1,000
8	8	1	1	10,000	1,000
9	8	1	1	10,000	1,000
10	8	1	1	10,000	1,000
1	9	0	1	1,000	1,000
2	9	1	1	10,000	1,000
3	9	1	1	10,000	1,000
4	9	1	1	10,000	1,000
5	9	1	1	10,000	1,000
6	9	1	1	10,000	1,000
7	9	1	1	10,000	1,000
8	9	1	1	10,000	1,000
9	9	1	1	10,000	1,000
10	9	1	1	10,000	1,000
1	10	0	1	1,000	1,000
2	10	1	1	10,000	1,000

3	10	1	1	10,000	1,000
4	10	1	1	10,000	1,000
5	10	1	1	10,000	1,000
6	10	1	1	10,000	1,000
7	10	1	1	10,000	1,000
8	10	1	1	10,000	1,000
9	10	1	1	10,000	1,000
10	10	1	1	10,000	1,000

2. Variable demand data

Demand can be estimated more accurately in the short-run, but it is difficult to do so for periods far into the future due to a variety of factors including changes in trends, customer preferences, seasonality and others. Because the dynamic model only considers the current and next periods, the solution provided by the two models will be different. The dynamic approach closes a facility immediately if there is no demand in the next period and if the fixed costs are significant. A disadvantage of the dynamic model is that it makes myopic decisions based on the just next period's demand. For example, it may close a facility because there is no demand in the next period and opens it again in a subsequent future period when the demand is high. Once it is closed, it will only be opened if the costs can be offset by demand in the next period. The robust approach shows a different solution because demand for multiple periods into the future is assumed to be known.

A comparison of objective function values is difficult as the dynamic

approach always generates an objective value for two consecutive periods. The following table shows the difference for open status and total capacity for the robust approach and the dynamic approach, respectively using a model size of 10x10x10 with a fixed cost of 50,000.

Table 5 Results for alternating demand for dynamic and robust approach

Facility	Period	Open status		Capacity	
		Robust	Dynamic	Robust	Dynamic
1	1	1	1	1.000	1.000
2	1	1	1	1.000	1.000
3	1	1	1	1.000	1.000
4	1	1	1	1.000	1.000
5	1	1	1	1.000	1.000
6	1	1	1	1.000	1.000
7	1	1	1	1.000	1.000
8	1	1	1	1.000	1.000
9	1	1	1	1.000	1.000
10	1	1	1	1.000	1.000
1	2	0	0	1.000	1.000
2	2	0	0	20000	1.000
3	2	0	0	10000	1.000
4	2	0	0	10000	1.000
5	2	0	0	10000	1.000
6	2	0	0	10000	1.000
7	2	0	0	10000	1.000
8	2	0	0	10000	1.000
9	2	0	0	10000	1.000
10	2	0	0	10000	1.000
1	3	0	0	1.000	1.000

2	3	1	1	20000	1.000
3	3	1	1	10000	1.000
4	3	1	1	10000	1.000
5	3	1	1	10000	1.000
6	3	1	1	10000	1.000
7	3	1	1	10000	1.000
8	3	1	1	10000	1.000
9	3	1	1	10000	1.000
10	3	1	1	10000	1.000
1	4	0	0	1.000	1.000
2	4	0	0	20000	1.000
3	4	0	0	10000	1.000
4	4	0	0	10000	1.000
5	4	0	0	10000	1.000
6	4	0	0	10000	1.000
7	4	0	0	10000	1.000
8	4	0	0	10000	1.000
9	4	0	0	10000	1.000
10	4	0	0	10000	1.000
1	5	0	0	1.000	1.000
2	5	1	1	20000	1.000
3	5	1	1	10000	1.000
4	5	1	1	10000	1.000
5	5	1	1	10000	1.000
6	5	1	1	10000	1.000
7	5	1	1	10000	1.000
8	5	1	1	10000	1.000
9	5	1	1	10000	1.000
10	5	1	1	10000	1.000
1	6	0	0	1.000	1.000
2	6	0	0	20000	1.000
3	6	0	0	10000	1.000

4	6	0	0	10000	1.000
5	6	0	0	10000	1.000
6	6	0	0	10000	1.000
7	6	0	0	10000	1.000
8	6	0	0	10000	1.000
9	6	0	0	10000	1.000
10	6	0	0	10000	1.000
1	7	0	1	1.000	1.000
2	7	1	1	20000	1.000
3	7	1	1	10000	1.000
4	7	1	1	10000	1.000
5	7	1	1	10000	1.000
6	7	1	1	10000	1.000
7	7	1	1	10000	1.000
8	7	1	1	10000	1.000
9	7	1	1	10000	1.000
10	7	1	1	10000	1.000
1	8	0	0	1.000	1.000
2	8	0	0	20000	1.000
3	8	0	0	10000	1.000
4	8	0	0	10000	1.000
5	8	0	0	10000	1.000
6	8	0	0	10000	1.000
7	8	0	0	10000	1.000
8	8	0	0	10000	1.000
9	8	0	0	10000	1.000
10	8	0	0	10000	1.000
1	9	0	1	1.000	1.000
2	9	1	1	20000	1.000
3	9	1	1	10000	1.000
4	9	1	1	10000	1.000
5	9	1	1	10000	1.000

6	9	1	1	10000	1.000
7	9	1	1	10000	1.000
8	9	1	1	10000	1.000
9	9	1	1	10000	1.000
10	9	1	1	10000	1.000
1	10	0	0	1.000	1.000
2	10	0	0	20000	1.000
3	10	0	0	10000	1.000
4	10	0	0	10000	1.000
5	10	0	0	10000	1.000
6	10	0	0	10000	1.000
7	10	0	0	10000	1.000
8	10	0	0	10000	1.000
9	10	0	0	10000	1.000
10	10	0	0	10000	1.000

3. Variability in net profit

A variability in net profit shows an expansion of capacity in the robust approach. The facility is closed down when net profit decreases and opened again when the net profit increases again. The dynamic approach keeps the facilities closed. Lowering opening and closing costs influences the behavior. If the opening and closing costs are relatively low, facilities will be closed and reopened. The following table shows the results for high opening, closing and fixed costs. Demand is assigned only to one single facility in periods when net profit is high enough to cover the variable cost.

Table 6 Results for alternating prices for dynamic and robust approach

Facility	Period	Open status		Capacity	
		Robust	Dynamic	Robust	Dynamic
1	1	1	1	1,000	1,000
2	1	1	1	1,000	1,000
3	1	1	1	1,000	1,000
4	1	1	1	1,000	1,000
5	1	1	1	1,000	1,000
6	1	1	1	1,000	1,000
7	1	1	1	1,000	1,000
8	1	1	1	1,000	1,000
9	1	1	1	1,000	1,000
10	1	1	1	1,000	1,000
1	2	0	0	1,000	1,000
2	2	0	0	1000000	1,000
3	2	0	0	1,000	1,000
4	2	0	0	1,000	1,000
5	2	0	0	1,000	1,000
6	2	0	0	1,000	1,000
7	2	0	0	1,000	1,000
8	2	0	0	1,000	1,000
9	2	0	0	1,000	1,000
10	2	0	0	1,000	1,000
1	3	0	0	1,000	1,000
2	3	1	0	1000000	1,000
3	3	0	0	1,000	1,000
4	3	0	0	1,000	1,000
5	3	0	0	1,000	1,000
6	3	0	0	1,000	1,000
7	3	0	0	1,000	1,000
8	3	0	0	1,000	1,000

9	3	0	0	1.000	1,000
10	3	0	0	1.000	1,000
1	4	0	0	1.000	1,000
2	4	0	0	1000000	1,000
3	4	0	0	1.000	1,000
4	4	0	0	1.000	1,000
5	4	0	0	1.000	1,000
6	4	0	0	1.000	1,000
7	4	0	0	1.000	1,000
8	4	0	0	1.000	1,000
9	4	0	0	1.000	1,000
10	4	0	0	1.000	1,000
1	5	0	0	1.000	1,000
2	5	1	0	1000000	1,000
3	5	0	0	1.000	1,000
4	5	0	0	1.000	1,000
5	5	0	0	1.000	1,000
6	5	0	0	1.000	1,000
7	5	0	0	1.000	1,000
8	5	0	0	1.000	1,000
9	5	0	0	1.000	1,000
10	5	0	0	1.000	1,000
1	6	0	0	1.000	1,000
2	6	0	0	1000000	1,000
3	6	0	0	1.000	1,000
4	6	0	0	1.000	1,000
5	6	0	0	1.000	1,000
6	6	0	0	1.000	1,000
7	6	0	0	1.000	1,000
8	6	0	0	1.000	1,000
9	6	0	0	1.000	1,000
10	6	0	0	1.000	1,000

1	7	0	0	1.000	1,000
2	7	1	0	1000000	1,000
3	7	0	0	1.000	1,000
4	7	0	0	1.000	1,000
5	7	0	0	1.000	1,000
6	7	0	0	1.000	1,000
7	7	0	0	1.000	1,000
8	7	0	0	1.000	1,000
9	7	0	0	1.000	1,000
10	7	0	0	1.000	1,000
1	8	0	0	1.000	1,000
2	8	0	0	1000000	1,000
3	8	0	0	1.000	1,000
4	8	0	0	1.000	1,000
5	8	0	0	1.000	1,000
6	8	0	0	1.000	1,000
7	8	0	0	1.000	1,000
8	8	0	0	1.000	1,000
9	8	0	0	1.000	1,000
10	8	0	0	1.000	1,000
1	9	0	0	1.000	1,000
2	9	1	0	1000000	1,000
3	9	0	0	1.000	1,000
4	9	0	0	1.000	1,000
5	9	0	0	1.000	1,000
6	9	0	0	1.000	1,000
7	9	0	0	1.000	1,000
8	9	0	0	1.000	1,000
9	9	0	0	1.000	1,000
10	9	0	0	1.000	1,000
1	10	0	0	1.000	1,000
2	10	0	0	1000000	1,000

3	10	0	0	1.000	1,000
4	10	0	0	1.000	1,000
5	10	0	0	1.000	1,000
6	10	0	0	1.000	1,000
7	10	0	0	1.000	1,000
8	10	0	0	1.000	1,000
9	10	0	0	1.000	1,000
10	10	0	0	1.000	1,000

4. Demand is constantly decreasing

The behavior of the model in terms of constantly decreasing demand again depends on the net profit. If the net profit is relatively high, the robust model extends the capacity to be able to satisfy higher demand in the earlier periods. All facilities are kept open the entire planning horizon. If net profit is relatively low, then those facilities with highest cost are closed first. The dynamic model shows a different behavior. Though there is a higher demand than capacity at the beginning, no expansion is made, even in terms of higher net profit. If net profit decreases, the model closes only that facility with the smallest net profit first. Later all facilities will be closed at the same time.

In a nutshell the dynamic model is much more sensitive towards variability in demand. The better knowledge of future demands will enable to satisfy higher demands in earlier periods when using the robust approach.

The initial data used for the model is shown in Appendix C.

3.3 Model 2: Allowing multiple products and additional constraints

Considering multiple products resulted in a significant increase in the number of variables. The second improvement was to add additional constraints to the model in order to allow stepwise cost. This was necessary, because a facility will be shrunk only if operation with higher capacity is more expensive than operation with lower capacity. Another modification that was tested was to add budget constraints for the expansion of the model. When applying it to the real-world, it is to be expected that the model will always need to be modified to suit the situation at hand.

The subsequent subchapters describe the mathematical formulation of the model and display mathematical results. As the complexity of the data grew, an additional subchapter for database setup and the corresponding challenges is added to this dissertation.

3.3.1 Mathematical formulation

Indexes

i	Set of facilities
j	Set of demand
t	Set of time periods
p	Set of products

Model parameters

$initialCapacity_{ip}$	The initial capacity of facility i in period I for product p
$initialOpen_i$	The initial status of facility i in period I
$price_{itp}$	The price for selling a single unit of product p at facility i in period t
$variableCost_{itp}$	The unit cost for selling a single unit of product p at facility i in period t
$fixCost_{it}$	The cost for having open facility i in period t
$openingCost_{it}$	The cost for opening facility i in period t
$closingCost_{it}$	The cost for closing facility i in period t
$attraction_{ijtp}$	Attraction value for product p of a company's own facility i with respect to demand node j in period t
$competitorsAttraction_{jtp}$	The highest attraction for product p of competitors for demand node j in period t
$expansionCost_{it}$	Fix cost for expansion of facility i in period t
$shrinkageCost_{it}$	Fix cost for contraction of facility i in period t
$unitExpansionCost_{itp}$	Unit (variable) cost for expansion of product p at facility i in period t
$unitShrinkageCost_{itp}$	Unit (variable) cost for contraing of product p at facility i in period t
$demand_{jtp}$	Demand for product p at demand node j in period t

$step < X >_{itp}$ The step (capacity) when a modification of fix cost is necessary. As there might be several steps, X indicates the number of the step.

$stepCost < X >_{itp}$ The additional fixed cost for a certain step if the capacity is expanded for the corresponding product p at facility i in period t

$$assignment_{ijtp} = \begin{cases} 1, & \text{if } attraction_{ijtp} \geq comp_{jtp} \\ 0, & \text{else} \end{cases}$$

This parameter is set to one if the attraction for a product p of a company's own facility i relative to a demand node j is greater or equal than the highest attraction for that or a competitive product of the competitor with respect to node j in period t . It is used for the assignment of products and facilities to a demand node in that period.

Model variables

$$open_{it} = \begin{cases} 1, & \text{if facility } i \text{ remains open in } t \\ 0, & \text{if facility } i \text{ is closed in } t \end{cases}$$

$$opened_{it} = \begin{cases} 1, & \text{if facility } i \text{ is opened in } t \\ 0, & \text{else} \end{cases}$$

$$closed_{it} = \begin{cases} 1, & \text{if facility } i \text{ is closed in } t \\ 0, & \text{else} \end{cases}$$

$$expanded_{itp} = \begin{cases} 1, & \text{if facility } i \text{ is expanded in } t \text{ for product } p \\ 0, & \text{else} \end{cases}$$

$$shrunk_{itp} = \begin{cases} 1, & \text{if facility } i \text{ is shrunk in } t \text{ for product } p \\ 0, & \text{else} \end{cases}$$

$uPlus_{itp}$	Expansion of product p in units at facility i in t
$uMinus_{itp}$	Contraction of product p in units at facility i in t
$totalCapacity_{itp}$	Maximum capacity in units of product p at i in period t
$usedCapacity_{itp}$	Capacity utilized in units of product p at i in period t
$totalFixCost_{it}$	The fix cost for operating facility i in period t depending on the step-wise cost function
$threshold < X >_{itp}$	The variable that is indicating whether the costs for a certain threshold in terms of capacity have to be incorporated in the model. X is again used for the corresponding step
$thresholdSwitch < X >_{itp}$	This variable is used for modeling purposes in order to set the threshold correctly for facility i in period t and product p

Objective function

$$\begin{aligned}
& \max \sum_i \sum_t \sum_p (usedCapacity_{itp} * (price_{itp} - variableCost_{itp}) - totalFixCost_{it} - \\
& closingCost_{it} * closed_{it} - openingCost_{it} * opened_{it}) - \sum_i \sum_t \sum_p expansionCost_{it} * \\
& expanded_{itp} - \sum_i \sum_t \sum_p unitExpansionCost_{itp} * uPlus_{itp} - \\
& \sum_i \sum_t \sum_p shrinkageCost_{it} * shrunk_{itp} - \sum_i \sum_t \sum_p unitShrinkageCost_{itp} * uMinus_{itp}
\end{aligned}$$

Subject to

$$1) \quad open_{i1} = initialOpen_i, \quad \forall i$$

- 2) $totalCapacity_{i1p} = initialCapacity_{ip}, \quad \forall i, p$
- 3) $usedCapacity_{itp} \leq totalCapacity_{itp}, \quad \forall i, t, p$
- 4) $usedCapacity_{itp} \leq \sum_j assignment_{ijt} * demand_{jtp}, \quad \forall i, t, p$
- 5) $totalCapacity_{it+1p} = totalCapacity_{itp} + uPlus_{itp} - uMinus_{itp}, \quad \forall i, t, p$
- 6) $open_{it+1} = open_{it} + opened_{it} - closed_{it}, \forall i, t$
- 7) $M * expanded_{itp} \geq uPlus_{itp}, \forall i, t, p$
- 8) $M * shrunk_{itp} \geq uMinus_{itp}, \forall i, t, p$
- 9) $totalCapacity_{itp} \geq 0, \forall i, t, p$
- 10) $totalCapacity_{itp} \geq uMinus_{itp}, \forall i, t, p$
- 11) $expanded_{itp} \leq open_{itp}, \forall i, t, p$
- 12) $shrunk_{itp} \leq open_{it}, \forall i, t, p$
- 13) $M * open_{it} \geq usedCapacity_{it}, \forall i, t$
- 14) $\sum_i assignment_{ijt} \leq 1, \forall j, t$
- 15) $M * threshold < X >_{itp} \leq (totalCapacity_{itp} / step < X + 1 >_{itp}) - 1, \forall i, t, p$
where $1 \leq X \leq N$
- 16) $threshold < X >_{itp} + open_{it} + thresholdSwitch < X >_{itp} \leq 2, \forall i, t, p$
where $1 \leq X \leq N$
- 17) $threshold < X >_{itp} - thresholdSwitch < X >_{itp} \geq 0$

- 18) $totalFixCost_{it} = open_{it} * fixCost_{it} + \sum_p (stepCost1_{itp} * (threshold1 - thresholdSwitch1_{itp}) + stepCost2_{itp} (threshold2 - thresholdSwitch2_{itp}) + \dots$
- 22) $assignment_{ijtp}, expanded_{itp}, shrunk_{itp}, open_{it}, opened_{it}, closed_{it} \in \{0; 1\}, \forall i, j, t, p$
- 23) $uPlus_{itp}, uMinus_{itp}, totalCapacity_{itp}, usedCapacity_{itp} \in \mathbb{N}, \forall i, t$

The objective function maximizes the profit. It consists of six terms. The first term computes the net profit. The second term includes the fixed cost component when a facility is open, which is also determined by the stepwise cost function. The third and fourth terms are for assigning a closing or opening cost, respectively. The last two terms consider the expansion and contraction costs. Constraints (1) and (2) are used to initialize the open status of each facility as well as the initial capacity. The usage must never be greater than the capacity. This is ensured by constraint (3). Constraint (4) ensures that the usage cannot exceed the assigned demands. The capacity of the facilities might vary over time. Constraint (5) computes the capacity for the next period by adding the units of expansion or subtracting the units of contraction from the current capacity. Constraint (6) is a key constraint and is used for setting the status of a facility to open or close. Constraint (7) sets $expanded_{itp}$ to 1 if the facility is expanded with respect to product p . Constraint (8) does the same for the contraction case. Constraint (9) ensures that the capacity will never become negative and constraint (10) states that the capacity will always be greater or equal than a potential shrinkage. Constraints (11) and (12) assert that only open facilities can be contracted or expanded, respectively. Constraint (13) ensures that capacity will only be used if a facility is open. Constraint (14) guarantees a

maximum of one assignment per demand. Constraints (15), (16), (17) are used to determine the values for modification of the total cost function due to a change in capacity. Finally the total fixed costs are calculated in constraint (18).

3.3.2 Computational results

The model was solved with five different settings and a LINGO solver. The corresponding LINGO Model is provided in Appendix D . Fictitious data was used for tests and read from the oracle database.

Table 7 Variables and runtimes for different datasets for extended model

Data Set	Number of variables	Elapsed runtime
3x3x3x3	657	< 1 sec.
5x5x5x5	3,945	22 sec.
10x10x10x10	30,290	n.a.
50x50x50x50	8,757,450	error
50x50x12x50	2,099,250	n.a.

The model can be solved relatively easily for specific problem instances with 1.7 million variables. Larger models need too much memory, and the LINGO solver crashed when these problems were solved. Although the model could be solved optimally for some very large problem instances, for example, those with 1 million variables, there were other

problems that were much smaller that could not be solved optimally. The ability of the solver to solve problems depends upon the specific dataset used, whether or not the data matrix is sparse and other factors.

3.3.3 Additional modifications

The complexity of the model was raised by adding more constraints. The budget constraint provides a limited budget for expansion in a given period. A second constraint limits the capacity to a maximum and also requires a certain minimum capacity of a certain product. In real world problems, expansion to infinity is not possible, thus the maximum size of a facility will limit the potential expansion to a certain level. The LINGO model was also modified. The corresponding file is in Appendix E.

Additional parameters

expansionBudget_{itp} Available budget for expansion at facility i in period t for product p

minimumCapacity_{itp} The minimum capacity of product p at facility i in period t

maximumCapacity_{itp} The maximum capacity of product p at facility i in period t

Additional constraints

- a)
$$\sum_i \sum_p (expanded_{itp} * expansionCost_{it} + uPlus_{itp} * unitExpansionCost_{itp}) \leq expansionBudget_{itp}, \forall t$$
- b) $totalCapacity_{itp} \leq maximumCapacity_{itp}, \forall i, t, p$
- c) $totalCapacity_{itp} \geq minimumCapacity_{itp}, \forall i, t, p$

3.3.4 Some thoughts on the attraction parameter

Allowing multiple products in the model raised the question whether the attraction parameter is to be considered as a facility-based or a product-based factor. The facility-based option would not allow a product-related consideration afterwards and the overall attraction of a facility very much depends on the product mix. The extended model considers a product-based attraction. This results in an assignment of a specific product at a facility only if the attraction value is higher than that of the competitors with the highest attraction for that product. A facility-related attraction can either be realized by setting all parameters to the same value for all products at the relevant facility or by removing the product indexation for the attraction parameter.

3.3.5 Computational results

Several tests for solving the extended model were performed. The tests were again performed on different model sizes. The model extensions added complexity to the model.

Solving the model for 10x10x10x10 could not be carried out in a reasonable time. The LINGO solver had to be interrupted after 100 hours. Smaller problem instances were solved quickly.

Figure 4 LINGO solver status for extended model 2

3.3.6 Complexity in real world applications

Real world application settings are very much depending on the industry. For retail businesses, a network of 50 or more facilities is typical and a forecast for 5 years (60 months) is reasonable. Even the number of demand locations will exceed 50. Financial institutions such as banks or insurance companies will also have several facilities. Big

banks or insurance companies can have hundreds or thousands of locations. A time horizon of 5-10 years is often considered in the location analysis process. The product mix very often varies and is not limited to capacities. Production companies do usually not have dozens of facilities, but very often a very high variety of products or even product groups. Thus, real-world applications will exceed the size of the problems discussed in this dissertation and different approaches for solution generation are required.

3.3.7 Technical implementation and corresponding challenges

Utilizing Excel for a large model was not appropriate, requiring the data to be input from a database. An ODBC connection to an Oracle Database was established and a data structure both for the model input and output was created. Writing to a databases requires the table to have exactly the number of required rows. The setup of the tables is also very important as the data is read in exactly the order of the indexes. A change in the order will result in a wrong initialization of the model. This turned out to be a significant issue when views are created from the data as the order of the data might get mixed up. For each model, four different database tables are used. The following tables display the structure.

Table 8 Database view "modeldata_mp_10_view"

	COLUMN_NAME	DATA_TYPE
1	VARIABLECOST	FLOAT(20)
2	DEMAND	FLOAT(20)
3	ATTRACTIVITY	FLOAT(20)
4	COMPETITORSATTRACTION	FLOAT(20)
5	INDEXI	NUMBER(3)
6	PRICE	FLOAT(20)
7	INDEXJ	NUMBER(3)

8	INDEXT	NUMBER(3)
9	INDEXP	NUMBER(3)
10	UNITSHRINKAGECOST	FLOAT(126)
11	UNITEXPANSIONCOST	FLOAT(126)

Table 9 Database table "modeldata_mp_10_initials_cap"

	COLUMN_NAME	DATA_TYPE
1	INITIALCAP	FLOAT
2	INDEXI	FLOAT
3	INDEXP	FLOAT

Table 10 Database table "modeldata_mp_10_initials"

	COLUMN_NAME	DATA_TYPE
1	INITIALOPEN	FLOAT

Table 11 Database table "modeldata_mp_10_facilities_npr"

	COLUMN_NAME	DATA_TYPE
1	EXPANSIONCOST	FLOAT
2	SHRINKAGECOST	FLOAT
3	FIXCOST	FLOAT
4	OPENINGCOST	FLOAT
5	CLOSINGCOST	FLOAT

Table 8 contains all product relevant data. It is created as database view to ensure the correct order of the data. The view is created from a database table containing exactly the

same data. Table 9 is used for initialization of capacities in period 1 for product p at facility i .

Table 10 provides the open status for each facility in period 1.

Table 11 contains all not product related data, such as fix cost, opening, closing, expansion and shrinkage cost information for the relevant facilities and periods.

The database turned out to be much faster in data access than Excel files.

3.3.8 Retrieving the model's bounds

In order to get a bound on the optimal OFV, the relaxed problem was solved by removing all binary and integer constraints from the variables. The following table shows the relaxed solutions and the corresponding objective function values if available for the model. This is only useful as a first estimate. If the model is not too big (i.e., starting LINGO solver is possible but will not generate a solution) we were able to retrieve at least some upper bounds for orientation for the non-relaxed problem. Using the data from the models above the following bounds were obtained. It shows that only for the 5x5x5x5 model the relaxed solution is close to the solution for the original problem.

Table 12 The model's bounds for the extended model

Model	Original problem	Relaxed solution
3x3x3x3	2.41038E10	1.70044E9

5x5x5x5	2.95213E8	2.98255E8
10x10x10x10	Not available	Not available
50x50x50x50	Not available	Not available

3.3.9 Modification of parameters

In order to get a better understanding of the models behaviour, several parameter modifications were made. All tests were performed on the 5x5x5x5 model.

1. Starting with too big initial capacity

The first modification was starting with a much too high initial capacity. Much too high was given by multiplying the cumulated demand of one product at all demand points over all periods by five. As the fix cost function will automatically adjust the fix cost to the corresponding facility size this modification causes a massive cost explosion. The change was made at first for one single product at one facility, than for all products at one facility and finally for all products at all facilities.

The model's behaviour is very much depending on the underlying cost structure. If operating costs are too high the solver will immediately close down the facilities. If the difference is not too big, the facility's capacity with respect to that certain product is shrunk.

2. Letting the variable cost explode

A second modification was made by a change in the variable cost. The variable costs were changed over time and raised progressively. Again this modification was made at first for only one single product at one facility, second for all products at one facility and finally for all products at all facilities.

The first observation was that the solving time for the problem rose significantly. All other problems with 5x5x5x5 dimension could be solved within 2 minutes. The current problem took almost 40 minutes to be solved. In period one the first shrinkage is to be observed. All facilities are finally closed when the variable cost exceed the price.

3. Letting opening cost explode

A similar modification was made to the opening cost. The opening cost were set to the fivefold of the net profit made over all products made in that period. This modification was made for one, two and three periods at two facilities. After running the model it could be observed that despite of higher demand than capacity the facilities were not expanded. This is obvious as it is the objective of the model to maximize the profit over all time periods. If there are only some facilities assigned very high expansion cost, the demand is transferred to those facilities which have either not been open before or can be expanded for a price that is very much below.

3.4 Summary

A single product model has been developed and extended. The first part of this chapter has demonstrated that the model provides reasonable results. It is necessary to extend the model in such a way that multiple products are taken into consideration. This extension makes the model much more complex. This complexity requires a technology for data management that covers larger data than excel can. Thus an oracle database has been used for further computations. Several tests were performed and have shown that it is not possible to solve the model with standard software for problem sizes that are expected in real world applications. A dynamic solving procedure could be a first try to get an approximate solution but it will neglect the advantages of the original model, taking multiple time periods into consideration. For models using fewer (<5) time periods this approach could be appropriate. Different solution approaches have to be developed to get either a good approximate solution or better to find a way to get the optimal solution for bigger problems that will arise for retail companies, financial companies, rentals, logistic companies and so on. A heuristic based on a decomposition approach therefore will be developed in the next chapter and will be applied to the model.

CHAPTER 4

A DECOMPOSITION APPROACH

4.1 Introduction

Trials to solve the model using LINGO have shown that it is too complex to be solved using standard software. The complexity increased when multiple products were introduced in the model. The dynamic solution approach has already shown that splitting up the problem into smaller sub problems is a realistic approach for gathering a solution, but will result in different solutions than a robust approach and likely not lead to optimal results. Thus another decomposition approach is developed. The original problem is decomposed into sub-problems such that each sub-problem is for a specific product. A disadvantage of this approach is that it solves each sub-problem in isolation and ignores the fact that the products compete for the same set of resources. However, it allows us to develop separate solutions – one for each product – that can be combined in a subsequent step to include resource and capacity constraints that impact each of the sub-problems. The following section depicts how the problem was decomposed and the technical implementation. Then the computational results are provided and the results are combined so as to provide a solution the original problem.

4.2 Decomposition and database realization

The model was decomposed by products. This resulted in a sub model for each of the products. The mathematical model looked like the original model, only the number of facilities was set to 1 (refer to 3.3 Model 2: Allowing multiple products and additional constraints). The following code fragment shows the modification for a 5x5x5x5 model.

```
!Set of facilities;
locations /1..5/:initialOpen;

!Set of demand points;
demands /1..5/;

!Set of time periods;
periods /1..5/;

!Set of products;
products /1..1/;
```

In order to be able to compare the results, the data used in Chapter 3 was used for the decomposed problem. For the sub problem data, database views were created. In order to be able to work on the correct data set, the original data was enriched by columns containing the indices of the relevant row. This made it possible to work on the correct order of the sub data sets, because first trials have shown that the creation of the views resulted in a different order of the data than expected. The following SQL statements show the view creation and the corresponding tables depict the data structure for the sub problems.

```
SELECT
  MODELDATA_MP_5_VIEW.DEMAND,
  MODELDATA_MP_5_VIEW.COMPETITORSATTRACTION,
  MODELDATA_MP_5_VIEW.INDEXJ,
  MODELDATA_MP_5_VIEW.INDEXT,
  MODELDATA_MP_5_VIEW.INDEXP
```

```

FROM
    MODELDATA_MP_5_VIEW
WHERE
    MODELDATA_MP_5_VIEW.INDEXP = 1
ORDER BY
    MODELDATA_MP_5_VIEW.INDEXJ ASC ,
    MODELDATA_MP_5_VIEW.INDEXT ASC

-----

SELECT
    MODELDATA_MP_5_VIEW.UNITSHRINKAGECOST,
    MODELDATA_MP_5_VIEW.UNITEXPANSIONCOST,
    MODELDATA_MP_5_VIEW.PRICE,
    MODELDATA_MP_5_VIEW.ATTRACTIVITY,
    MODELDATA_MP_5_VIEW.VARIABLECOST,
    MODELDATA_MP_5_VIEW.INDEXI,
    MODELDATA_MP_5_VIEW.INDEXT,
    MODELDATA_MP_5_VIEW.INDEXP
FROM
    MODELDATA_MP_5_VIEW
WHERE
    MODELDATA_MP_5_VIEW.INDEXP = 1
ORDER BY
    MODELDATA_MP_5_VIEW.INDEXI ASC ,
    MODELDATA_MP_5_VIEW.INDEXT ASC

-----

SELECT
    MODELDATA_MP_5_INITIALS_CAP.INITIALCAP INITIALCAP
FROM
    MODELDATA_MP_5_INITIALS_CAP
WHERE
    MODELDATA_MP_5_INITIALS_CAP.INDEXP = 1
ORDER BY
    MODELDATA_MP_5_INITIALS_CAP.INDEXI ASC

```

Table 13 Database view MP_5_DEMANDS_P01_VIEW

	COLUMN_NAME	DATA_TYPE
1	DEMAND	FLOAT(20)
2	COMPETITORSATTRACTION	FLOAT(20)
3	INDEXJ	NUMBER(3)
4	INDEXT	NUMBER(3)
5	INDEXP	NUMBER(3)

Table 14 Database view MP_5_FACILITIES_P02_VIEW

1	UNITEXPANSIONCOST	FLOAT(126)
2	PRICE	FLOAT(20)
3	ATTRACTIVITY	FLOAT(20)
4	VARIABLECOST	FLOAT(20)
5	INDEXI	NUMBER(3)
6	UNITSHRINKAGECOST	FLOAT(126)
7	INDEXT	NUMBER(3)
8	INDEXP	NUMBER(3)

Table 15 Database view MP_5_INITIALS_CAP_P03_VIEW

MP_5_INITIALS_CAP_P01_VIEW		
1	INITIALCAP	FLOAT(20)

All statements and tables refer to the 5x5x5x5 problem. The data structure for problems of other sizes is identical. Table 12 contains the demands for the relevant product (product 1 in this case) for each period and the corresponding demand point. The table also contains the value for competitors' attraction levels. Table 13 contains all variables depending on the facility as unitexpansioncost, unitshrinkagecost, price, attractivity and variablecost. The last created view is Table 14. It contains the initial capacities for the product.

4.3 Computational results

All models were solved separately using LINGO. The results for all variables were written in the database. The solution time for each sub problem differs. This difference is only recognizable for very large problems.

Table 16 Solving times for 5x5x5x5 sub problems in seconds

For each product, the value for open status was written into a database table. The corresponding indices can be found in the columns *indexi* and *indext*. A value of 0 means facility *i* was closed in period *t* in the corresponding sub model.

Table 17 shows the opening status for certain products for various time periods.

Table 17 Results tableau for opening status of facilities depending on the sub problem

P01	P02	P03	P04	P05	indexi	indext
1	1	1	1	1	1	1
0	1	1	1	1	1	2
0	0	0	1	1	1	3
0	0	0	0	0	1	4
0	0	0	0	0	1	5
1	1	1	1	1	2	1
0	0	0	0	0	2	2
0	1	0	0	0	2	3
0	0	0	0	0	2	4
0	0	0	0	0	2	5
1	1	1	1	1	3	1
1	0	0	0	0	3	2
0	0	1	0	0	3	3
0	0	0	0	0	3	4
0	0	0	0	0	3	5
1	1	1	1	1	4	1
0	0	0	0	0	4	2
0	0	0	0	0	4	3
0	0	0	0	0	4	4
0	0	0	0	0	4	5
1	1	1	1	1	5	1
1	0	0	0	0	5	2
1	0	0	0	0	5	3
0	0	0	0	0	5	4
0	0	0	0	0	5	5

4.4 Combining different solutions

After solving all the sub-problems separately, the results had to be combined and applied to the original problem. In order to get a better understanding of which procedure is best, different selection procedures were used. The selection strategies were tested on the models of size 5x5x5x5 and 10x10x10x10. The first strategy selects all facilities where the status is *open* for all sub problems with respect to a certain period. Selecting all

closed facilities was not included. The following strategies select those facilities where at least some number of sub-problems requires the facility to be open in a certain period

The relevant information was added as constraint to the original model and the model was solved. The results were compared with the original problem. The following flow chart shows the general procedure of the decomposition algorithm.

Figure 5 Procedure of decomposition algorithm

4.4.1 Selection strategy 1: Select if open for all products

First, the facilities that were open for all the sub-problems were selected. Facilities that were closed in all the sub-problems were not considered in the first run. The lines corresponding to the open facilities are marked in Table 18.

Table 18 Result tableau for sub problems with five products

P01	P02	P03	P04	P05	indexi	indext
1	1	1	1	1	1	1
0	1	1	1	1	1	2
0	0	0	1	1	1	3
0	0	0	0	0	1	4
0	0	0	0	0	1	5
1	1	1	1	1	2	1
0	0	0	0	0	2	2
0	1	0	0	0	2	3
0	0	0	0	0	2	4
0	0	0	0	0	2	5
1	1	1	1	1	3	1
1	0	0	0	0	3	2
0	0	1	0	0	3	3
0	0	0	0	0	3	4
0	0	0	0	0	3	5
1	1	1	1	1	4	1
0	0	0	0	0	4	2
0	0	0	0	0	4	3
0	0	0	0	0	4	4
0	0	0	0	0	4	5
1	1	1	1	1	5	1
1	0	0	0	0	5	2
1	0	0	0	0	5	3
0	0	0	0	0	5	4
0	0	0	0	0	5	5

Because only the lines containing “1” as entries are initialized with theses values, there was no need to add constraints to the model.

Another run was made by selecting all the lines where the open status for all products equals zero with respect to a facility and period. The corresponding lines are marked in blue.

Table 19 Selection of all lines with open status equals 0

P01	P02	P03	P04	P05	indexi	indext
1	1	1	1	1	1	1
0	1	1	1	1	1	2
0	0	0	1	1	1	3
0	0	0	0	0	1	4
0	0	0	0	0	1	5
1	1	1	1	1	2	1
0	0	0	0	0	2	2
0	1	0	0	0	2	3
0	0	0	0	0	2	4
0	0	0	0	0	2	5
1	1	1	1	1	3	1
1	0	0	0	0	3	2
0	0	1	0	0	3	3
0	0	0	0	0	3	4
0	0	0	0	0	3	5
1	1	1	1	1	4	1
0	0	0	0	0	4	2
0	0	0	0	0	4	3
0	0	0	0	0	4	4
0	0	0	0	0	4	5
1	1	1	1	1	5	1
1	0	0	0	0	5	2
1	0	0	0	0	5	3
0	0	0	0	0	5	4
0	0	0	0	0	5	5

This selection resulted in the addition of the following constraints to the model.

```

open(1,4) = 0;
open(1,5) = 0;
open(2,2) = 0;

```

```

open (2, 4) = 0;
open (2, 5) = 0;
open (3, 4) = 0;
open (3, 5) = 0;
open (4, 2) = 0;
open (4, 3) = 0;
open (4, 4) = 0;
open (4, 5) = 0;
open (5, 4) = 0;
open (5, 5) = 0;

```

The model was solved and a solution could be obtained. Because the 5x5x5x5 problem is small enough to be solved optimally for the original (non-decomposed) model, we were able to compare the OFV of the suboptimal decomposition approach with that of the optimal solution.

Table 20 Objective function value for 5x5x5x5 model using different selection strategies

Selection strategy	1	0
Original model	2.95213E08	
All values	Not available	2.95213E08
At least one	2.95044E08	1.82156E08
At least two	2.95213E08	2.69256E08
At least three	Not available	2.69256E08
At least four	Not available	2.81741E08

The decomposition procedure was also applied to the model size of 10x10x10x10. The

complexity for the data structure grew, but the sub problems could be solved quickly. The solution times for each of the different sub problems were less than 35 seconds.

Figure 6 Solution times for 10x10x10x10 sub problems (in seconds)

Table 21 Results tableau for 10x10x10x10 model and its sub problems

P01	P02	P03	P04	P05	P06	P07	P08	P09	P10	indexi	indext
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	0	0	0	0	0	1	2
1	0	1	1	1	0	0	0	0	0	1	3
0	0	0	0	0	0	0	0	0	0	1	4
0	0	0	0	0	0	0	0	0	0	1	5
0	0	0	0	0	0	0	0	0	0	1	6
0	0	0	0	0	0	0	0	0	0	1	7
0	0	0	0	0	0	0	1	0	0	1	8
0	0	0	0	0	0	0	0	0	0	1	9
0	0	0	0	0	0	0	0	0	0	1	10
1	1	1	1	1	1	1	1	1	1	2	1
0	0	0	0	1	0	0	0	0	0	2	2
0	1	1	1	1	0	0	0	0	0	2	3

0	0	0	0	0	0	0	0	0	0	2	4
0	0	0	0	0	0	0	0	0	0	2	5
0	0	0	0	0	0	0	0	0	0	2	6
0	0	0	0	0	0	0	0	0	0	2	7
0	0	0	0	0	0	0	0	0	0	2	8
0	0	0	0	0	0	0	0	0	0	2	9
0	0	0	0	0	0	0	0	0	0	2	10
1	1	1	1	1	1	1	1	1	1	3	1
1	1	1	0	1	0	0	0	0	0	3	2
0	1	1	1	0	0	0	0	0	0	3	3
0	0	0	0	0	0	0	0	0	0	3	4
0	0	0	0	0	0	0	0	0	0	3	5
0	0	0	0	0	0	0	0	0	0	3	6
0	0	0	0	0	0	0	0	0	0	3	7
0	0	0	0	0	0	0	0	0	0	3	8
0	1	0	1	1	0	0	0	0	0	3	9
0	0	0	0	0	1	0	0	0	0	3	10
1	1	1	1	1	1	1	1	1	1	4	1
0	0	1	1	1	0	0	0	0	0	4	2
1	0	1	0	1	0	0	0	0	1	4	3
0	0	0	0	0	0	0	0	0	0	4	4
0	0	0	0	0	0	0	0	0	0	4	5
1	0	0	0	0	1	1	0	0	0	4	6
0	0	0	0	1	0	0	0	0	1	4	7
0	0	0	0	0	0	0	0	0	0	4	8
0	0	1	0	0	0	0	0	0	0	4	9
0	0	0	0	0	0	0	0	0	0	4	10
1	1	1	1	1	1	1	1	1	1	5	1
0	1	1	1	1	0	0	0	0	0	5	2
1	0	0	1	1	0	0	0	0	0	5	3
0	0	0	0	0	0	0	0	0	0	5	4
0	0	0	0	0	0	0	0	0	1	5	5
0	0	0	0	0	0	0	0	0	0	5	6
0	0	0	0	0	0	0	0	0	0	5	7
0	0	0	0	0	0	0	0	1	0	5	8
0	0	0	0	1	1	0	0	0	0	5	9
0	0	0	0	0	0	0	0	0	0	5	10
1	1	1	1	1	1	1	1	1	1	6	1
0	0	0	0	1	0	1	1	1	0	6	2
0	0	0	0	0	0	0	0	0	0	6	3
0	0	0	1	0	0	0	0	0	0	6	4
0	0	0	0	0	0	0	0	0	0	6	5
0	0	0	0	0	0	1	0	0	0	6	6

0	0	1	0	1	0	0	0	0	0	6	7
0	0	0	0	0	0	0	0	0	0	6	8
0	0	0	0	1	1	0	0	0	0	6	9
0	0	0	0	0	0	0	0	0	0	6	10
1	1	1	1	1	1	1	1	1	1	7	1
1	0	0	0	0	0	0	0	0	0	7	2
0	0	0	0	0	0	0	0	0	0	7	3
0	1	0	1	0	0	0	0	0	0	7	4
0	0	0	0	0	0	0	0	0	0	7	5
0	0	0	1	0	0	0	0	0	0	7	6
0	0	0	0	0	0	0	0	0	0	7	7
0	1	0	1	0	0	0	0	0	1	7	8
0	0	0	0	0	0	0	0	0	0	7	9
0	0	0	0	0	0	0	0	0	0	7	10
1	1	1	1	1	1	1	1	1	1	8	1
0	0	0	0	0	0	0	1	0	0	8	2
0	0	0	0	1	0	0	0	0	0	8	3
0	0	1	0	0	1	0	0	0	1	8	4
0	0	1	0	0	0	0	0	0	0	8	5
0	1	0	0	0	0	0	0	0	0	8	6
0	0	0	1	0	0	0	0	0	0	8	7
0	0	0	0	0	1	0	0	0	0	8	8
0	0	0	1	1	0	0	0	0	0	8	9
0	0	0	0	0	1	1	1	1	0	8	10
1	1	1	1	1	1	1	1	1	1	9	1
0	0	0	0	0	0	0	0	0	0	9	2
0	0	0	0	0	0	0	0	0	0	9	3
0	0	0	0	0	1	0	0	0	1	9	4
0	0	0	0	0	0	0	0	0	0	9	5
0	0	0	0	0	0	0	1	0	0	9	6
0	0	0	0	0	0	0	0	0	0	9	7
0	0	1	0	0	0	0	0	0	0	9	8
0	0	0	1	0	1	0	1	0	0	9	9
1	0	0	0	0	0	0	0	0	0	9	10
1	1	1	1	1	1	1	1	1	1	10	1
0	0	0	0	0	0	1	0	0	0	10	2
0	0	0	0	0	0	0	0	0	0	10	3
0	0	1	0	1	0	0	0	0	0	10	4
0	0	0	0	0	0	0	1	0	0	10	5
1	0	0	0	0	0	0	0	0	0	10	6
0	0	0	0	0	0	0	0	0	0	10	7
0	0	0	0	0	1	0	0	0	0	10	8
0	0	0	0	0	0	0	0	0	0	10	9

0	0	0	0	0	0	0	0	0	0	0	10	10
---	---	---	---	---	---	---	---	---	---	---	----	----

A selection of lines where all sub problems show an open status returned an empty result.

Though there are 10 lines with all 1's, these values result from initialization and are already set. So the selection of all lines with only zeros was made and the following constraints were added to the model.

```

open(1,4) = 0;
open(1,5) = 0;
open(1,6) = 0;
open(1,7) = 0;
open(1,9) = 0;
open(1,10) = 0;
open(2,4) = 0;
open(2,5) = 0;
open(2,6) = 0;
open(2,7) = 0;
open(2,8) = 0;
open(2,9) = 0;
open(2,10) = 0;
open(3,4) = 0;
open(3,5) = 0;
open(3,6) = 0;
open(3,7) = 0;
open(3,8) = 0;
open(4,4) = 0;
open(4,5) = 0;
open(4,8) = 0;
open(4,10) = 0;
open(5,4) = 0;
open(5,6) = 0;
open(5,7) = 0;
open(5,10) = 0;
open(6,3) = 0;
open(6,5) = 0;
open(6,8) = 0;
open(6,10) = 0;
open(7,3) = 0;
open(7,5) = 0;
open(7,7) = 0;
open(7,9) = 0;
open(7,10) = 0;
open(9,2) = 0;
open(9,3) = 0;
open(9,5) = 0;
open(9,7) = 0;
open(10,3) = 0;

```

```

open(10,7) = 0;
open(10,9) = 0;
open(10,10) = 0;

```

The complexity of the model was reduced by 43 variables (30,247 variables) but a solution could be obtained quickly. Comparing the original problem solution's objective function value to the solution including the result of the sub problems, the objective function value is the same as value from the original problem (2.61799E8).

Three more selection strategies were tested.

4.4.2 Selection where at least three facilities are open / closed

In real applications one might not want to solve all subproblems. Thus, the next selection was made when at least three facilities were declared to be open (grey shadowed) or closed (in italics) as shown in Table 22.

Table 22 Results tableau with at least three facilities open for 10x10x10x10 model

P01	P02	P03	P04	P05	P06	P07	P08	P09	P10	indexi	indext
1	1	1	1	1	1	1	1	1	1	1	1
<i>1</i>	<i>1</i>	<i>1</i>	<i>1</i>	<i>1</i>	0	0	0	0	0	1	2
<i>1</i>	0	<i>1</i>	<i>1</i>	<i>1</i>	0	0	0	0	0	1	3
0	0	0	0	0	0	0	0	0	0	1	4
0	0	0	0	0	0	0	0	0	0	1	5
0	0	0	0	0	0	0	0	0	0	1	6
0	0	0	0	0	0	0	0	0	0	1	7
0	0	0	0	0	0	0	<i>1</i>	0	0	1	8
0	0	0	0	0	0	0	0	0	0	1	9
0	0	0	0	0	0	0	0	0	0	1	10
1	1	1	1	1	1	1	1	1	1	2	1
0	0	0	0	<i>1</i>	0	0	0	0	0	2	2
<i>0</i>	<i>1</i>	<i>1</i>	<i>1</i>	<i>1</i>	0	0	0	0	0	2	3
0	0	0	0	0	0	0	0	0	0	2	4

0	0	0	0	0	0	0	0	0	0	2	5
0	0	0	0	0	0	0	0	0	0	2	6
0	0	0	0	0	0	0	0	0	0	2	7
0	0	0	0	0	0	0	0	0	0	2	8
0	0	0	0	0	0	0	0	0	0	2	9
0	0	0	0	0	0	0	0	0	0	2	10
1	1	1	1	1	1	1	1	1	1	3	1
1	1	1	0	1	0	0	0	0	0	3	2
0	1	1	1	0	0	0	0	0	0	3	3
0	0	0	0	0	0	0	0	0	0	3	4
0	0	0	0	0	0	0	0	0	0	3	5
0	0	0	0	0	0	0	0	0	0	3	6
0	0	0	0	0	0	0	0	0	0	3	7
0	0	0	0	0	0	0	0	0	0	3	8
0	1	0	1	1	0	0	0	0	0	3	9
0	0	0	0	0	1	0	0	0	0	3	10
1	1	1	1	1	1	1	1	1	1	4	1
0	0	1	1	1	0	0	0	0	0	4	2
1	0	1	0	1	0	0	0	0	1	4	3
0	0	0	0	0	0	0	0	0	0	4	4
0	0	0	0	0	0	0	0	0	0	4	5
1	0	0	0	0	1	1	0	0	0	4	6
0	0	0	0	1	0	0	0	0	1	4	7
0	0	0	0	0	0	0	0	0	0	4	8
0	0	1	0	0	0	0	0	0	0	4	9
0	0	0	0	0	0	0	0	0	0	4	10
1	1	1	1	1	1	1	1	1	1	5	1
0	1	1	1	1	0	0	0	0	0	5	2
1	0	0	1	1	0	0	0	0	0	5	3
0	0	0	0	0	0	0	0	0	0	5	4
0	0	0	0	0	0	0	0	0	1	5	5
0	0	0	0	0	0	0	0	0	0	5	6
0	0	0	0	0	0	0	0	0	0	5	7
0	0	0	0	0	0	0	0	1	0	5	8
0	0	0	0	1	1	0	0	0	0	5	9
0	0	0	0	0	0	0	0	0	0	5	10
1	1	1	1	1	1	1	1	1	1	6	1
0	0	0	0	1	0	1	1	1	0	6	2
0	0	0	0	0	0	0	0	0	0	6	3
0	0	0	1	0	0	0	0	0	0	6	4
0	0	0	0	0	0	0	0	0	0	6	5
0	0	0	0	0	0	1	0	0	0	6	6
0	0	1	0	1	0	0	0	0	0	6	7
0	0	0	0	0	0	0	0	0	0	6	8
0	0	0	0	1	1	0	0	0	0	6	9

0	0	0	0	0	0	0	0	0	0	6	10
1	1	1	1	1	1	1	1	1	1	7	1
1	0	0	0	0	0	0	0	0	0	7	2
0	0	0	0	0	0	0	0	0	0	7	3
0	1	0	1	0	0	0	0	0	0	7	4
0	0	0	0	0	0	0	0	0	0	7	5
0	0	0	1	0	0	0	0	0	0	7	6
0	0	0	0	0	0	0	0	0	0	7	7
0	1	0	1	0	0	0	0	0	1	7	8
0	0	0	0	0	0	0	0	0	0	7	9
0	0	0	0	0	0	0	0	0	0	7	10
1	1	1	1	1	1	1	1	1	1	8	1
0	0	0	0	0	0	0	1	0	0	8	2
0	0	0	0	1	0	0	0	0	0	8	3
0	0	1	0	0	1	0	0	0	1	8	4
0	0	1	0	0	0	0	0	0	0	8	5
0	1	0	0	0	0	0	0	0	0	8	6
0	0	0	1	0	0	0	0	0	0	8	7
0	0	0	0	0	1	0	0	0	0	8	8
0	0	0	1	1	0	0	0	0	0	8	9
0	0	0	0	0	1	1	1	1	0	8	10
1	1	1	1	1	1	1	1	1	1	9	1
0	0	0	0	0	0	0	0	0	0	9	2
0	0	0	0	0	0	0	0	0	0	9	3
0	0	0	0	0	1	0	0	0	1	9	4
0	0	0	0	0	0	0	0	0	0	9	5
0	0	0	0	0	0	0	1	0	0	9	6
0	0	0	0	0	0	0	0	0	0	9	7
0	0	1	0	0	0	0	0	0	0	9	8
0	0	0	1	0	1	0	1	0	0	9	9
1	0	0	0	0	0	0	0	0	0	9	10
1	1	1	1	1	1	1	1	1	1	10	1
0	0	0	0	0	0	1	0	0	0	10	2
0	0	0	0	0	0	0	0	0	0	10	3
0	0	1	0	1	0	0	0	0	0	10	4
0	0	0	0	0	0	0	1	0	0	10	5
1	0	0	0	0	0	0	0	0	0	10	6
0	0	0	0	0	0	0	0	0	0	10	7
0	0	0	0	0	1	0	0	0	0	10	8
0	0	0	0	0	0	0	0	0	0	10	9
0	0	0	0	0	0	0	0	0	0	10	10

The following constraints were added to the model with at least three facilities open.

```

open(1,2) = 1;
open(1,3) = 1;
open(2,3) = 1;
open(3,2) = 1;
open(3,3) = 1;
open(3,9) = 1;
open(4,3) = 1;
open(4,6) = 1;
open(5,2) = 1;
open(5,3) = 1;
open(6,2) = 1;
open(7,8) = 1;
open(8,4) = 1;
open(8,10) = 1;
open(9,9) = 1;
open(10,1) = 1;

```

The objective function value did not change (2.61799E08).

Adding constraints for closed facilities resulted in closing down all facilities after the first (initial) period.

```

open(1,4) = 0;
open(1,5) = 0;
open(1,6) = 0;
open(1,7) = 0;
open(1,8) = 0;
open(1,9) = 0;
open(1,10) = 0;
open(2,4) = 0;
open(2,5) = 0;
open(2,6) = 0;
open(2,7) = 0;
open(2,8) = 0;
open(2,9) = 0;
open(2,10) = 0;
open(3,4) = 0;
open(3,5) = 0;
open(3,6) = 0;
open(3,7) = 0;
open(3,8) = 0;
open(3,10) = 0;
open(4,4) = 0;
open(4,5) = 0;
open(4,7) = 0;
open(4,8) = 0;
open(4,9) = 0;
open(4,10) = 0;

```

```

open(5,4) = 0;
open(5,5) = 0;
open(5,6) = 0;
open(5,7) = 0;
open(5,8) = 0;
open(5,9) = 0;
open(5,10) = 0;
open(6,3) = 0;
open(6,4) = 0;
open(6,5) = 0;
open(6,6) = 0;
open(6,7) = 0;
open(6,8) = 0;
open(6,9) = 0;
open(6,10) = 0;
open(7,3) = 0;
open(7,4) = 0;
open(7,5) = 0;
open(7,6) = 0;
open(7,7) = 0;
open(7,9) = 0;
open(7,10) = 0;
open(8,2) = 0;
open(8,3) = 0;
open(8,5) = 0;
open(8,6) = 0;
open(8,7) = 0;
open(8,8) = 0;
open(8,9) = 0;
open(9,2) = 0;
open(9,3) = 0;
open(9,4) = 0;
open(9,5) = 0;
open(9,6) = 0;
open(9,7) = 0;
open(9,8) = 0;
open(9,10) = 0;
open(10,2) = 0;
open(10,3) = 0;
open(10,4) = 0;
open(10,5) = 0;
open(10,6) = 0;
open(10,7) = 0;
open(10,8) = 0;
open(10,9) = 0;
open(10,10) = 0;

```

The model was solved and the objective function value fell down rapidly (1.5499*1E08).

4.4.3 Selection where at least five facilities are open / closed

The same procedure was made selecting where at least five facilities were open or closed.

The following constraint was added to the model

$\text{open}(1,2) = 1;$

The objective function value was the same as for the original problem. Another run for 0-line was not made because the relevant lines are exactly the same as for the selection strategy “at least three”.

4.4.4 Selection where at least eight facilities are open /closed

This selection was not possible with the data, as there is no row with at least eight “1” entries. Selecting the lines with at least eight “0” entries yields the result shown in Table 23.

Table 23 Results tableau with at least eight facilities open for 10x10x10x10 model

P01	P02	P03	P04	P05	P06	P07	P08	P09	P10	indexi	indext
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	0	0	0	0	0	1	2
1	0	1	1	1	0	0	0	0	0	1	3
0	0	0	0	0	0	0	0	0	0	1	4
0	0	0	0	0	0	0	0	0	0	1	5
0	0	0	0	0	0	0	0	0	0	1	6
0	0	0	0	0	0	0	0	0	0	1	7
0	0	0	0	0	0	0	1	0	0	1	8
0	0	0	0	0	0	0	0	0	0	1	9
0	0	0	0	0	0	0	0	0	0	1	10
1	1	1	1	1	1	1	1	1	1	2	1
0	0	0	0	1	0	0	0	0	0	2	2
0	1	1	1	1	0	0	0	0	0	2	3
0	0	0	0	0	0	0	0	0	0	2	4
0	0	0	0	0	0	0	0	0	0	2	5
0	0	0	0	0	0	0	0	0	0	2	6
0	0	0	0	0	0	0	0	0	0	2	7
0	0	0	0	0	0	0	0	0	0	2	8

0	0	0	0	0	0	0	0	0	0	2	9
0	0	0	0	0	0	0	0	0	0	2	10
1	1	1	1	1	1	1	1	1	1	3	1
1	1	1	0	1	0	0	0	0	0	3	2
0	1	1	1	0	0	0	0	0	0	3	3
0	0	0	0	0	0	0	0	0	0	3	4
0	0	0	0	0	0	0	0	0	0	3	5
0	0	0	0	0	0	0	0	0	0	3	6
0	0	0	0	0	0	0	0	0	0	3	7
0	0	0	0	0	0	0	0	0	0	3	8
0	1	0	1	1	0	0	0	0	0	3	9
0	0	0	0	0	1	0	0	0	0	3	10
1	1	1	1	1	1	1	1	1	1	4	1
0	0	1	1	1	0	0	0	0	0	4	2
1	0	1	0	1	0	0	0	0	1	4	3
0	0	0	0	0	0	0	0	0	0	4	4
0	0	0	0	0	0	0	0	0	0	4	5
1	0	0	0	0	1	1	0	0	0	4	6
0	0	0	0	1	0	0	0	0	1	4	7
0	0	0	0	0	0	0	0	0	0	4	8
0	0	1	0	0	0	0	0	0	0	4	9
0	0	0	0	0	0	0	0	0	0	4	10
1	1	1	1	1	1	1	1	1	1	5	1
0	1	1	1	1	0	0	0	0	0	5	2
1	0	0	1	1	0	0	0	0	0	5	3
0	0	0	0	0	0	0	0	0	0	5	4
0	0	0	0	0	0	0	0	0	1	5	5
0	0	0	0	0	0	0	0	0	0	5	6
0	0	0	0	0	0	0	0	0	0	5	7
0	0	0	0	0	0	0	0	1	0	5	8
0	0	0	0	1	1	0	0	0	0	5	9
0	0	0	0	0	0	0	0	0	0	5	10
1	1	1	1	1	1	1	1	1	1	6	1
0	0	0	0	1	0	1	1	1	0	6	2
0	0	0	0	0	0	0	0	0	0	6	3
0	0	0	1	0	0	0	0	0	0	6	4
0	0	0	0	0	0	0	0	0	0	6	5
0	0	0	0	0	0	1	0	0	0	6	6
0	0	1	0	1	0	0	0	0	0	6	7
0	0	0	0	0	0	0	0	0	0	6	8
0	0	0	0	1	1	0	0	0	0	6	9
0	0	0	0	0	0	0	0	0	0	6	10
1	1	1	1	1	1	1	1	1	1	7	1
1	0	0	0	0	0	0	0	0	0	7	2
0	0	0	0	0	0	0	0	0	0	7	3

0	1	0	1	0	0	0	0	0	0	7	4
0	0	0	0	0	0	0	0	0	0	7	5
0	0	0	1	0	0	0	0	0	0	7	6
0	0	0	0	0	0	0	0	0	0	7	7
0	1	0	1	0	0	0	0	0	1	7	8
0	0	0	0	0	0	0	0	0	0	7	9
0	0	0	0	0	0	0	0	0	0	7	10
1	1	1	1	1	1	1	1	1	1	8	1
0	0	0	0	0	0	0	1	0	0	8	2
0	0	0	0	1	0	0	0	0	0	8	3
0	0	1	0	0	1	0	0	0	1	8	4
0	0	1	0	0	0	0	0	0	0	8	5
0	1	0	0	0	0	0	0	0	0	8	6
0	0	0	1	0	0	0	0	0	0	8	7
0	0	0	0	0	1	0	0	0	0	8	8
0	0	0	1	1	0	0	0	0	0	8	9
0	0	0	0	0	1	1	1	1	0	8	10
1	1	1	1	1	1	1	1	1	1	9	1
0	0	0	0	0	0	0	0	0	0	9	2
0	0	0	0	0	0	0	0	0	0	9	3
0	0	0	0	0	1	0	0	0	1	9	4
0	0	0	0	0	0	0	0	0	0	9	5
0	0	0	0	0	0	0	1	0	0	9	6
0	0	0	0	0	0	0	0	0	0	9	7
0	0	1	0	0	0	0	0	0	0	9	8
0	0	0	1	0	1	0	1	0	0	9	9
1	0	0	0	0	0	0	0	0	0	9	10
1	1	1	1	1	1	1	1	1	1	10	1
0	0	0	0	0	0	1	0	0	0	10	2
0	0	0	0	0	0	0	0	0	0	10	3
0	0	1	0	1	0	0	0	0	0	10	4
0	0	0	0	0	0	0	1	0	0	10	5
1	0	0	0	0	0	0	0	0	0	10	6
0	0	0	0	0	0	0	0	0	0	10	7
0	0	0	0	0	1	0	0	0	0	10	8
0	0	0	0	0	0	0	0	0	0	10	9
0	0	0	0	0	0	0	0	0	0	10	10

A selection of at least eight was not possible, as there were no relevant lines. For the selection of at least eight entries with 0, the following constraints were added.

```
open(1,4) = 0;
open(1,5) = 0;
```

```
open(1,6) = 0;
open(1,7) = 0;
open(1,8) = 0;
open(1,9) = 0;
open(1,10) = 0;
open(2,4) = 0;
open(2,5) = 0;
open(2,6) = 0;
open(2,7) = 0;
open(2,8) = 0;
open(2,9) = 0;
open(2,10) = 0;
open(3,4) = 0;
open(3,5) = 0;
open(3,6) = 0;
open(3,7) = 0;
open(3,8) = 0;
open(3,10) = 0;
open(4,4) = 0;
open(4,5) = 0;
open(4,8) = 0;
open(4,9) = 0;
open(4,10) = 0;
open(5,4) = 0;
open(5,5) = 0;
open(5,6) = 0;
open(5,7) = 0;
open(5,8) = 0;
open(5,10) = 0;
open(6,3) = 0;
open(6,4) = 0;
open(6,5) = 0;
open(6,6) = 0;
open(6,8) = 0;
open(6,10) = 0;
open(7,2) = 0;
open(7,3) = 0;
open(7,5) = 0;
open(7,6) = 0;
open(7,7) = 0;
open(7,9) = 0;
open(7,10) = 0;
open(8,2) = 0;
open(8,3) = 0;
open(8,5) = 0;
open(8,6) = 0;
open(8,7) = 0;
open(8,8) = 0;
open(9,2) = 0;
open(9,3) = 0;
open(9,5) = 0;
open(9,6) = 0;
open(9,7) = 0;
open(9,8) = 0;
open(9,10) = 0;
open(10,2) = 0;
open(10,3) = 0;
```

```

open(10,5) = 0;
open(10,6) = 0;
open(10,7) = 0;
open(10,8) = 0;
open(10,9) = 0;
open(10,10) = 0;

```

The following table shows the objective function values of the different selection strategies for the 10x10x10x10 model.

Table 24 Objective function values depending on selection strategy

Selection strategy	1	0
Original model	2.61799E08	
All values	Not available	2.61799E08
At least three	2.61799E08	1.549897E08
At least five	2.61799E08	1.549897E08
At least eight	Not available	2.60496E08

The table shows that it is better to focus on open facilities and not on closed facilities for the objective function value. For a good result it is also better to have knowledge about as many products as possible. If there is a high number of closed facilities, the selection of those might be a good approach.

4.5 Summary

The decomposition approach reduces the complexity of the original model significantly. Depending on the selection strategy, different results are obtained. Based on our experimentation with the 5x5x5x5 problem (for which an optimal solution can be obtained), the sub-problem solutions lead to the optimal solution of the original problem, if the selection is made for those facilities where all sub problems require the facilities to be opened or closed respectively. With respect to the 10x10x10x10 problem, the results were different. In general, it is important to look at the data carefully, because selecting “open” and adding open constraints to the original problem provide better results than focusing on closed facilities. The more facilities we know that are open for the relevant sub problems, the better the objective function value. Sometimes the problems might become so large that a fully decomposition is too time consuming, In that case a selection based on a subset of products could be made to get some good results.

The solving times of the sub problem and the modified original problem are fast. For larger problems, this procedure is time consuming as the data has to be prepared and each sub problem has to be solved separately. Finally, the decomposition approach provides an algorithm for obtaining an optimal solution from the original problem with some effort. It must be checked whether a preprocessing or an alternative decomposition approach will generate a better result.

CHAPTER 5

A SIMULATED ANNEALING ALGORITHM

5.1 Introduction

In real world applications, companies deal with many more than 10 products. Therefore, it is important to try alternatives in solving the optimization problem, as a decomposition approach is likely to be ineffective when we have a large number of products and there is significant interaction amongst them. Simulated annealing (SA) is known to be an effective approach for large-scale combinatorial optimization problems. It is a heuristic technique for finding an approximate solution, when it is not possible to find an optimal solution due to computational complexity. SA replicates the gradual cooling of molten material. Slow annealing ensures that atoms have enough time to find a stable position to build crystals. A state of low energy close to the optimum is reached. Translated to the optimization problem, the SA explores better solutions gradually ensuring a sufficient number of solutions are evaluated at each temperature epoch. Not only does the algorithm move to better solutions like a steepest descent 2-opt algorithm, it also moves to a worse solution with a certain probability that depends upon how much worse that solution is (Press, Teukolsky, Vetterling, & Flannery, 2007). SA is applied to solve the models in Chpaters 3 and 4.

5.2 Modification of algorithm

In the SA algorithm applied to solve to the facility location problem considered in this dissertation, an array with a dimension corresponding to the product of the number of facilities and time periods considered is used. It is named configuration and contains binary values 0 (facility is closed in the corresponding time period) and 1 (facility is open). In each step of the SA algorithm, the value of only one element in the array is changed – from 0 to 1 or vice-versa. The array is then passed to the LINGO solver and the objective value is obtained. The rest of the procedure is the same as that proposed in the original annealing algorithm.

The following pseudo code demonstrates how the algorithm works.

Algorithm simulatedAnnealing (temperature, cooling Rate)

Begin

Step 1, Initialization

startConfiguration = createNewConfiguration(facilities, periods);

bestEnergy = getEnergy(startConfiguration);

bestConfiguration = startConfiguration;

currentConfiguration = startConfiguration;

currentEnergy = bestEnergy;

Step 2, Annealing

While temperature > 1 **Do**

neighborConfiguration = createNewConfiguration(currentConfiguration);

neighborEnergy = getEnergy(neighborConfiguration);


```

If getAcceptanceProbability > random number Then
    currentConfiguration = neighborConfiguration;
    currentEnergy = neighborEnergy;
End If

If currentEnergy > bestEnergy Then
    bestConfiguration = currentConfiguration;
    bestEnergy = currentEnergy;

End If

temperature *= 1-coolingRate;
End While

Return bestEnergy, bestConfiguration;

End

```

Procedure getAcceptanceProbability (energy, newEnergy, temperature)

Begin Procedure

If newEnergy > energy **then**

Return 1.0

Else

Return $e^{\frac{-(energy - newEnergy)}{temperature}}$

End if

End Procedure

Procedure createNewConfiguration (currentConfiguration)

indexi = *RandomNumber**facilities;

indext = 0;

```

While indext = 0 Do
    indext = RandonNumber*periods;
End While

newConfiguration = currentConfiguration;
if newConfiguration[indexi][indext] = 0) Then
    newConfiguration[indexi][indext] = 1;
Else If
    newConfiguration[indexi][indext] = 0;
End If

    return newConfiguration;
End Procedure

```

```

Procedure getEnergy(configuration)
    Load the LINGO model;
    Retrieve variable indices;
    For each variableIndex Do
        Add constraint from configuration;
    End For

    Solve the model;
    Return objective function value;
End Procedure

```

The corresponding JAVA source code can be found in Appendix F.

5.3 Technical challenges

Using the LINDO API in Java raised several technical challenges. The model needed to be saved as an MPS file in advance so that the API could read it. For the computation the

LINDO API with unlimited variables is necessary. To be able to change the model's constraints, it was necessary to obtain the variable indices. This was possible using one of the functions provided in LINDO. Additionally several JAVA error messages had be resolved, especially setting the PATH environment was difficult. Finally the source code was written using Eclipse with SDK 8.

5.4 Computational results

The annealing algorithm was tested on four different problem instances (5x5x5x5, 10x10x10x10, 50x50x50x50 and 50x50x12x50). Mixed integer problems could be solved for problems with fewer than 4,000 variables. Relaxing the model resulted in durations of less than 100 milliseconds for each step for smaller models, while it took up to ten hours for the largest models.

5.4.1 Solution times

As expected, the solution times were highly dependent on the problem sizes. First, the SA algorithm was applied to the mixed-integer problem, and then to the relaxed model.

The following table shows the average computational times for each annealing step for the mixed-integer case. The SA algorithm can barely be applied to a problem of size 5x5x5x5. The standard deviation of the solution times for the mixed-integer problems is larger than the mean which indicates a strong dependence of the configuration.

Table 25 Computational times for each step of the SA algorithm with respect to problem size for the mixed-integer model

	Mean	Standard deviation	# of variables
	[<i>sec</i> * 10^{-3}]		
5x5x5x5	9901938	14740967	3,945
10x10x10x10	n.a.	n.a.	30,290
50x50x12x50	n.a.	n.a.	2,099,850
50x50x50x50	n.a.	n.a.	8,757,450

The application of the SA algorithm works much better for the relaxed model. This shows that application of the annealing algorithm is a good option for model sizes up to 2 million variables when using the relaxed model.

Table 26 Computational times for each step of the SA algorithm with respect to problem size for the relaxed model

	Mean	Standard deviation	# of variables
	[<i>sec</i> * 10^{-3}]	[<i>sec</i> * 10^{-3}]	
5x5x5x5	87	8	3,945
10x10x10x10	806	67	30,290
50x50x12x50	34,917,794	n.a.	2,099,850

50x50x50x50	16,060,523	n.a.	8,757,450
-------------	------------	------	-----------

The results show again that large models require considerable solution time for each step, although the specific data also influences runtime. Therefore the right choice of temperature and cooling rate is of great importance. The 50x50x50x50 model could only be run after a RAM extension to the computer from 8GB to 16GB was made as the application required too much working memory. This shows that even for the SA algorithm applied to the relaxed model, there are technical restrictions of the problem size that can be solved.

Figure 7 Average runtime per annealing step for the relaxed model

5.4.2 The choice of temperature and cooling rate

For each model, potentially $2^{\text{locations} \cdot (\text{periods} - 1)}$ configurations are possible. Taking the different problem sizes considered, this would result in the following solution times if all possible configurations had to be tested to obtain the optimal solution.

Table 27 Potential solving times for all configurations

	Potential configurations	Total solving time
5x5x5x5	$2^{5 \cdot (5-1)} = 1,048,576$	3.6 hours
10x10x10x10	$2^{10 \cdot (10-1)}$	> 28499000143365870103 years
50x50x12x50	$2^{50 \cdot (12-1)}$	Too big to display
50x50x50x50	$2^{50 \cdot (50-1)}$	Too big to display

It is obvious that it is not possible to test all these configurations. Therefore, the SA algorithm was applied. A higher temperature will cause both a longer run of the entire annealing procedure and a higher probability accepting lower energy values at the beginning. The cooling rate also influences the length of the annealing procedure. Several runs with different energy levels were done. The following table shows the number of corresponding steps.

Table 28 Annealing steps depending on the choice of cooling rate and temperature

		Temperature						
		100	1,000	10,000	100,000	1,000,000	10,000,000	100,000,000
Cooling Rate	0.001	4,603	6,905	9,206	11,508	13,809	16,111	18,412
	0.01	459	688	917	1,146	1,375	1,604	1,866
	0.05	90	135	180	225	270	315	360
	0.1	44	66	88	110	132	153	175
	0.2	21	31	42	52	62	73	83
	0.25	7	10	14	17	20	24	27
	0.75	4	5	7	9	10	12	14

As the number of steps grows rapidly with lower cooling rates, attention was particularly drawn to the change of energy levels. The following figures show the development of the energy levels over time for the 10x10x10x10 model with different settings of temperature and cooling rate.

Figure 8 Simulated annealing for 10x10x10x10 model temperature 10^6
and cooling rate 0.05

Figure 9 Simulated annealing for 10x10x10x10 model temperature 100
and cooling rate 0.001

Figure 10 Simulated annealing for 10x10x10x10 model temperature 10^7 and cooling rate 0.01

Figure 11 Simulated annealing for 10x10x10x10 model temperature 10,000 and cooling rate 0.01

These charts have been created for all the different combinations (temperature / cooling rate) They demonstrate that the best results for the SA algorithm are to be found when the temperature is about $1/10^{\text{th}}$ of the objective function value and a cooling rate of 1%. Lower cooling rates ($\ll 0.01$) will only cause a longer run of the annealing algorithm without significant improvement of the objective function value.

5.5 Pre-processing

Comparing the results of the annealing algorithm to the results from the decomposition approach, it is obvious that the objective function values are worse. A preprocessing test was made to improve the objective function value. The following preprocessing approaches were carried out.

1. Setting all facilities to open.
2. Setting all facilities to close.
3. Random opening or closing the facilities.
4. Setting those facilities to open in a period during which the net profit is positive for more than half of the products in that period.
5. Setting those facilities to open in a period during which the attraction for more than half of the products is greater than the attraction of competitor in that period.

The SA algorithm was started after the preprocessing with a temperature of 10,000,000 and cooling rate 0.01 using both the 5x5x5x5 and the 10x10x10x10 problem instances. Each of the different approaches was run 10 times for both models. The objective

function values were obtained and mean and standard deviation were computed. A two-sample *t*-test was performed to test for significant difference in the energy levels. The following table shows the results for the means and standard deviations for both models and the pre-processing approach. For the 5x5x5x5 problem, the values did not improve after the sixteenth and second annealing step for the net profit and attraction pre-setting, respectively. Therefore the best value obtained were taken from the simulation.

Table 29 Means and standard deviations for objective function values using different initial configurations

	5x5x5x5		10x10x10x10	
	Mean	Standard deviation	Mean	Standard deviation
All open	298,470,150	0	224,786,154	2,750,762
All closed	297,812,850	1,562,038	222,801,956	512,0944
Random	298,328,881	57,201	223,662,044	4,487,364
Net profit	298,355,119	17,236	242,092,760	429,700
Attraction	298,337,036	49,097	224,586,090	2,475,947

Because the objective function values did not differ greatly for the 5x5x5x5 problem, no further tests were performed. The values for the 10x10x10x10 showed a difference. A *t*-test on the mean was performed.

Table 30 p-values for *t*-test on means for 5x5x5x5

p-value	All open	All closed	Random	Net profit	Attraction
All open		23.85%	0.00%	0.00%	0.00%
All closed			34.79%	32.49%	34.06%
Random				21.70%	74.95%
Net profit					31.95%
Attraction					

Table 31 p-values for *t*-test on means for 10x10x10x10

p-value	All open	All closed	Random	Net profit	Attraction
All open		32.45%	53.21%	0.00%	87.31%
All closed			70.94%	0.00%	36.53%
Random				0.00%	59.71%
Net profit					0.00%
Attraction					

The t -test shows that there is a significance difference in the objective function values when using a start configuration based on the net profit. The value turns out to be more than 30% better in this model.

5.6 Conclusion

Simulated annealing is used in general to obtain fast and good solutions. The original algorithm had to be modified. Several modifications were made and the simulation was run on different model sizes. First, the mixed integer model was solved. The model works up to a size of about 4,000 variables. Second the algorithm was applied to the relaxed model. The limitations of the simulation are essentially determined by the values of temperature, cooling rate, time periods and facilities. Therefore it is better to use the annealing when a model has relatively fewer time periods. The algorithm provides a good approach for finding better solutions, knowing not to have reached the global optimum, but getting a near-optimal solution. A guideline for selecting the cooling rate and temperature can be derived from the results, in addition to the observation that selection of a good initial configuration also helps to improve the objective function value.

CHAPTER 6

CASE STUDY

6.1 Introduction

Banking business has changed significantly in the last two decades. Internet banking, low interest rates, , and monetary policies have brought many changes to the banking industry. Branch banking has lost market share and is faced with operational costs that are higher than cost in direct banking business. The cooperative financial associations also have similar problems. High operational costs are especially attributed to facilities and employees. On the other hand for distribution purposes, a high density of facilities is the key for business, especially for financial products requiring personal consulting. Almost all banks in Germany are currently reviewing their facility strategy – their strategy for spatial distribution. The model developed in this dissertation is applied to the facility network of Dortmunder Volksbank. Several scenarios are considered and a recommendation is developed based on the results. The next section describes the situation of Dortmunder Volksbank with respect to the German and the local market conditions including the objective of the bank's facility configuration. Next the model is developed. It is based on spatial data and data of the facilities as well on the products that are offered.

6.2 About Dortmund Volksbank

Dortmunder Volksbank has a history of more than 100 years. The bank's business is built on four pillars.

1. Direct contact with customers in facilities
2. Consultancy on special topics
3. Online banking and brokerage
4. ATMs and service terminals.

Dortmunder Volksbank currently operates 61 facilities in 5 different cities. 33 are located in a core area of Dortmund city. A set of additional ATMs is also available. With a balance sheet total of 6,321 billion EUR in 2015, the bank is ranked 11th out of 1,045 cooperative banks in Germany.

The goal of this case study is to define the strategic facility configuration for Dortmund Volksbank for the next 10 years. The bank has already started an initiative of facility restructuring, including the closure of three facilities. Currently the bank operates 33 facilities in the Dortmund region, but several mergers have made the bank grow in smaller cities around Dortmund. Originally Dortmund had 36 facilities but the recent closure of three, has resulted in the current 33. The location of ATMs and self-service terminals are not included in the model, but should be part of an additional analysis, beyond the scope of this thesis. 13 typical financial products are included and it is assumed that all employees are qualified equally and no special qualifications are

necessary for selling the products. It is also assumed that each employee has a capacity to sell 600 products a year. Facilities are rented and fixed costs are paid for rent. Costs occur for additional employees. The number of products sold per year and facilities were set to the capacity of the model. Demand occurs at 170 demand locations, which are the different districts of the city.

Figure 12 City of Dortmund and its districts (Stadt Dortmund, 3/Dez - Stabsstelle Dortmunder Statistik, 2015)

Three different scenarios were carried out.

Scenario 1: The model run was carried out for all 36 locations, including those already closed. Additional facilities were not included.

Scenario 2: The model run was performed for all 33 existing facilities. Additional facilities were not included.

Scenario 3: The model was run with all 33 existing facilities. 10 potential new locations were included.

6.3 The model

6.3.1 Gathering model data and database implementation

1) Demand data

Demand points have geographical coordinates. These coordinates were obtained manually from Google maps. Demands were taken from statistical data provided by the administration of the city of Dortmund (Stadt Dortmund, 3/Dez - Stabsstelle Dortmunder Statistik, 2015) and (Eckert, 2014). The sources contain relevant information on migration, changes in population, (un)employment rates, age structure, affinity towards certain banking products, amount of money invested in banking products and the need to be consulted for investment decisions.

2) Location data

All facilities have certain location data, such as rent, the number of employees and geographic coordinates. This data was taken to determine the fixed cost, opening cost, closing cost, expansion cost, shrinking cost and finally the attraction parameter. The

attraction parameter was generated by using the inverse of the squared (driving) distance between facility i and demand location j . The matrix was created using Google API. The script used for the generation³ was coded in Ruby. Appendix G has the script for generating the distance information.

3) Product data

The 13 relevant products are checking accounts, saving accounts, cash value life insurances, building savings agreements, mortgaging, call money, funds, fixed term deposits, savings plans, stocks, fixed-interest securities, gold and certificates. The demand data can be found in Appenix H.

4) Competitors data

Seven competitors with a total of 68 facilities are incorporated in the model. The highest attraction parameter with respect to a demand point was determined. Competitors are Sparkasse, Deutsche Bank, Targo Bank and Commerzbank. The relevant data can be found in Appendix H Data for case study.

The data was loaded into seven data base tables. The database structure used for the other models had to be modified as the number of locations, periods, demands and products were not equal. The data base structure is listed below.

³ <https://developers.google.com/maps/?hl=en>

Table 32 Database view " CS_COMPETITORS_VIEW"

	COLUMN_NAME	DATA_TYPE
1	COMPETITORSATTRACTION	FLOAT(126)
2	INDEXJ	NUMBER(3)
3	INDEXT	NUMBER(3)

Table 33 Database table " CS_ATTRACTIVITY_VIEW"

	COLUMN_NAME	DATA_TYPE
1	ATTRACTIVITY	NUMBER(30)
2	INDEXI	NUMBER(3)
3	INDEXJ	NUMBER(3)
4	INDEXT	NUMBER(3)

Table 34 Database table "CASE_STUDY_DATA_INITIALS_CAP"

	COLUMN_NAME	DATA_TYPE
1	INITIALCAP	FLOAT
2	INDEXI	FLOAT

Table 35 Database table "CASE_STUDY_INITIALS"

	COLUMN_NAME	DATA_TYPE
1	INITIALOPEN	FLOAT

Table 36 Database table "CS_PRODUCT_VIEW"

	COLUMN_NAME	DATA_TYPE
1	PRICE	FLOAT(20)
2	VARIABLECOST	FLOAT(20)
3	INDEXI	NUMBER(3)
4	INDEXT	NUMBER(3)
5	INDEXP	NUMBER(3)
6	UNITSHRINKAGECOST	FLOAT(126)
7	UNITEXPANSIONCOST	FLOAT(126)

Table 37 Database table "CASE_STUDY_DATA_FACILITIES_NPR"

	COLUMN_NAME	DATA_TYPE
1	EXPANSIONCOST	FLOAT

2	SHRINKAGECOST	FLOAT
3	FIXCOST	FLOAT
4	OPENINGCOST	FLOAT
5	CLOSINGCOST	FLOAT
6	NAME	VARCHAR2(60 BYTE)

Table 38 Database table "CS_DEMAND_VIEW"

	COLUMN_NAME	DATA_TYPE
1	DEMAND	NUMBER(30)
2	INDEXP	NUMBER(3)
3	INDEXT	NUMBER(3)
4	INDEXJ	NUMBER(3)

6.3.2 Mathematical formulation

Indexes

i	Set of facilities
j	Set of demand nodes
t	Set of time periods
p	Set of products

Model parameters

$initialCapacity_i$	The initial capacity of facility i in period I . The capacity represents the number of personal consultations that can be carried out
$initialOpen_i$	The initial status of facility i in period I
$price_{itp}$	The price for selling one unit of product p at facility i in period t
$variableCost_{itp}$	The unit cost for selling product p at facility i in period t
$fixCost_i$	The cost for having open facility i – assumed to be constant over all periods. The fix cost component contains especially the rent for the office space
$openingCost_i$	The cost for opening facility i . It is assumed that opening cost will not change over time
$closingCost_i$	The cost for closing facility i . It is assumed that closing cost will not change over time
$attraction_{ijt}$	Attraction value of own facility i with respect to demand node j in period t computed by $\frac{1}{d^2}$ where d represents the driving distance in minutes from demand node j to facility i in period t

$competitorsAttraction_{jt}$	The highest attraction of competitors for demand node j in period t . The value is computed using the same formula as for the attraction value
$expansionCost_i$	Fixed cost for expansion of facility i . The value represents basically recruiting cost and training.
$shrinkageCost_i$	Fixed cost for shrinking of facility i . This value contains cost for compensation
$unitExpansionCost_{itp}$	Unit (variable) cost for expansion of product p at facility i in period t , assumed to be 0
$unitShrinkageCost_{itp}$	Unit (variable) cost for shrinking of product p at facility i in period t , assumed to be 0
$demand_{jtp}$	Demand for product p at demand node j in period t
$step < X >_{it}$	The step-wise (capacity) when a modification of fix cost is necessary. The model incorporates ten potential steps, each representing an additional employee for the relevant facility.
$stepCost < X >_{it}$	The additional fix cost for a certain step if the capacity is expanded for the corresponding product p at facility i in period t , representing the annual wage
$assignment_{ijt}$	$= \begin{cases} 1, & \text{if } attraction_{ijt} \geq comp_{jt} \\ 0, & \text{else} \end{cases}$

This parameter is set to one if the attraction of a company's own facility i relative to a demand node j is greater or equal than the highest attraction of the competitor with respect to node j in period t . It is used for the assignment of facilities to a demand node in that period.

Model variables

$$open_{it} = \begin{cases} 1, & \text{if facility } i \text{ remains open in } t \\ 0, & \text{if facility } i \text{ is closed in } t \end{cases}$$

$$opened_{it} = \begin{cases} 1, & \text{if facility } i \text{ is opened in } t \\ 0, & \text{else} \end{cases}$$

$$closed_{it} = \begin{cases} 1, & \text{if facility } i \text{ is closed in } t \\ 0, & \text{else} \end{cases}$$

$$expanded_{it} = \begin{cases} 1, & \text{if facility } i \text{ is expanded in } t \\ 0, & \text{else} \end{cases}$$

$$shrunk_{it} = \begin{cases} 1, & \text{if facility } i \text{ is shrunk in } t \\ 0, & \text{else} \end{cases}$$

$$uPlus_{it} \quad \text{Expansion of capacity at facility } i \text{ in } t$$

$$uMinus_{it} \quad \text{Shrinkage of capacity at facility } i \text{ in } t$$

$$totalCapacity_{it} \quad \text{Maximum capacity at } i \text{ in period } t$$

$$usedCapacity_{itp} \quad \text{Used capacity in units of product } p \text{ at } i \text{ in period } t$$

$$totalFixCost_{it} \quad \text{The fixed cost for operating facility } i \text{ in period } t$$

$threshold < X >_{it}$ The variable that is indicating whether the costs for a certain threshold in terms of capacity have to be incorporated in the model. X is used for the corresponding step.

$thresholdSwitch < X >_{it}$ This variable is used for modeling purpose to set the threshold correctly for facility i in period t .

Objective function

$$\begin{aligned}
& \max \sum_{i=1}^{36} \sum_{t=1}^{10} \sum_{p=1}^{13} \left(usedCapacity_{itp} * (price_{itp} - variableCost_{itp}) \right) - \\
& \sum_{i=1}^{36} \sum_{t=1}^{10} (totalFixCost_{it} + closingCost_i * closed_{it} + openingCost_i * opened_{it}) - \\
& \sum_{i=1}^{36} \sum_{t=1}^{10} expansionCost_i * expanded_{it} - \sum_{i=1}^{36} \sum_{t=1}^{10} \sum_{p=1}^{13} unitExpansionCost_{itp} * \\
& uPlus_{it} - \\
& \sum_{i=1}^{36} \sum_{t=1}^{10} shrinkageCost_{it} * shrunk_{it} - \sum_{i=1}^{36} \sum_{t=1}^{10} \sum_{p=1}^{13} unitShrinkageCost_{itp} * \\
& uMinus_{it}
\end{aligned}$$

Subject to

- 1) $open_{i1} = initialOpen_i, \forall i$
- 2) $totalCapacity_{i1} = initialCapacity_i, \forall i$
- 3a) $step1_{it} = 600, \quad \forall i, t$
- 3b) $step2_{it} = 1200, \quad \forall i, t$
- 3c) $step3_{it} = 1800, \quad \forall i, t$
- 3d) $step4_{it} = 2400, \quad \forall i, t$

- 3e) $step5_{it} = 3000, \quad \forall i, t$
- 3f) $step6_{it} = 3600, \quad \forall i, t$
- 3g) $step7_{it} = 4200, \quad \forall i, t$
- 3h) $step8_{it} = 4800, \quad \forall i, t$
- 3i) $step9_{it} = 5400, \quad \forall i, t$
- 3j) $step10_{it} = 6000, \quad \forall i, t$
- 4a) $9999999 * threshold1_{it} \leq (totalCapacity_{it}/step1_{it}) - 1, \forall i, t$
- 4b) $9999999 * threshold2_{it} \leq (totalCapacity_{it}/step2_{it}) - 1, \forall i, t$
- 4c) $9999999 * threshold3_{it} \leq (totalCapacity_{it}/step3_{it}) - 1, \forall i, t$
- 4d) $9999999 * threshold4_{it} \leq (totalCapacity_{it}/step4_{it}) - 1, \forall i, t$
- 4e) $9999999 * threshold5_{it} \leq (totalCapacity_{it}/step5_{it}) - 1, \forall i, t$
- 4f) $9999999 * threshold6_{it} \leq (totalCapacity_{it}/step6_{it}) - 1, \forall i, t$
- 4g) $9999999 * threshold7_{it} \leq (totalCapacity_{it}/step7_{it}) - 1, \forall i, t$
- 4h) $9999999 * threshold8_{it} \leq (totalCapacity_{it}/step8_{it}) - 1, \forall i, t$
- 4i) $9999999 * threshold9_{it} \leq (totalCapacity_{it}/step9_{it}) - 1, \forall i, t$
- 4j) $9999999 * threshold10_{it} \leq (totalCapacity_{it}/step10_{it}) - 1, \forall i, t$
- 5a) $threshold1_{it} + open_{it} + thresholdSwitch1_{it} \leq 2, \forall i, t$
- 5b) $threshold2_{it} + open_{it} + thresholdSwitch2_{it} \leq 2, \forall i, t$
- 5c) $threshold3_{it} + open_{it} + thresholdSwitch3_{it} \leq 2, \forall i, t$
- 5d) $threshold4_{it} + open_{it} + thresholdSwitch4_{it} \leq 2, \forall i, t$

- 5e) $threshold5_{it} + open_{it} + thresholdSwitch5_{it} \leq 2, \forall i, t$
- 5f) $threshold6_{it} + open_{it} + thresholdSwitch6_{it} \leq 2, \forall i, t$
- 5g) $threshold7_{it} + open_{it} + thresholdSwitch7_{it} \leq 2, \forall i, t$
- 5h) $threshold8_{it} + open_{it} + thresholdSwitch8_{it} \leq 2, \forall i, t$
- 5i) $threshold9_{it} + open_{it} + thresholdSwitch9_{it} \leq 2, \forall i, t$
- 5j) $threshold10_{it} + open_{it} + thresholdSwitch10_{it} \leq 2, \forall i, t$
- 6a) $threshold1_{it} - thresholdSwitch1_{it} \geq 0, \forall i, t$
- 6b) $threshold2_{it} - thresholdSwitch2_{it} \geq 0, \forall i, t$
- 6c) $threshold3_{it} - thresholdSwitch3_{it} \geq 0, \forall i, t$
- 6d) $threshold4_{it} - thresholdSwitch4_{it} \geq 0, \forall i, t$
- 6e) $threshold5_{it} - thresholdSwitch5_{it} \geq 0, \forall i, t$
- 6f) $threshold6_{it} - thresholdSwitch6_{it} \geq 0, \forall i, t$
- 6g) $threshold7_{it} - thresholdSwitch7_{it} \geq 0, \forall i, t$
- 6h) $threshold8_{it} - thresholdSwitch8_{it} \geq 0, \forall i, t$
- 6i) $threshold9_{it} - thresholdSwitch9_{it} \geq 0, \forall i, t$
- 6j) $threshold10_{it} - thresholdSwitch10_{it} \geq 0, \forall i, t$

$$7) \quad totalFixCost_{it}$$

$$= open_{it} * fixCost_i + stepCost1_{it} * (threshold1_{it} - thresholdSwitch1_{it}) + stepCost2_{it} * (threshold2_{it} - thresholdSwitch2_{it}) + stepCost3_{it} * (threshold3_{it} - thresholdSwitch3_{it}) + stepCost4_{it} * (threshold4_{it} - thresholdSwitch4_{it}) + stepCost5_{it} * (threshold5_{it} - thresholdSwitch5_{it}) + stepCost6_{it} * (threshold6_{it} - thresholdSwitch6_{it}) + stepCost7_{it} * (threshold7_{it} - thresholdSwitch7_{it}) + stepCost8_{it} * (threshold8_{it} - thresholdSwitch8_{it}) + stepCost9_{it} * (threshold9_{it} - thresholdSwitch9_{it}) + stepCost10_{it} * (threshold10_{it} - thresholdSwitch10_{it}), \forall i, t$$

$$7) \quad \sum_{p=1}^{13} usedCapacity_{itp} \leq totalCapacity_{it}, \forall i, t$$

$$9) \quad usedCapacity_{itp} \leq \sum_{j=1}^{170} assignment_{ijt} * demand_{jtp}, \quad \forall i, t, p$$

$$10) \quad totalCapacity_{it+1} = totalCapacity_{it} + uPlus_{it} - uMinus_{it}, \quad \forall i, t$$

$$11) \quad open_{it+1} = open_{it} + opened_{it} - closed_{it}, \forall i, t$$

$$12) \quad 9999999 * expanded_{it} \geq uPlus_{it}, \forall i, t$$

$$13) \quad 9999999 * shrunk_{it} \geq uMinus_{it}, \forall i, t$$

$$14) \quad totalCapacity_{it} \geq uMinus_{it}, \forall i, t, p$$

- 15) $expanded_{it} \leq open_{it}, \forall i, t, p$
- 16) $shrunk_{itp} \leq open_{it}, \forall i, t, p$
- 17) $9999999 * open_{it} \geq usedCapacity_{it}, \forall i, t$
- 18) $\sum_{i=1}^{36} assignment_{ijt} \leq 1 \forall j, t$
- 19) $assignment_{ijt}, expanded_{itp}, shrunk_{itp}, open_{it}, opened_{it}, closed_{it}$
- 20) $uPlus_{itp}, uMinus_{itp}, totalCapacity_{itp}, usedCapacity_{itp} \in \mathbb{N}, \forall i, t$

The objective function maximizes the profit. It consists of six terms. The first term computes the net profit. The second term includes the fixed cost component when a facility is open, which is also determined by the step cost function. The third and fourth terms are for assigning a closing or opening cost that occurs if a facility is opened or closed, respectively. The last two terms consider the expansion or shrinkage cost. Constraints (1) and (2) are used to initialize the open status of each facility as well as the initial capacity. Constraint (3), (4), (5) and (6) are used for the step cost function. The total cost occurring in the relevant period t at facility i is calculated in constraint (7). Usage must never be greater than the capacity. This is ensured by constraint (8). This constraint is modified slightly such that all products are competing for the whole capacity in a facility. Constraint (9) addresses the usage again given that usage must never exceed the assigned demands. The capacity of the facilities might vary over time. Constraint (10) computes the capacity for the following period by adding the units of expansion or subtracting the units of shrinkage from the current. Constraint (11) is a key constraint and

is used for setting the status of a facility to open or close. Constraint (12) sets $expanded_{it}$ to 1 if the facility is expanded. Constraint (13) does the same for the contraction case. Constraint (14) ensures that the capacity will never become negative. Constraints (15) and (16) assert that only open facilities can be contracted or expanded, respectively. Constraint (17) ensures that capacity will only be used if a facility is open. Constraint (18) guarantees a maximum of one assignment per demand.

The corresponding LINGO model can be found in Appendix I LINGO file for case study model.

6.3.3 Computational results

Scenario 1: The problem uses 76,239 variables. It could not be solved in a reasonable time. First a decomposition approach was applied by splitting up the problem into 13 sub-problems and trying to solve for each product separately. The decomposition did not greatly reduce complexity. Each of the sub-models still consists of 71,919 variables because most of the complexity is derived by facilities and demand points, and not products. As discussed in Chapter 4, decomposition by demand will adjust the capacity of the facilities to the demand of the certain demand point. Thus, a decomposition approach by facilities was applied. 36 sub-models were created, except for facility 1 (2,111 variables) consisting of 2,120 variables. Solving times for each of the sub-models differ greatly. Several sub-models could not be solved in less than a day. The model assumes a facility cannot be reopened again once it has been closed. Thus 10 potential solutions for the open status are possible. Each of the configurations was applied to the sub-model

by adding additional constraints for the opening status, the solutions were obtained and the configuration with the highest objective function value was taken.

Configuration	period									
	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	1	1	1	0	0	0
5	1	1	1	1	1	1	0	0	0	0
6	1	1	1	1	1	0	0	0	0	0
7	1	1	1	1	0	0	0	0	0	0
8	1	1	1	0	0	0	0	0	0	0
9	1	1	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0

The following flow chart describes the procedure.

Figure 13 Procedure for solving sub-problems in case study

The solutions for the sub-problems were obtained. The results for the corresponding open statuses were written into the database.

Table 39 Open status for sub problems

INDEXT	1	2	3	4	5	6	7	8	9	10
F01	1	1	1	1	1	1	1	1	1	1
F02	1	1	1	1	1	1	1	1	1	1
F03	1	1	1	1	1	1	1	1	1	1
F04	1	1	1	1	1	1	1	1	1	1
F05	1	1	1	1	1	1	1	1	1	1
F06	1	1	1	1	1	1	1	1	1	1
F07	1	1	1	1	1	1	1	1	1	1
F08	1	1	1	1	1	1	1	1	1	1
F09	1	1	1	1	1	1	1	1	1	1
F10	1	1	1	1	1	1	1	1	1	1
F11	1	1	1	1	1	1	1	1	1	1
F12	1	1	1	1	1	1	1	1	1	1
F13	1	1	1	1	1	1	1	1	1	1
F14	1	1	1	1	1	1	1	1	0	0
F15	1	0	0	0	0	0	0	0	0	0
F16	1	0	0	0	0	0	0	0	0	0
F17	1	1	1	1	1	1	1	1	1	1
F18	1	1	1	1	1	1	1	1	1	1
F19	1	1	1	1	1	1	1	1	1	1
F20	1	0	0	0	0	0	0	0	0	0
F21	1	0	0	0	0	0	0	0	0	0
F22	1	0	0	0	0	0	0	0	0	0
F23	1	1	1	1	1	1	1	1	1	1
F24	1	1	1	1	1	1	1	1	1	1
F25	1	0	0	0	0	0	0	0	0	0
F26	1	1	1	1	1	1	1	1	1	1
F27	1	1	1	1	1	1	1	1	1	1
F28	1	1	1	1	1	1	1	1	1	1
F29	1	1	1	1	1	1	1	1	1	1
F30	1	1	1	1	1	1	1	1	1	1
F31	1	1	1	1	1	1	1	1	1	1
F32	1	1	1	1	1	1	1	1	1	1
F33	1	0	0	0	0	0	0	0	0	0
F34	1	1	1	1	1	1	1	1	1	1
F35	1	1	1	1	1	1	1	1	1	1
F36	1	1	1	1	1	1	1	1	1	1

The open statuses for all 36 sub-problems were added to the original problem and the original problem was solved. The following graphs illustrate the facility configuration in period 1 and in period 10. The locations are colored in blue, demand points are colored in violet. Assignments are illustrated by the use of edges. Grey colored nodes point out closed facilities. The size of the nodes visualizes the total capacity. The figures were created by means Gephi⁴.

Figure 14 Scenario 1: Facility configuration and assigned demands in period 1

⁴ <https://gephi.org/>

Figure 15 Scenario 1: Facility configuration and assigned demands in period 10

The model closes down 8 facilities in the sub-models. As the results of the sub-models require certain facilities to remain open, several facilities are contracted in the complete model and operated with the lowest capacity possible. The closing decisions differ from those made by Dortmunder Volksbank. The relevant facilities are still assigned demands even in period 10 of the model.

Scenario 2: The problem uses 69,885 variables. It could also not be solved in a reasonable amount of time. The open status of the relevant 33 remaining facilities was taken from the decomposition made for Scenario 1. The model was run including the solutions of the sub-problems. A solution was obtained in less than 5 minutes. Demand assignment differs over the periods. The facility sizes change over time and are almost all reduced to the minimum size by the end of period 10.

Table 40 Open status for sub-problems

INDEXT	1	2	3	4	5	6	7	8	9	10
F01	1	1	1	1	1	1	1	1	1	1
F02	1	1	1	1	1	1	1	1	1	1
F03	1	1	1	1	1	1	1	1	1	1
F04	1	1	1	1	1	1	1	1	1	1
F05	1	1	1	1	1	1	1	1	1	1
F06	1	1	1	1	1	1	1	1	1	1
F07	1	1	1	1	1	1	1	1	1	1
F08	1	1	1	1	1	1	1	1	1	1
F09	1	1	1	1	1	1	1	1	1	1
F10	1	1	1	1	1	1	1	1	1	1
F11	1	1	1	1	1	1	1	1	1	1
F12	1	1	1	1	1	1	1	1	1	1
F13	1	1	1	1	1	1	1	1	1	1
F14	1	1	1	1	1	1	1	1	0	0
F15	1	0	0	0	0	0	0	0	0	0
F16	1	0	0	0	0	0	0	0	0	0
F17	1	1	1	1	1	1	1	1	1	1
F18	1	1	1	1	1	1	1	1	1	1
F19	1	1	1	1	1	1	1	1	1	1
F20	1	0	0	0	0	0	0	0	0	0
F21	1	0	0	0	0	0	0	0	0	0
F22	1	0	0	0	0	0	0	0	0	0
F23	1	1	1	1	1	1	1	1	1	1
F24	1	1	1	1	1	1	1	1	1	1
F25	1	0	0	0	0	0	0	0	0	0
F26	1	1	1	1	1	1	1	1	1	1
F27	1	1	1	1	1	1	1	1	1	1
F28	1	1	1	1	1	1	1	1	1	1
F29	1	1	1	1	1	1	1	1	1	1

F30	1	1	1	1	1	1	1	1	1
F31	1	1	1	1	1	1	1	1	1
F32	1	1	1	1	1	1	1	1	1
F33	1	0	0	0	0	0	0	0	0
F34	1	1	1	1	1	1	1	1	1
F35	1	1	1	1	1	1	1	1	1
F36	1	1	1	1	1	1	1	1	1

Figure 16 Scenario 2: Facility configuration and assigned demands in period 1

Figure 17 Scenario 2: Facility configuration and assigned demands in period 5

Figure 18 Scenario 2: Facility configuration and assigned demands in period 10

Scenario 3: The problem consists of 91,065 variables. A feasible solution was gathered using the decomposition and LINGO. The following table shows the result of the sub-problems for the additional locations.

Table 41 Open status for potential new locations

INDEX	T	1	2	3	4	5	6	7	8	9	10
NL 1		1	1	1	1	1	1	1	1	1	1
NL 2		1	0	0	0	0	0	0	0	0	0
NL 3		1	1	1	1	1	1	1	1	1	1
NL 4		1	1	1	1	1	1	1	1	1	1
NL 5		1	0	0	0	0	0	0	0	0	0
NL 6		1	1	1	1	1	1	1	1	1	1
NL 7		1	1	1	1	1	1	1	1	1	1
NL 8		1	1	1	1	1	1	1	1	1	1
NL 9		1	1	1	1	1	1	1	1	1	1
NL 10		1	1	1	1	1	1	1	1	1	1

The feasible solution shows an objective function value of 0.31E08 which is better than the objective function value of the value for scenario 2, also ending up with low capacities for the new facilities.

6.3.4 Applying simulated annealing

Another attempt to solve the problem was made using simulated annealing. The SA algorithm developed in Chapter 5 allows a facility to be reopened again if it has previously been closed. The banking application does not allow a facility to be opened if it was closed in all the following periods. The SA algorithm had to be modified respectively.

The following pseudo code demonstrates how the adjusted algorithm works.

Algorithm simulatedAnnealing (temperature, cooling Rate)

Begin

 Step 1, Initialization

```
startConfiguration = feasibleConfiguration;  
bestEnergy = getEnergy(startConfiguration);  
bestConfiguration = startConfiguration;  
currentConfiguration = startConfiguration;  
currentEnergy = bestEnergy;
```

Step 2, Annealing

While temperature > 1 **Do**

```
    neighborConfiguration = createNewConfiguration(currentConfiguration);  
    neighborEnergy = getEnergy(neighborConfiguration);
```

If getAcceptanceProbability > *random number* **Then**

```
    currentConfiguration = neighborConfiguration;  
    currentEnergy = neighborEnergy;
```

End If

If currentEnergy > bestEnergy **Then**

```
    bestConfiguration = currentConfiguration;  
    bestEnergy = currentEnergy;
```

End If

```
temperature *= 1-coolingRate;
```

End While

Return bestEnergy, bestConfiguration;

End

Procedure getAcceptanceProbability (energy, newEnergy, temperature)

Begin Procedure

If newEnergy > energy **then**

Return 1.0

Else

Return $e^{\frac{-(energy - newEnergy)}{temperature}}$

End if

End Procedure

Procedure createNewConfiguration (currentConfiguration)

indexi = *RandomNumber**facilities;

indext = 0;

While indext = 0 **Do**

indext = *RandomNumber**periods;

End While

newConfiguration = currentConfiguration;

For each t1 := indext; t1 < periods **Do**

newConfiguration[indexi][t1] = 0;

End For

For each t2 := indext; t1 <= indext **Do**

newConfiguration[indexi][t2] = 1;

End For

return newConfiguration;

End Procedure

Procedure getEnergy(configuration)

Load the LINGO model;

Retrieve variable indices;

For each variableIndex **Do**

Add constraint from configuration;

End For

Solve the model;

Return objective function value;

End Procedure

The corresponding JAVA code can be found in Appendix J.

Solving the problem was not possible within 100 hours. In order to get an upper bound, the relaxed model was passed to the SA algorithm. The SA algorithm was run using two different initial configurations. The first declared all facilities open for all periods. The second declared all facilities closed for all periods except period 1.. The headquarter ($i = 1$) was also declared to be open for all periods.

The SA algorithm was run with a temperature of 30,000,000 and a cooling rate of 0.01. It is obvious that the objective function value having all facilities open is much better than starting with having all facilities except the headquarter closed after the first period.

6.4 Conclusion

Dortmunder Volksbank is faced with a significant change of business and as a consequence, their entire facility landscape. The results show that the facility model is complex. Closure decisions in the past are not confirmed by the model. The complete model depends on the competition. As soon as a competitor is removed, the model shows

a different behavior. The same thing happens after closure of one of the bank's own facility.

Solving the model for each facility using a decomposition approach could be run in a reasonable time. It is important to combine the solutions carefully. Solving for each facility resulted in assigning several demands more than once. Thus, it could be part of an additional analysis to compose the solutions of the sub-problems in a different way.

Keeping the option for opening additional facilities gives the entire model more degrees of freedom. If a facility has to be closed, in most cases the best objective value can be obtained when the closure is done as soon as possible.

Simulated annealing is not easy to apply to the model, as the problem is complex. The algorithm had to be modified when a facility is not re-opened.

Furthermore it is to be expected that customers will patronize a facility not only because of its proximity. Several other aspects need to be considered and are currently not incorporated in the model. Finally the model can show certain recommendations for the facility settings.

CHAPTER 7

CONCLUSIONS

7.1 Contributions

This dissertation provides several contributions.

- 1) First, the models presented provide an extension of existing facility location models that can be applied to a variety of location problems in commercial and industry sectors that need to make their decisions considering future time periods and competitors. An initial model focusing on a single product was developed. The model was extended by including multiple products. The developed heuristic shows multiple options for solving the problem including the advantages and disadvantages, respectively. The JAVA code and LINGO fragments thus developed can be used to provide easy access for related problems.
- 2) The case study shows the application of the problem in a real world environment. Slight modifications make it easy to customize the model to the area of interest. Given the real data of Dortmunder Volksbank, several solution approaches and scenarios give detailed advice for the bank's facility decisions.

7.2 Further Works

The research in this dissertation gives direction to further topics that are of great interest and can be very useful for the daily application of the model.

7.2.1 Extension of the attraction function

The attraction parameter plays a central role in the assignment of a facility. The model takes it as a given parameter. The case study computes the parameter using the distances. Facility attraction depends on several factors. The model can be extended by setting up an attraction function which incorporates additional parameters and may change over time – depending on the decisions made by the model. A utility function can be used for model purposes.

7.2.2 Incorporating additional constraints

The current model uses several constraints. With respect to certain applications, additional constraints may become necessary, such as subsidies, taxes, space limitations, laws, etc. The development of these constraints will make the model more realistic, but for every problem the model needs to be adjusted to the relevant use case.

7.2.3 Combination of solutions

Taking additional analysis into account – such as the location of ATMs, the combination of analyses will give the model and other analyses additional value. The location of ATMs was not incorporated in the case study, but it is of greater interest to find out

where they may located best. A facility location will undoubtedly also influence the location of an ATM. Dependency of the two models is to be expected.

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Appendix A The Lingo File of model 1

Model:

```
!Variables

assignment: Assignment variable
expanded: Set to 1 if facility is expanded
shrunk: Set to 1 if facility is shrunk
uPlus: Units of expansion
uMinus: Units of shrinkage
totalCapacity: Capacity of facility
open: Have facility open
opened: Open facility at end of period
closed: Close down facility at end of period
usedCapacity: The used capacity at the facility in certain period ;

!Parameters
initialCapacity: Initial Capacity of facility in period 1
initialOpen: Initial setting for having open facility in period 1
price: Price for selling
variableCost: Unit cost for selling
openingCost: Cost for opening
closingCost: Cost for closing
fixCost: cost for an open facility in a period
attraction: Attraction index of own facility
competitorsAttraction: Attraction index of competitors
expansionCost: Fix cost for expansion
shrinkageCost: Fix cost for shrinkage
unitExpansionCost: unit (variable) cost for expanding
unitShrinkageCost: unit (variable) cost for shrinkage
demand: demand at demand location;

Sets:
!Set of facilities;
facilities /1..10/ : initialOpen, initialCapacity;

!Set of demand points;
demands /1..10/;

!Set of time periods;
periods /1..10/;

indexijt (facilities, demands, periods): assignment, attraction;
indexij (facilities, demands);
indexit (facilities, demands): expanded, shrunk, totalCapacity,
expansionCost, shrinkageCost, unitExpansionCost, unitShrinkageCost,
```

```

uPlus, uMinus, usedCapacity, price, variableCost, fixCost, openingCost,
closingCost, open, opened, closed;
    indexjt (demands, periods): competitorsAttraction, demand;

```

Endsets

```

@for(facilities(i):
    @for(demands(j):
        @for(periods(t):
            @bin(assignment(i,j,t));
            @bin(expanded(i,t));
            @bin(shrunk(i,t));
            @bin(open(i,t));
            @bin(opened(i,t));
            @bin(closed(i,t));
            @gin(uPlus(i,t));
            @gin(uMinus(i,t));
            @gin(totalCapacity(i,t));

        ));

```

Data:

```

attraction = @OLE('GD_10x10x10_VER1.xls');
competitorsAttraction = @OLE('GD_10x10x10_VER1.xls');
initialCapacity = @OLE('GD_10x10x10_VER1.xls');
initialOpen = @OLE('GD_10x10x10_VER1.xls');
variableCost = @OLE('GD_10x10x10_VER1.xls');
fixCost = @OLE('GD_10x10x10_VER1.xls');
openingCost = @OLE('GD_10x10x10_VER1.xls');
closingCost = @OLE('GD_10x10x10_VER1.xls');
unitExpansionCost = @OLE('GD_10x10x10_VER1.xls');
expansionCost = @OLE('GD_10x10x10_VER1.xls');
unitShrinkageCost = @OLE('GD_10x10x10_VER1.xls');
shrinkageCost = @OLE('GD_10x10x10_VER1.xls');
price = @OLE('GD_10x10x10_VER1.xls');
demand = @OLE('GD_10x10x10_VER1.xls');

```

enddata

```

    max = @sum(indexit(i,t): usedCapacity(i,t)*(price(i,t) -
variableCost(i,t)) - fixCost(i,t)*open(i,t) -
closingcost(i,t)*closed(i,t) - openingcost(i,t)*opened(i,t)) -
@sum(indexit(i,t): expansionCost(i,t)*expanded(i,t)) -
@sum(indexit(i,t): unitExpansionCost(i,t)*uPlus(i,t)) -
@sum(indexit(i,t): shrinkageCost(i,t)*shrunk(i,t)) - @sum(indexit(i,t):
unitShrinkageCost(i,t)*uMinus(i,t)) ;

```

```

!Initializing open and Capacity data;
@for(facilities(i):
    open(i,1)= initialOpen;
    totalCapacity(i,1) = initialCapacity);

!Only assign if higher attraction level than competitor;
@for(indexijt(i,j,t):

```

```

        (assignment(i,j,t))*(attraction(i,j,t)-
competitorsAttraction(j,t))>= 0);

!Usage does not exceed the capacity at i in t;
@for(facilities(i):
    @for( periods(t): usedCapacity(i,t) <= totalCapacity(i,t)));

!Total demand will always be greater or equal than total usage;
@for(facilities(i):
    @for( periods(t): usedCapacity(i,t) <= @sum(demands(j):
(assignment(i,j,t)*demand(j,t)))));

!Capacity if the facility is determined by expansion or shrinking
in the previous period;
@for(facilities(i):
    @for( periods(t)|(t) #NE#10 :
        totalCapacity(i,t+1) = totalCapacity(i,t) + uPlus(i,t)
- uMinus(i,t)));

!Central setting of open, opening or closing down;
@for(facilities(i):
    @for( periods(t)|(t) #NE#10 : open(i,t+1) = open(i,t) +
opened(i,t) - closed(i,t)));

!If is expanded, expanded must be set to 1;
@for(facilities(i):
    @for( periods(t):
        10000000*expanded(i,t) >= uPlus(i,t)));

!If is shrunk, shrunk must be set to 1;
@for(facilities(i):
    @for( periods(t):
        10000000*shrunk(i,t) >= uMinus(i,t)));

!Capacity must not be negative;
@for(facilities(i):
    @for( periods(t):
        totalCapacity(i,t) >= 0));

!Capacity must be bigger than shrinkage;
@for(facilities(i):
    @for( periods(t):
        totalCapacity(i,t) >= uMinus(i,t)));

!Only open facilities can be expanded;
@for(facilities(i):
    @for( periods(t):
        expanded(i,t) <= open(i,t)));

!Only open facilities can be shrunk;
@for(facilities(i):
    @for( periods(t):
        shrunk(i,t) <= open(i,t)));

!Only have used capacity if open ;

```

```

@for(facilities(i):
    @for( periods(t):
        10000000*open(i,t) >= usedCapacity(i,t));

!Only have a maximum of one assignment ;
@for(demands(i):
    @for( periods(t):
        @sum(facilities(i): assignment(i,j,t)) <= 1));

DATA:

@OLE('output_robust.xlsx', 'OPEN') = open;
@OLE('output_robust.xlsx', 'TOTALCAPACITY') = totalCapacity;

ENDDATA

```

Appendix B The Lingo file of model 1 using a dynamic approach

Model:

```
!Variables

assignment: Assignment variable
expanded: Set to 1 if facility is expanded
shrunk: Set to 1 if facility is shrunk
uPlus: Units of expansion
uMinus: Units of shrinkage
totalCapacity: Capacity of facility
open: Have facility open
opened: Open facility at end of period
closed: Close down facility at end of period
usedCapacity: The used capacity at the facility in certain period ;

!Parameters
initialCapacity: Initial Capacity of facility in period 1
initialOpen: Initial setting for having open facility in period 1
price: Price for selling
variableCost: Unit cost for selling
openingCost: Cost for opening
closingCost: Cost for closing
fixCost: cost for an open facility in a period
attraction: Attraction index of own facility
competitorsAttraction: Attraction index of competitors
expansionCost: Fix cost for expansion
shrinkageCost: Fix cost for shrinkage
unitExpansionCost: unit (variable) cost for expanding
unitShrinkageCost: unit (variable) cost for shrinkage
demand: demand at demand location;

Sets:
!Set of facilities;
facilities /1..10/ : initialOpen1, initialCapacity1;

!Set of demand points;
demands /1..10/;

!Set of time periods;
periods /1..2/;

indexijt (facilities, demands, periods): assignment, attraction1;
indexij (facilities, demands);
indexit (facilities, periods): expanded, shrunk, totalCapacity,
expansionCost1, shrinkageCost1, unitExpansionCost1, unitShrinkageCost1,
```

```

uPlus, uMinus, usedCapacity, price1, variableCost1, fixCost1,
openingCost1, closingCost1, open, opened, closed;
    indexjt (demands, periods): competitorsattraction1, demand1;

```

Endsets

```

@for(facilities(i):
    @for(demands(j):
        @for(periods(t):
            @bin(assignment(i,j,t));
            @bin(expanded(i,t));
            @bin(shrunk(i,t));
            @bin(open(i,t));
            @bin(opened(i,t));
            @bin(closed(i,t));
            @gin(uPlus(i,t));
            @gin(uMinus(i,t));
            @gin(totalCapacity(i,t));

        ));

```

Data:

```

attraction1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
competitorsAttraction1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
initialCapacity1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
initialOpen1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
variableCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
fixCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
openingCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
closingCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
unitExpansionCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
expansionCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
unitShrinkageCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
shrinkageCost1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
price1 = @OLE('GD_10x10x10_VER1_dynamic.xls');
demand1 = @OLE('GD_10x10x10_VER1_dynamic.xls');

```

enddata

```

    max = @sum(indexit(i,t): usedCapacity(i,t)*(price1(i,t) -
variableCost1(i,t)) - fixCost1(i,t)*open(i,t) -
closingCost1(i,t)*closed(i,t) - openingCost1(i,t)*opened(i,t)) -
@sum(indexit(i,t): expansionCost1(i,t)*expanded(i,t)) -
@sum(indexit(i,t): unitExpansionCost1(i,t)*uPlus(i,t)) -
@sum(indexit(i,t): shrinkageCost1(i,t)*shrunk(i,t)) - @sum(indexit(i,t):
unitShrinkageCost1(i,t)*uMinus(i,t)) ;

```

```

!Initializing open and Capacity data;

```

```

@for(facilities(i):
    open(i,1)= initialOpen1;
    totalCapacity(i,1) = initialCapacity1);

```

```

!Only assign if higher attraction level than competitor;

```

```

@for(indexijt(i,j,t):

```

```

        (assignment(i,j,t))*(attraction1(i,j,t)-
competitorsattraction1(j,t))>= 0);

!Usage does not exceed the capacity at i in t;
@for(facilities(i):
    @for( periods(t): usedCapacity(i,t) <= totalCapacity(i,t)));

!Total demand will always be greater or equal than total usage;
@for(facilities(i):
    @for( periods(t): usedCapacity(i,t) <= @sum(demands(j):
(assignment(i,j,t)*demand1(j,t)))));

!Capacity if the facility is determined by expansion or shrinking
in the previous period;
@for(facilities(i):
    @for( periods(t)|(t) #NE#2 :
        totalCapacity(i,t+1) = totalCapacity(i,t) + uPlus(i,t)
- uMinus(i,t)));

!Central setting of open, opening or closing down;
@for(facilities(i):
    @for( periods(t)|(t) #NE#2 : open(i,t+1) = open(i,t) +
opened(i,t) - closed(i,t)));

!If is expanded, expanded must be set to 1;
@for(facilities(i):
    @for( periods(t):
        10000000*expanded(i,t) >= uPlus(i,t)));

!If is shrunk, shrunk must be set to 1;
@for(facilities(i):
    @for( periods(t):
        10000000*shrunk(i,t) >= uMinus(i,t)));

!Capacity must not be negative;
@for(facilities(i):
    @for( periods(t):
        totalCapacity(i,t) >= 0));

!Capacity must be bigger than shrinkage;
@for(facilities(i):
    @for( periods(t):
        totalCapacity(i,t) >= uMinus(i,t)));

!Only open facilities can be expanded;
@for(facilities(i):
    @for( periods(t):
        expanded(i,t) <= open(i,t)));

!Only open facilities can be shrunk;
@for(facilities(i):
    @for( periods(t):
        shrunk(i,t) <= open(i,t)));

!Only have used capacity if open ;

```

```

@for(facilities(i):
    @for( periods(t):
        10000000*open(i,t) >= usedCapacity(i,t));

!Only have a maximum of one assignment ;
@for(demands(j):
    @for( periods(t):
        @sum(facilities(i): assignment(i,j,t)) <= 1));

DATA:

    @OLE('output_dynamic_VER1.xlsx', 'OPEN2') = open;
    @OLE('output_dynamic_VER1.xlsx', 'TOTALCAPACITY2') =
totalCapacity;

ENDDATA
END

```


Appendix C Data used for comparison of robust and dynamic solution approach

Table 42 Data used for first test - keeping demand and prices constant over time

Initial Capacity	
initialCap	
Facility (i)	InitialCapacity
1	5000
2	5000
3	5000
4	5000
5	5000
6	5000
7	5000
8	5000
9	5000
10	5000

Open settings for period 1	
initialOpen	
Facility (i)	initalOpen
1	1
2	1
3	1
4	1
5	1
6	1
7	1
8	1
9	1
10	1

Price for selling one unit at i in t										
Facility (i)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	250	250	250	250	250	250	250	250	250	250
2	250	250	250	250	250	250	250	250	250	250
3	250	250	250	250	250	250	250	250	250	250
4	250	250	250	250	250	250	250	250	250	250
5	250	250	250	250	250	250	250	250	250	250
6	250	250	250	250	250	250	250	250	250	250
7	250	250	250	250	250	250	250	250	250	250
8	250	250	250	250	250	250	250	250	250	250
9	250	250	250	250	250	250	250	250	250	250
10	250	250	250	250	250	250	250	250	250	250

Variable cost for selling in period t										
Facility (i)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	150	150	150	150	150	150	150	150	150	150
2	150	150	150	150	150	150	150	150	150	150
3	150	150	150	150	150	150	150	150	150	150
4	150	150	150	150	150	150	150	150	150	150
5	150	150	150	150	150	150	150	150	150	150
6	150	150	150	150	150	150	150	150	150	150
7	150	150	150	150	150	150	150	150	150	150
8	150	150	150	150	150	150	150	150	150	150
9	150	150	150	150	150	150	150	150	150	150
10	150	150	150	150	150	150	150	150	150	150

Unit cost for expanding i in period t										
Facility (i)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
2	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

3	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
4	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
5	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
6	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
7	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
8	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
9	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
10	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

Unit cost for shrinking i in period t

unit
Shrink
ageCos
t

Period (t)

Facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	10	10	10	10	10	10	10	10	10	10
2	10	10	10	10	10	10	10	10	10	10
3	10	10	10	10	10	10	10	10	10	10
4	10	10	10	10	10	10	10	10	10	10
5	10	10	10	10	10	10	10	10	10	10
6	10	10	10	10	10	10	10	10	10	10
7	10	10	10	10	10	10	10	10	10	10
8	10	10	10	10	10	10	10	10	10	10
9	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10

Expansion cost for i in t

expans
ionCos
t

Period (t)

Facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
2	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
3	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
4	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
5	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
6	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000

7	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
8	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
9	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
10	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000

Shrinkage cost for i in t										
Facility (i)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
2	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
3	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
4	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
5	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
6	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
7	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
8	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
9	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
10	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000

Demand at j in t										
Demand (j)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
2	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
3	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
4	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
5	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
6	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
7	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
8	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
9	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
10	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000

Attraction value for facility i for demand point j in period t										
	demand 1									
	Period (t)									
facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	30	30	30	30	30	30	30	30	30	30
2	80	81	81	81	81	81	81	81	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
	demand 2									
	Period (t)									
facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	40	40	40	40	40	40	40	40	40	40
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
	demand 3									
	Period (t)									
facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	50	50	50	50	50	50	50	50	50	50
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
	demand 4									
	Period (t)									
facilit	1	2	3	4	5	6	7	8	9	10

y (i)										
1	60	60	60	60	60	60	60	60	60	60
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2

demand 5

Period (t)

facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	70	70	70	70	70	70	70	70	70	70
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2

demand 6

Period (t)

facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	80	80	80	80	80	80	80	80	80	80
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2

demand 7

Period (t)

facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	90	90	90	90	90	90	90	90	90	90
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60

4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
	demand 8									
	Period (t)									

facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	90	90	90	90	90	90	90	90	90	90
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
	demand 9									
	Period (t)									

facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	90	90	90	90	90	90	90	90	90	90
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2
8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2
	demand 10									
	Period (t)									

facilit y (i)	1	2	3	4	5	6	7	8	9	10
1	90	90	90	90	90	90	90	90	90	90
2	80	80	80	80	80	80	80	80	80	80
3	60	60	60	60	60	60	60	60	60	60
4	60	60	60	60	60	60	60	60	60	60
5	60	60	60	60	60	60	60	60	60	60
6	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2

8	2	2	2	2	2	2	2	2	2	2
9	2	2	2	2	2	2	2	2	2	2
10	2	2	2	2	2	2	2	2	2	2

Highest attraction value of competitor for demand j in period t										
competitorsAttraction demand (j)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	53	53	53	53	53	53	53	53	53	53
2	73	73	73	73	73	73	73	73	73	73
3	53	53	53	53	53	53	53	53	53	53
4	79	79	79	79	79	79	79	79	79	79
5	11	11	11	11	11	11	11	11	11	11
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1

Fix cost for running facility i in period t										
Facility (i)	Period (t)									
	1	2	3	4	5	6	7	8	9	10
1	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
2	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
3	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
4	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
5	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
6	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
7	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
8	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
9	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000
10	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000	500,000

Opening cost for i in t										
opening										
Cost										
Period (t)										
Facility										
(i)	1	2	3	4	5	6	7	8	9	10
1	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
2	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
3	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
4	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
5	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
6	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
7	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
8	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
9	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000
10	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000

Closing cost for i in t										
closing										
Cost										
Period (t)										
Facility										
(i)	1	2	3	4	5	6	7	8	9	10
1	4,961	5,940	4,741	5,119	4,411	5,071	5,132	4,631	5,810	4,421
2	5,596	4,687	5,266	4,528	5,273	4,428	5,307	4,910	5,194	5,023
3	5,246	5,389	4,460	4,682	5,455	4,644	5,056	5,114	4,570	5,243
4	4,679	4,182	4,388	5,076	4,271	5,008	4,655	4,896	5,371	4,712
5	3,989	4,854	5,192	4,905	5,128	4,154	5,454	5,007	5,180	5,294
6	5,293	5,356	5,436	5,244	5,044	4,783	5,798	5,362	4,547	5,379
7	5,732	5,190	4,799	4,312	5,241	4,713	4,625	5,023	5,172	4,959
8	4,507	5,268	5,543	4,665	6,141	4,990	5,181	4,559	4,712	6,372
9	4,049	4,526	4,059	4,637	5,092	4,193	4,691	4,567	4,582	4,737
10	5,005	5,024	5,869	5,114	5,033	4,451	5,109	4,925	5,065	5,077

Table 43 Data used for second test - Alternating demand and keeping prices constant over time

Demand at j in t										
demand										
Period (t)										
demand										
(j)	1	2	3	4	5	6	7	8	9	10
1	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0

2	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
3	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
4	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
5	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
6	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
7	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
8	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
9	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0
10	10,000	0	10,000	0	10,000	0	10,000	0	10,000	0

Table 44 Data used for third test - Alternating prices and keeping demand constant over time

Price for selling one unit at i in t										
price		Period (t)								
Facility										
(i)	1	2	3	4	5	6	7	8	9	10
1	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
2	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
3	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
4	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
5	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
6	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
7	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
8	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
9	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50
10	1,100	50	1,100	50	1,100	50	1,100	50	1,100	50

Table 45 Data used for forth test - Contantly sinking demand

Demand at j in t										
demand		Period (t)								
demand										
(j)	1	2	3	4	5	6	7	8	9	10
1	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
2	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
3	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
4	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
5	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
6	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000

7	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
8	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
9	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000
10	10,000	9,000	8,000	7,000	6,000	5,000	4,000	3,000	2,000	1,000

Appendix D The Lingo File of model 2

Model:

!Variables

assignment: Assignment variable
expanded: Expand
shrunk: Shrink
uPlus: Units of expansion
uMinus: Units of shrinkage
totalCapacity: totalCapacity of facility
open: Have facility open
opened: Open facility at end of period
closed: Close down facility at end of period
usedCapacity: The used Capacity at the facility in certain period ;

!Parameters

initalcapacity: Initial Capacity of faciltiy in period 1
initialOpen: Initial setting for having open facility in period 1
price: Price for selling
variableCost: Unit cost for selling
openingCost: Cost for opening
closingCost: Cost for closing
fixCost: cost for an open facility in a period
attraction: Attraction index of own facility
competitorsAttraction: Attraction index of competitors
expansionCost: Fix cost for expansion
shrinkageCost: Fix cost for shrinkage
unitExpansionCost: unit (variable) cost for expanding
unitShrinkageCost: unit (variable) cost for shrinkage
demand: demand at demand location ;

Sets:

!Set of facilities;
locations /1..10/:initialOpen;

!Set of demand points;
demands /1..10/;

!Set of time periods;
periods /1..10/;

!Set of products;
products /1..10/;

indexip (locations, products): initialCapacity;

```

indexijtp (locations, demands, periods, products): assignment;
indexijp (locations, demands, products);
indexitp (locations, periods, products): expanded, shrunk,
attraction, unitExpansionCost, totalCapacity, unitShrinkageCost, uPlus,
uMinus, usedCapacity, price, variableCost;
indexjtp (demands, periods, products): competitorsAttraction,
demand;
indexit (locations, periods): open, opened, closed, fixCost,
expansionCost, shrinkageCost, openingCost, closingCost;

```

Endsets

```

@for(locations(i):
    @for(demands(j):
        @for(periods(t):
            @for(products(p):
                @bin(assignment(i,j,t,p));
                @bin(expanded(i,t,p));
                @bin(shrunk(i,t,p));
                @bin(open(i,t));
                @bin(opened(i,t));
                @bin(closed(i,t));
                @gin(uPlus(i,t,p));
                @gin(uMinus(i,t,p));
                @gin(totalCapacity(i,t,p));
            )
        )
    )
);

```

Data:

```

competitorsAttraction =
@odbc('dissertation','modeldata_mp_10_view','COMPETITORSATTRACTION');
attraction =
@odbc('dissertation','modeldata_mp_10_view','ATTRACTIVITY');
initialCapacity =
@odbc('dissertation','modeldata_mp_10_INITIALS_CAP','INITIALCAP');
initialOpen =
@odbc('dissertation','modeldata_mp_10_INITIALS','INITIALOPEN');
variableCost =
@odbc('dissertation','modeldata_mp_10_view','VARIABLECOST');
fixCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','FIXCOST');
openingCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','OPENINGCOST');
closingCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','CLOSINGCOST');
unitExpansionCost =
@odbc('dissertation','modeldata_mp_10_view','UNITEXPANSIONCOST');

```

```

        expansionCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','EXPANSIONCOST');
        unitShrinkageCost =
@odbc('dissertation','modeldata_mp_10_view','UNITSHRINKAGECOST');
        shrinkageCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','SHRINKAGECOST');
        price = @odbc('dissertation','modeldata_mp_10_view','PRICE');
        demand = @odbc('dissertation','modeldata_mp_10_view','DEMAND');

enddata

        max = @sum(indexitp(i,t,p): usedCapacity(i,t,p)*(price(i,t,p) -
variableCost(i,t,p)) - fixCost(i,t)*open(i,t) -
closingcost(i,t)*closed(i,t) - openingcost(i,t)*opened(i,t)) -
@sum(indexitp(i,t,p): expansionCost(i,t)*expanded(i,t,p)) -
@sum(indexitp(i,t,p): unitExpansionCost(i,t,p)*uPlus(i,t,p)) -
@sum(indexitp(i,t,p): shrinkageCost(i,t)*shrunk(i,t,p)) -
@sum(indexitp(i,t,p): unitShrinkageCost(i,t,p)*uMinus(i,t,p));

        !Initializing open and totalCapacity data;
@for(locations(i):
            open(i,1)= initialOpen;
        );

@for(locations(i):
            @for(products(p):

                totalCapacity(i,1,p) = initialCapacity;

            )
        );

        !Only assign if higher attraction level than competitor;
@for(indexijtp(i,j,t,p):
            (assignment(i,j,t,p))*(attraction(i,t,p)-
competitorsAttraction(j,t,p))>= 0);

        !Usage does not exceed the totalCapacity at i in t;
@for(locations(i):
            @for(periods(t):
                @for(products(p):
                    usedCapacity(i,t,p) <= totalCapacity(i,t,p)
                )
            )
        );

        !Usage will never be greater than demand;
@for(locations(i):
            @for(periods(t):
                @for(products(p):
                    usedCapacity(i,t,p) <= @sum(demands(j):
(assignment(i,j,t,p)*demand(j,t,p))
                )
            )
        );

```

```

    )
);

!totalCapacity if the facility is determined by expansion or
shrinking in the previous period;
@for(locations(i):
    @for( periods(t) | (t) #NE#10:
        @for( products(p):
            totalCapacity(i,t+1,p) = totalCapacity(i,t,p) +
uPlus(i,t,p) - uMinus(i,t,p)
        )
    )
);

!Central setting of open, opening or closing down;
@for(locations(i):
    @for( periods(t) | (t) #NE#10: open(i,t+1) = open(i,t) +
opened(i,t) - closed(i,t)));

!If is expanded, expanded must be set to 1;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            999999*expanded(i,t,p) >= uPlus(i,t,p)
        )
    )
);

!If is shrunk, shrunk must be set to 1;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            999999*shrunk(i,t,p) >= uMinus(i,t,p)
        )
    )
);

!totalCapacity must be bigger than shrinkage;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            totalCapacity(i,t,p) >= uMinus(i,t,p)
        )
    )
);

!Only open facilities can be expanded;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            expanded(i,t,p) <= open(i,t)
        )
    )
);

```

```

    )
);

!Only open facilities can be shrunk;
@for(locations(i):
    @for(periods(t):
        @for(products(p):
            shrunk(i,t,p) <= open(i,t)
        )
    )
);

!Only have used Capacity if open ;
@for(locations(i):
    @for(periods(t):
        @for(products(p):
            999999*open(i,t) >= usedCapacity(i,t,p)
        )
    )
);

!Only have a maximum of one assignment ;
@for(demands(j):
    @for(periods(t):
        @for(products(p):
            @sum(locations(i): assignment(i,j,t,p)) <= 1));

end

```


Appendix E The Lingo File of model 2 including extensions

Model:

!Variables

assignment: Assignment variable
expanded: Expand
shrunk: Shrink
uPlus: Units of expansion
uMinus: Units of shrinkage
totalCapacity: totalCapacity of facility
open: Have facility open
opened: Open facility at end of period
closed: Close down facility at end of period
usedCapacity: The used Capacity at the facility in certain period ;

!Parameters

initalcapacity: Initial Capacity of faciltiy in period 1
initialOpen: Initial setting for having open facility in period 1
price: Price for selling
variableCost: Unit cost for selling
openingCost: Cost for opening
closingCost: Cost for closing
fixCost: cost for an open facility in a period
attraction: Attraction index of own facility
competitorsAttraction: Attraction index of competitors
expansionCost: Fix cost for expansion
shrinkageCost: Fix cost for shrinkage
unitExpansionCost: unit (variable) cost for expanding
unitShrinkageCost: unit (variable) cost for shrinkage
demand: demand at demand location ;

Sets:

!Set of facilities;
locations /1..10/:initialOpen;

!Set of demand points;
demands /1..10/;

!Set of time periods;
periods /1..10/;

!Set of products;
products /1..10/;

indexip (locations, products): initialCapacity;

```

        indexijtp (locations, demands, periods, products): assignment;
        indexijp (locations, demands, products);
        indexitp (locations, periods, products): expanded, shrunk,
attraction, unitExpansionCost, totalCapacity, unitShrinkageCost, uPlus,
uMinus, usedCapacity, price, variableCost, threshold1, threshold2,
threshold3, threshold4, threshold5, threshold6, threshold7,
thresholdSwitch1, thresholdSwitch2, thresholdSwitch3, thresholdSwitch4,
thresholdSwitch5, thresholdSwitch6, thresholdSwitch7, step1, step2,
step3, step4, step5, step6, step7, stepCost1, stepCost2, stepCost3,
stepCost4, stepCost5, stepCost6, stepCost7;
        indexjtp (demands, periods, products): competitorsAttraction,
demand;
        indexit (locations, periods): open, opened, closed, fixCost,
expansionCost, shrinkageCost, openingCost, closingCost, totalFixCost;

```

Endsets

```

@for(locations(i):
    @for(demands(j):
        @for(periods(t):
            @for(products(p):
                @bin(assignment(i,j,t,p));
                @bin(expanded(i,t,p));
                @bin(shrunk(i,t,p));
                @bin(open(i,t));
                @bin(opened(i,t));
                @bin(closed(i,t));
                @gin(uPlus(i,t,p));
                @gin(uMinus(i,t,p));
                @gin(totalCapacity(i,t,p));
                @gin(usedCapacity(i,t,p));
                @gin(totalFixCost(i,t));
                @bin(threshold1(i,t,p));
                @bin(threshold2(i,t,p));
                @bin(threshold3(i,t,p));
                @bin(threshold4(i,t,p));
                @bin(threshold5(i,t,p));
                @bin(threshold6(i,t,p));
                @bin(threshold7(i,t,p));
                @bin(thresholdSwitch1(i,t,p));
                @bin(thresholdSwitch2(i,t,p));
                @bin(thresholdSwitch3(i,t,p));
                @bin(thresholdSwitch4(i,t,p));
                @bin(thresholdSwitch5(i,t,p));
                @bin(thresholdSwitch6(i,t,p));
                @bin(thresholdSwitch7(i,t,p));
            )
        )
    )
);

```

Data:

```

        competitorsAttraction =
@odbc('dissertation','modeldata_mp_10_view','COMPETITORSATTRACTION');
        attraction =
@odbc('dissertation','modeldata_mp_10_view','ATTRACTIVITY');
        initialCapacity =
@odbc('dissertation','modeldata_mp_10_INITIALS_CAP','INITIALCAP');
        initialOpen =
@odbc('dissertation','modeldata_mp_10_INITIALS','INITIALOPEN');
        variableCost =
@odbc('dissertation','modeldata_mp_10_view','VARIABLECOST');
        fixCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','FIXCOST');
        openingCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','OPENINGCOST');
        closingCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','CLOSINGCOST');
        unitExpansionCost =
@odbc('dissertation','modeldata_mp_10_view','UNITEXPANSIONCOST');
        expansionCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','EXPANSIONCOST');
        unitShrinkageCost =
@odbc('dissertation','modeldata_mp_10_view','UNITSHRINKAGECOST');
        shrinkageCost =
@odbc('dissertation','modeldata_mp_10_facilities_npr','SHRINKAGECOST');
        price = @odbc('dissertation','modeldata_mp_10_view','PRICE');
        demand = @odbc('dissertation','modeldata_mp_10_view','DEMAND');

```

enddata

```

        max = @sum(indexitp(i,t,p): usedCapacity(i,t,p)*(price(i,t,p) -
variableCost(i,t,p)) - totalFixCost(i,t) - closingcost(i,t)*closed(i,t)
- openingcost(i,t)*opened(i,t)) - @sum(indexitp(i,t,p):
expansionCost(i,t)*expanded(i,t,p)) - @sum(indexitp(i,t,p):
unitExpansionCost(i,t,p)*uPlus(i,t,p)) - @sum(indexitp(i,t,p):
shrinkageCost(i,t)*shrunk(i,t,p)) - @sum(indexitp(i,t,p):
unitShrinkageCost(i,t,p)*uMinus(i,t,p));

```

!Initializing open and totalCapacity data;

```

@for(locations(i):
    open(i,1)= initialOpen;
);

```

```

@for(locations(i):
    @for(products(p):

        totalCapacity(i,1,p) = initialCapacity(i,p);

    )
);

```

```

@for(locations(i):
    @for( periods(t):
        @for( products(p):
            step1(i,t,p) = 1000;

```

```

        step2(i,t,p) = 2000;
        step3(i,t,p) = 5000;
        step4(i,t,p) = 10000;
        step5(i,t,p) = 15000;
        step6(i,t,p) = 30000;
        step7(i,t,p) = 80000;

        stepCost1(i,t,p) = 40;
        stepCost2(i,t,p) = 20;
        stepCost3(i,t,p) = 10;
        stepCost4(i,t,p) = 5;
        stepCost5(i,t,p) = 2.5;
        stepCost6(i,t,p) = 1.25;
        stepCost7(i,t,p) = 1;

    )
)

!Modification of fix cost in terms of change in facility size;
@for(locations(i):
    @for(periods(t):
        @for(products(p):

            step1(i,t,p))-1;
            step2(i,t,p))-1;
            step3(i,t,p))-1;
            step4(i,t,p))-1;
            step5(i,t,p))-1;
            step6(i,t,p))-1;
            step7(i,t,p))-1;

            thresholdSwitch1(i,t,p)*9999999 >= (1-
threshold1(i,t,p))+(1-open(i,t));
            threshold1(i,t,p) + open(i,t) +
thresholdSwitch1(i,t,p) <= 2;

            thresholdSwitch2(i,t,p)*9999999 >= (1-
threshold2(i,t,p))+(1-open(i,t));
            threshold2(i,t,p) + open(i,t) +
thresholdSwitch2(i,t,p) <= 2;

            thresholdSwitch3(i,t,p)*9999999 >= (1-
threshold3(i,t,p))+(1-open(i,t));
            threshold3(i,t,p) + open(i,t) +
thresholdSwitch3(i,t,p) <= 2;

            thresholdSwitch4(i,t,p)*9999999 >= (1-
threshold4(i,t,p))+(1-open(i,t));

```

```

        threshold4(i,t,p) + open(i,t) +
thresholdSwitch4(i,t,p) <= 2;

        thresholdSwitch5(i,t,p)*9999999 >= (1-
threshold5(i,t,p))+(1-open(i,t));
        threshold5(i,t,p) + open(i,t) +
thresholdSwitch5(i,t,p) <= 2;

        thresholdSwitch6(i,t,p)*9999999 >= (1-
threshold6(i,t,p))+(1-open(i,t));
        threshold6(i,t,p) + open(i,t) +
thresholdSwitch6(i,t,p) <= 2;

        thresholdSwitch7(i,t,p)*9999999 >= (1-
threshold7(i,t,p))+(1-open(i,t));
        threshold7(i,t,p) + open(i,t) +
thresholdSwitch7(i,t,p) <= 2;

    )
)

);

@for(locations(i):
    @for( periods(t):

        totalFixCost(i,t) = open(i,t)*fixCost(i,t) +
@sum(products(p) : stepCost1(i,t,p)*(threshold1-
thresholdSwitch1(i,t,p)) + stepCost2(i,t,p)*(threshold2(i,t,p)-
thresholdSwitch2(i,t,p)) + stepCost3(i,t,p)*(threshold3(i,t,p)-
thresholdSwitch3(i,t,p)) + stepCost4(i,t,p)*(threshold4(i,t,p)-
thresholdSwitch4(i,t,p))+ stepCost5(i,t,p)*(threshold4(i,t,p)-
thresholdSwitch5(i,t,p)) + stepCost6(i,t,p)*(threshold6(i,t,p)-
thresholdSwitch6(i,t,p)) + stepCost7(i,t,p)*(threshold7(i,t,p)-
thresholdSwitch7(i,t,p)));
    )
);

!Only assign if higher attraction level than competitor;
@for(indexijtp(i,j,t,p):
    (assignment(i,j,t,p))*(attraction(i,t,p)-
competitorsAttraction(j,t,p))>= 0);

!Usage does not exceed the totalCapacity at i in t;
@for(locations(i):
    @for( periods(t):
        @for(products(p):
            usedCapacity(i,t,p) <= totalCapacity(i,t,p)
        )
    )
);

!Usage will never be greater than demand;
@for(locations(i):

```

```

        @for( periods(t) :
            @for( products(p) :
                usedCapacity(i,t,p) <= @sum( demands(j) :
                    (assignment(i,j,t,p)*demand(j,t,p))
                )
            )
        );

!totalCapacity if the facility is determined by expansion or
shrinking in the previous period;
@for( locations(i) :
    @for( periods(t) | (t) #NE#10 :
        @for( products(p) :
            totalCapacity(i,t+1,p) = totalCapacity(i,t,p) +
uPlus(i,t,p) - uMinus(i,t,p)
        )
    )
);

!Central setting of open, opening or closing down;
@for( locations(i) :
    @for( periods(t) | (t) #NE#10 : open(i,t+1) = open(i,t) +
opened(i,t) - closed(i,t));

!If is expanded, expanded must be set to 1;
@for( locations(i) :
    @for( periods(t) :
        @for( products(p) :
            9999999*expanded(i,t,p) >= uPlus(i,t,p)
        )
    )
);

!If is shrunk, shrunk must be set to 1;
@for( locations(i) :
    @for( periods(t) :
        @for( products(p) :
            9999999*shrunk(i,t,p) >= uMinus(i,t,p)
        )
    )
);

!totalCapacity must be bigger than shrinkage;
@for( locations(i) :
    @for( periods(t) :
        @for( products(p) :
            totalCapacity(i,t,p) >= uMinus(i,t,p)
        )
    )
);

```

```

!Only open facilities can be expanded;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            expanded(i,t,p) <= open(i,t)
        )
    )
);

!Only open facilities can be shrunk;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            shrunk(i,t,p) <= open(i,t)
        )
    )
);

!Only have used Capacity if open ;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            9999999*open(i,t) >= usedCapacity(i,t,p)
        )
    )
);

!NEW: Budget constraint for expanding;
@sum(indexip(i,p): expanded(i,1,p)*expansionCost(i,1) +
uPlus(i,1,p)*unitExpansionCost(i,1,p)) <= 50000;
@sum(indexip(i,p): expanded(i,2,p)*expansionCost(i,2) +
uPlus(i,2,p)*unitExpansionCost(i,2,p)) <= 40000;
@sum(indexip(i,p): expanded(i,3,p)*expansionCost(i,3) +
uPlus(i,3,p)*unitExpansionCost(i,3,p)) = 0;
@sum(indexip(i,p): expanded(i,4,p)*expansionCost(i,4) +
uPlus(i,4,p)*unitExpansionCost(i,4,p)) <= 50000;
@sum(indexip(i,p): expanded(i,5,p)*expansionCost(i,5) +
uPlus(i,5,p)*unitExpansionCost(i,5,p)) <= 40000;
@sum(indexip(i,p): expanded(i,6,p)*expansionCost(i,6) +
uPlus(i,6,p)*unitExpansionCost(i,6,p)) <= 0;
@sum(indexip(i,p): expanded(i,7,p)*expansionCost(i,7) +
uPlus(i,7,p)*unitExpansionCost(i,7,p)) <= 0;
@sum(indexip(i,p): expanded(i,8,p)*expansionCost(i,8) +
uPlus(i,8,p)*unitExpansionCost(i,8,p)) <= 50000;
@sum(indexip(i,p): expanded(i,9,p)*expansionCost(i,9) +
uPlus(i,9,p)*unitExpansionCost(i,9,p)) <= 400000;
@sum(indexip(i,p): expanded(i,10,p)*expansionCost(i,10) +
uPlus(i,10,p)*unitExpansionCost(i,10,p)) <= 400000;

!NEW: Size constraints;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            totalCapacity(i,t,p) <= 2000000;
            totalCapacity(i,t,p) >= open(i,t)*5000;
        )
    )
);

```

```

        )
    )
);

!Only have a maximum of one assignment ;
@for(demands(j):
    @for( periods(t):
        @for( products(p):
            @sum(locations(i): assignment(i,j,t,p)) <= 1)));

end

```


Appendix F The JAVA source code for the simulated annealing algorithm

```
package simulatedannealing;

import com.lindo.*;

import java.io.*;

public class SimulatedAnnealing extends Lindo {

    // Set initial temp
    public static double startTemp = 100;
    public static double temp = startTemp;
    // Cooling rate
    public static double coolingRate = 0.05;

    public static long startTime = 0;

    public static String energyTracker;
    public static String bestEnergyTracker;

    public static int counter = 0;
    public static int fileCounter = 0;
    //Retrieving variable indexes

    public static boolean initialized = false;

    public static int[][] startConfiguration;
    public static int[][] currentConfiguration;
    public static int[][] bestConfiguration;
    public static int[][] neighbourConfiguration;

    public static double startEnergy;
    public static double currentEnergy = 0;
    public static double bestEnergy;
    public static double neighbourEnergy;

    public static int facilities = 50;
    public static int periods = 12;

    public static int[][] variables = new int[facilities][periods];
```

```

        public static String license = "D:/Dropbox/PhD/Dissertation Sven
        Brose/09_Development/workspace/lib/lindoapi/license/ldapi100.lic";

        private static int nErrorCode[] = new int[1];
        private static StringBuffer cErrorMessage = new
        StringBuffer(LS_MAX_ERROR_MESSAGE_LENGTH);
        private static StringBuffer cLicenseKey = new
        StringBuffer(LS_MAX_ERROR_MESSAGE_LENGTH);
        double[][] array = null;
        double[] adC = null;
        double[] adB = null;

        /* Number of constraints, Number of variables */static int nM = 0,
        nN = 0;
        /* The length of each column. Since we aren't leaving any blanks
        in our matrix, we can set this to null */
        static int pnLenCol[] = null;

        /* The nonzero coefficients */static double adA[] = null;

        /* The row indices of the nonzero coefficients */static int anRowX[]
        = null;

        /* Simple upper and lower bounds on the variables.
        By default, all variables have a lower bound of zero
        and an upper bound of infinity. Therefore pass null
        pointers in order to use these default values. */
        static double pdLower[] = null, pdUpper[] = null;

        /* The number of nonzeros in the constraint matrix */static int nNZ
        = 0;
        /* The indices of the first nonzero in each column */static int
        anBegCol[] = null;
        /* The length of each column. Since there are no balnks can be null
        */static int[]
        Alencol = null;
        /* LB and UB of the variable, null means 0 LB and infinity UB
        */static double[]
        lb = null, ub = null;

        static String varnames[] = null, connames[] = null;

        public static void main(String[] args) {

            startTime = System.currentTimeMillis();
            System.out.println("Runtime in MilliSeconds: " +
            getElapsedTime());
            System.loadLibrary("lindojni");
            nErrorCode[0] = LSloadLicenseString(license, cLicenseKey);

            // Create startConfiguration (all Facilities open)

```

```

startConfiguration = new int[facilities][periods];

    for(int i = 0; i < facilities; i++)
    {
        for(int t = 0; t < periods; t++)
        {
            startConfiguration[i][t] =
(int)Math.round(Math.random());
        }
    }

    System.out.println("Starting with temperature " +temp+ " and
coolingRate " + coolingRate);

    // Initialize solution
    bestEnergy = getEnergy(startConfiguration);

    System.out.println("Initial solution: " + bestEnergy);

    // Set start as best and as current
    bestConfiguration = startConfiguration;
    currentConfiguration = startConfiguration;
    currentEnergy = bestEnergy;

    energyTracker = Double.toString(currentEnergy)+"\n";
    bestEnergyTracker = Double.toString(bestEnergy)+"\n";

    // Loop until system has cooled
    while (temp > 1) {

        // Create new configuration

        neighbourConfiguration =
createNewConfiguration(currentConfiguration);

        // Get energy of solutions
        //double currentEnergy = getEnergy(currentConfiguration);
        double neighbourEnergy = getEnergy(neighbourConfiguration);

        // Decide if we should accept the neighbour
        if (acceptanceProbability(currentEnergy, neighbourEnergy,
temp) > Math.random()) {
            currentConfiguration = neighbourConfiguration;
            currentEnergy = neighbourEnergy;
            System.out.println("accepted neighbour"+
neighbourEnergy);
        }

        // Keep track of the best solution found
        if (currentEnergy > bestEnergy) {
            bestConfiguration = currentConfiguration;
            bestEnergy = currentEnergy;
            System.out.println("New Best " + bestEnergy);

```

```

    }

    energyTracker = getElapsedTime() + ";" +
Double.toString(currentEnergy)+ ";" + temp;
    bestEnergyTracker = getElapsedTime() + ";" +
Double.toString(bestEnergy)+ ";" + temp;

    // Cool system
    temp *= 1-coolingRate;
    //temp = temp-coolingRate;
    System.out.println("Getting colder... ");
    System.out.println("New temperature is: "+temp);

    try{
        writeFile(bestEnergyTracker, "bestEnergies-
T"+startTemp+"-C"+coolingRate+".csv");
        writeFile(energyTracker, "allEnergies-T"+startTemp+"-
C"+coolingRate+".csv");

    }catch(Exception e)
    {
        e.toString();
    }

}

System.out.println("Final solution energy: " + bestEnergy);
System.out.println("Final solution configuration: ");
printConfiguration(bestConfiguration);

System.out.println("Configurations created: "+ counter);

} //main

// Generalized error Reporting function
private static void APIErrorCheck(Object pEnv) {
    if (0 != nErrorCode[0]) {
        LSgetErrorMessage(pEnv, nErrorCode[0], cErrorMessage);
        System.out.println("\nError " + nErrorCode[0] + ": " +
cErrorMessage);
        System.out.println();
        System.exit(1);
    }
}

// Calculate the acceptance probability
public static double acceptanceProbability(double energy, double
newEnergy, double temperature) {
    // If the new solution is better, accept it
    if (newEnergy < energy) {
        return 1.0;
    }
}

```

```

        }
        // If the new solution is worse, calculate an acceptance
probability
        double acceptanceProb = Math.exp(-(energy - newEnergy) /
temperature);

        System.out.println("Acceptance probability:"+acceptanceProb);

        return acceptanceProb;
    }

    public static void writeFile (String input, String fileName) throws
IOException
    {

        BufferedWriter out = new BufferedWriter(
            new OutputStreamWriter(
                new FileOutputStream(fileName, true)));

        out.write(input);
        out.newLine();
        out.close();

    }

    public static long getElapsedTime() {
        long elapsed;

        elapsed = (System.currentTimeMillis() - startTime);

        return elapsed;
    }

    public static double getEnergy(int[][] configuration){

        Object pEnv = null;
        Object pModel = null;

        pEnv = LScrtEnv(nErrorCode, cLicenseKey.toString());
        APIErrorCheck(pEnv);

        pModel = LScrtModel(pEnv, nErrorCode);
        APIErrorCheck(pEnv);
    }

```

```

        String fileName = "D:/Users/sbrose/Dropbox/PhD/Dissertation
Sven Brose/06_Lingo/Models/Model 2/50x50x50x12/50x50x50x12.mps";
        //System.out.println("Loading file ... ");
        //System.out.println(fileName);

        nErrorCode[0] = LSreadMPSFile(pModel, fileName,
LS_UNFORMATTED_MPS);
        APIErrorCheck(pEnv);

        int[] iVar = new int[1];
        iVar[0] = 1;

        for(int i = 0; i < facilities; i++)
        {
            for(int t = 0; t < periods; t++)
            {
                if(!initialized){

                    System.out.println("Initializing variable
index");

                    String varName = "OPEN("+ (i+1) + "," +
(t+1) + ")";
                    nErrorCode[0] = LSgetVariableIndex(pModel,
varName, iVar);
                    variables[i][t] = iVar[0];

                    System.out.println(varName);
                    System.out.println("index is: " +
iVar[0]);

                    System.out.println("Runtime in
Milliseconds: " + getElapsedTime());

                }

                int nCon = 1;
                String achCtype = "E";
                String apszCname[] = {"New constraint for "+i +",
" + t};

                int ia[]= new int [] {variables[i][t]};
                int ka[]= new int [] {0,1};
                double a[]= new double []{1};
                double rhs[]= new double []
{configuration[i][t]} ;

                if(variables[i][t] != -1)
                {
                    nErrorCode[0] = LSaddConstraints(pModel,
nCon, achCtype, apszCname, ka, a, ia, rhs);
                }
            }
        }

```

```

        }
        initialized = true;

        //b "nErrorCode[0] =
LWriteMPSFile(pModel,"createdModels/mod"+fileCounter+".mps",
LS_BASFILE_MPS);

        //time to call Lindo with configuration

        nErrorCode[0] = LSoptimize( pModel, LS_METHOD_PSIMPLEX, null);

        APIErrorCheck(pEnv);

        //nErrorCode[0] = LWriteSolution(pModel, "solution.txt");

        double dObj[] = new double[1];

        nErrorCode[0] = LSgetInfo(pModel,LS_DINFO_POBJ, dObj);

        APIErrorCheck(pEnv);
        System.out.println("Current Configuration");
        printConfiguration(configuration);
        System.out.println("Runtime in MilliSeconds: " +
getElapsedTime());

        System.out.println( "Objective Value for configuration = "+ "
"+ -dObj[0]);

        nErrorCode[0] = LSdeleteModel(pModel);
        nErrorCode[0] = LSdeleteEnv(pEnv);

        //System.out.print(nErrorCode[0]);

        APIErrorCheck(pEnv);

        fileCounter++;

        return -dObj[0];

    }

    public static int[][] createNewConfiguration(int[][]
currentConfiguration)
    {
        counter++;
        int[][] newConfiguration = new int[facilities][periods];

        int indexi = (int) (Math.random()*(facilities));
        int indext = 0;

        // Making sure that never for period 1 a configuration is
changed, because of initialization values.

```

```

        while(indext == 0)
        {
            indext = (int) (Math.random()*(periods));
        }

        newConfiguration = currentConfiguration;

        if(newConfiguration[indexi][indext] == 0){
            newConfiguration[indexi][indext] = 1;

        }else
        {
            newConfiguration[indexi][indext] = 0;
        }

        return newConfiguration;
    }

    public static void printConfiguration(int[][] configuration)
    {
        for(int i = 0; i < facilities ; i++)
        {
            System.out.println("");
            for(int t = 0;t < periods; t++)
            {
                System.out.print(configuration[i][t]);
            }
            System.out.println("");
        }
    }
}

```


Appendix G Ruby script for gathering distance information

```
require 'net/http'
require 'json'
require 'csv'

csvfile = File.open('distances.csv', "w")

banken = CSV.read('banks.csv')
kunden = CSV.read('demands.csv')

banken_25 = Array.new

banken.each_slice(25) {|a| banken_25 << a}

kunden.each do |kunde|

  csv_row = Array.new
  origin_address = String.new

  banken_25.each do |bank_25|

    bank_url = Array.new

    bank_25.each do |bank|

      #bank[1].gsub!(/[Öö]/, 'oe')
      #bank[1].gsub!(/[Üü]/, 'ue')
      #bank[1].gsub!(/[Ää]/, 'ae')
      #bank[1].gsub!('ß', 'ss')

      bank_url << "#{bank[2]}+#{bank[1].gsub(' ', '+')}+DE"

    end

    kunde_url = "#{kunde[0]},#{kunde[1]}"
    bank_url_25 = bank_url.join('|')

    base_url =
"http://maps.googleapis.com/maps/api/distancematrix/json?origins=#{kunde_url}&destinations=#{bank_url_25}+DE&mode=driving&language=de-DE&sensor=false"

    uri = URI(base_url)
    response = Net::HTTP.get(uri)

    parsed_result = JSON.parse(response)
```

```

    parsed_result["rows"][0]["elements"].each do |value|

      csv_row << value["distance"]["value"]

    end

    origin_address = parsed_result["origin_addresses"]

  end

  csv_row.insert(0,origin_address)
  puts csv_row.join('|')
  csvfile.puts csv_row.join('|')
  sleep(1)

end

csvfile.close

```

Appendix H Data for case study

Table 46 Initial capacity of facilities

facility	capacity
1	2400
2	600
3	1200
4	1200
5	600
6	1200
7	1200
8	600
9	600
10	600
11	600
12	600
13	600
14	1200
15	1200
16	1200
17	600
18	1200
19	1200
20	600
21	1200
22	600
23	600
24	1200
25	600
26	600
27	600
28	600
29	600
30	600
31	600
32	600
33	600
34	600

35	1200
36	600

Table 47 prices and variable cost

INDEXT	INDEXP	PRICE	VARIABLECOST
1	1	24	30
1	2	10	1.1
1	3	30	27.5
1	4	10	5.5
1	5	1000	550
1	6	20	11
1	7	50	27.5
1	8	20	11
1	9	10	1.1
1	10	25	11
1	11	10	5.5
1	12	10	5.5
1	13	10	5.5
2	1	24	30
2	2	10	1.2
2	3	30	30
2	4	10	6
2	5	1000	600
2	6	20	12
2	7	50	30
2	8	20	12
2	9	10	1.2
2	10	25	12
2	11	10	6
2	12	10	6
2	13	10	6
3	1	24	30
3	2	10	1.3
3	3	30	32.5
3	4	10	6.5
3	5	1000	650
3	6	20	13
3	7	50	32.5
3	8	20	13
3	9	10	1.3
3	10	25	13
3	11	10	6.5
3	12	10	6.5

3	13	10	6.5
4	1	24	30
4	2	10	1.4
4	3	30	35
4	4	10	7
4	5	1000	700
4	6	20	14
4	7	50	35
4	8	20	14
4	9	10	1.4
4	10	25	14
4	11	10	7
4	12	10	7
4	13	10	7
5	1	24	30
5	2	10	1.5
5	3	30	37.5
5	4	10	7.5
5	5	1000	750
5	6	20	15
5	7	50	37.5
5	8	20	15
5	9	10	1.5
5	10	25	15
5	11	10	7.5
5	12	10	7.5
5	13	10	7.5
6	1	24	30
6	2	10	1.6
6	3	30	40
6	4	10	8
6	5	1000	800
6	6	20	16
6	7	50	40
6	8	20	16
6	9	10	1.6
6	10	25	16
6	11	10	8
6	12	10	8
6	13	10	8
7	1	24	30
7	2	10	1.7
7	3	30	42.5
7	4	10	8.5
7	5	1000	850

7	6	20	17
7	7	50	42.5
7	8	20	17
7	9	10	1.7
7	10	25	17
7	11	10	8.5
7	12	10	8.5
7	13	10	8.5
8	1	24	30
8	2	10	1.8
8	3	30	45
8	4	10	9
8	5	1000	900
8	6	20	18
8	7	50	45
8	8	20	18
8	9	10	1.8
8	10	25	18
8	11	10	9
8	12	10	9
8	13	10	9
9	1	24	30
9	2	10	1.9
9	3	30	47.5
9	4	10	9.5
9	5	1000	950
9	6	20	19
9	7	50	47.5
9	8	20	19
9	9	10	1.9
9	10	25	19
9	11	10	9.5
9	12	10	9.5
9	13	10	9.5
10	1	24	30
10	2	10	2
10	3	30	50
10	4	10	10
10	5	1000	1000
10	6	20	20
10	7	50	50
10	8	20	20
10	9	10	2
10	10	25	20
10	11	10	10

10	12	10	10
10	13	10	10

Table 48 expansion, shrinkage, opening, closing and fixed cost

INDEXI	EXPANSION COST	SHRINKAGE COST	FIXED COST	OPENING COST	CLOSING COST
1	20000	25000	96000	5000000	500000
1	20000	25000	24000	100000	50000
1	20000	25000	36000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	36000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	48000	100000	50000
1	20000	25000	36000	100000	50000
1	20000	25000	36000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	36000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	48000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	9600	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	12000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	18000	100000	50000
1	20000	25000	24000	100000	50000
1	20000	25000	36000	100000	50000
1	20000	25000	12000	100000	50000

Table 49 Demand data for case study for all 13 products

period	demand	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2071	1225	1194	1013	854	881	749	654	557	445	299	207	138
1	2	553	313	290	242	215	224	188	174	144	117	83	55	40
1	3	2283	1362	1340	1144	956	979	832	716	613	488	324	225	149
1	4	1473	864	832	704	600	621	531	473	399	321	219	150	100
1	5	4745	2828	2712	2291	1964	2030	1725	1539	1297	1043	713	487	322
1	6	1737	1035	998	848	725	743	622	546	463	371	250	172	117
1	7	3766	2218	2101	1770	1537	1591	1342	1212	1016	820	567	385	261
1	8	2733	1628	1597	1364	1144	1170	982	842	722	574	380	265	180
1	9	4127	2433	2388	2040	1711	1749	1503	1305	1113	889	595	412	273
1	10	914	542	543	468	385	390	339	287	248	196	129	90	58
1	11	2597	1510	1446	1235	1068	1084	916	817	689	554	378	259	178
1	12	2868	1667	1606	1381	1193	1197	1024	911	768	618	421	288	196
1	13	4139	2433	2308	1968	1723	1745	1473	1330	1115	900	621	422	286
1	14	4858	2812	2755	2355	1980	2021	1743	1521	1294	1035	695	480	325
1	15	2710	1590	1577	1355	1125	1144	989	849	728	579	384	267	177
1	16	3394	1978	1949	1671	1398	1422	1214	1044	894	712	472	329	224
1	17	4786	2795	2761	2378	1989	2010	1739	1501	1284	1023	681	473	315
1	18	2374	1394	1393	1203	994	1003	873	744	640	508	334	234	153
1	19	5273	3076	3026	2599	2182	2211	1905	1651	1409	1125	751	521	349
1	20	2624	1542	1529	1318	1098	1110	955	819	702	559	370	258	171
1	21	4638	2703	2672	2301	1923	1944	1667	1432	1227	977	647	451	305
1	22	2175	1289	1291	1115	919	928	806	684	589	468	307	215	139
1	23	5446	3221	3112	2643	2251	2313	1974	1747	1477	1186	806	553	367
1	24	4565	2666	2579	2188	1859	1915	1642	1455	1230	987	671	460	309
1	25	6047	3527	3328	2815	2466	2530	2066	1848	1555	1252	858	586	423
1	26	2506	1468	1355	1137	1020	1051	866	801	665	540	380	255	180
1	27	4471	2591	2389	2028	1835	1856	1524	1410	1169	950	669	449	322
1	28	2904	1696	1555	1302	1176	1214	984	912	756	615	433	291	210
1	29	1345	795	720	608	561	569	468	444	364	298	214	142	98
1	30	4925	2930	2833	2405	2047	2105	1781	1568	1329	1065	721	496	331
1	31	3372	1993	1886	1590	1383	1429	1187	1067	896	722	497	338	233
1	32	3428	2020	1877	1610	1452	1448	1209	1116	926	752	529	355	243
1	33	2342	1376	1285	1098	981	986	825	758	630	511	358	241	165
1	34	2305	1329	1265	1084	948	954	794	709	597	480	328	224	160
1	35	2583	1478	1382	1171	1040	1059	845	757	637	513	352	240	184
1	36	7302	4241	4080	3492	3009	3045	2586	2297	1939	1558	1061	727	499
1	37	1034	601	584	501	428	432	370	326	277	222	150	103	70
1	38	278	165	153	132	119	118	97	88	74	60	42	28	20
1	39	3370	1977	1922	1657	1416	1420	1206	1055	897	718	483	333	226
1	40	586	345	325	277	244	247	207	187	157	127	88	60	41
1	41	3169	1834	1731	1485	1313	1315	1103	1000	837	676	468	317	222
1	42	403	240	226	195	174	172	144	131	110	89	61	42	28

1	43	1034	608	563	477	429	435	366	340	282	229	162	108	74
1	44	1663	970	906	780	700	695	587	540	449	364	255	172	118
1	45	767	450	428	366	321	323	270	242	204	164	113	77	53
1	46	619	364	330	281	260	260	217	206	169	138	99	66	45
1	47	1694	984	905	778	711	705	574	531	441	358	252	169	122
1	48	316	185	178	154	133	133	113	101	85	69	47	32	22
1	49	1913	1141	1090	944	829	819	680	603	510	409	279	191	130
1	50	1709	980	920	787	698	703	569	511	429	346	237	162	121
1	51	769	457	426	365	328	328	260	233	196	158	108	74	54
1	52	2143	1254	1141	970	893	898	713	662	548	446	315	211	156
1	53	2053	1189	1133	972	849	854	676	587	501	400	267	185	142
1	54	1008	584	531	451	415	418	349	332	272	223	160	106	74
1	55	7111	4072	3877	3346	2938	2922	2482	2236	1876	1513	1043	709	496
1	56	1869	1089	999	843	765	780	654	616	507	414	295	197	135
1	57	832	490	462	393	346	351	299	273	228	184	129	87	58
1	58	679	396	367	312	281	283	236	218	181	147	103	69	49
1	59	1751	1040	982	847	751	746	632	575	480	388	270	183	121
1	60	497	291	269	228	205	209	176	164	136	110	78	52	36
1	61	2229	1319	1236	1065	952	945	800	734	611	495	346	234	156
1	62	2791	1608	1456	1222	1123	1150	956	911	746	611	440	291	206
1	63	2323	1340	1253	1070	954	961	800	731	610	494	344	233	165
1	64	1932	1133	1040	887	808	811	668	623	515	419	297	199	139
1	65	612	356	333	284	253	255	214	197	164	133	93	63	43
1	66	1345	792	744	631	557	568	480	439	366	297	207	140	94
1	67	1253	729	680	581	520	522	442	408	339	275	194	130	89
1	68	1934	1108	1016	854	773	793	653	610	504	411	291	195	141
1	69	1580	918	860	727	643	658	555	509	424	344	240	162	112
1	70	1928	1131	1037	876	794	809	679	638	526	429	306	204	139
1	71	1421	859	857	747	623	618	539	461	396	314	207	145	90
1	72	3791	2180	2057	1762	1556	1563	1298	1170	982	792	546	371	266
1	73	2656	1543	1432	1217	1093	1105	916	844	701	569	399	269	190
1	74	1874	1084	1017	870	773	777	647	590	492	398	277	187	132
1	75	1338	780	742	635	555	560	474	427	358	289	199	135	93
1	76	1610	941	862	733	669	673	562	528	435	355	253	169	116
1	77	2168	1266	1180	1005	898	907	766	708	588	477	335	225	154
1	78	2467	1420	1305	1102	999	1017	853	801	660	539	383	256	179
1	79	1481	870	805	687	619	623	509	467	389	315	221	149	106
1	80	2297	1347	1252	1065	954	965	815	756	627	510	359	241	163
1	81	2760	1631	1488	1273	1174	1168	977	925	760	621	445	296	200
1	82	857	504	453	387	363	360	298	286	233	192	138	91	63
1	83	3984	2325	2136	1814	1649	1665	1378	1287	1063	866	614	410	288
1	84	2085	1237	1170	1006	888	887	749	679	568	459	318	216	144
1	85	1468	875	819	703	628	627	528	484	403	327	228	154	103
1	86	3112	1831	1729	1485	1314	1313	1118	1019	851	688	479	324	217
1	87	2732	1614	1542	1324	1153	1158	990	890	747	603	415	282	187

1	88	2524	1469	1348	1140	1035	1052	861	801	663	540	381	255	183
1	89	1206	708	662	562	499	507	410	370	310	250	173	117	85
1	90	2228	1317	1211	1040	952	943	776	722	597	486	343	230	160
1	91	1538	908	832	712	652	650	535	500	413	337	239	160	111
1	92	2982	1723	1581	1329	1202	1233	1015	946	782	637	451	302	216
1	93	2430	1432	1319	1127	1025	1025	833	768	638	518	364	245	174
1	94	534	313	299	256	222	225	188	167	141	113	77	53	37
1	95	496	290	259	217	203	207	157	146	121	99	70	47	37
1	96	4834	2817	2694	2296	1988	2022	1708	1525	1285	1034	708	483	332
1	97	2208	1286	1240	1057	905	924	766	670	570	456	307	212	150
1	98	2080	1206	1177	1004	848	867	734	638	544	434	291	201	140
1	99	3697	2129	2026	1713	1484	1528	1294	1165	978	789	543	370	257
1	100	1789	1005	956	823	722	721	611	550	461	372	256	174	126
1	101	1520	898	806	687	644	642	526	503	411	337	243	161	112
1	102	4087	2414	2232	1915	1738	1729	1397	1279	1066	863	603	407	291
1	103	586	349	322	275	249	250	200	182	152	123	86	58	42
1	104	332	198	183	157	142	142	117	108	90	73	51	34	24
1	105	2403	1412	1322	1125	1000	1012	825	746	625	504	348	237	169
1	106	2189	1282	1157	978	908	917	739	698	574	469	335	223	161
1	107	1564	932	864	737	665	668	546	501	417	338	236	159	111
1	108	756	451	423	362	322	323	272	248	207	167	117	79	53
1	109	348	207	195	167	147	148	122	110	92	74	51	35	24
1	110	2390	1403	1318	1117	985	1006	819	735	618	498	342	233	167
1	111	1090	641	604	509	447	460	383	347	290	235	162	110	76
1	112	309	184	175	149	130	132	110	99	83	67	46	31	21
1	113	1647	975	911	780	698	699	583	534	445	360	251	170	116
1	114	575	343	316	268	242	245	205	192	158	129	91	61	41
1	115	1938	1116	991	828	779	797	634	608	497	408	295	195	146
1	116	2884	1715	1629	1388	1213	1231	1012	900	760	611	417	285	198
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1	118	88	52	50	43	38	38	31	28	24	19	13	9	6
1	119	3620	2108	1995	1684	1467	1512	1286	1169	977	790	548	371	252
1	120	3311	1939	1815	1538	1363	1390	1161	1062	885	717	500	338	233
1	121	730	422	380	322	299	301	247	236	193	158	114	75	54
1	122	1257	737	679	582	531	528	443	415	342	279	198	132	90
1	123	778	463	437	378	337	332	268	238	201	161	110	75	54
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1	125	143	85	77	65	60	61	51	48	40	32	23	15	10
1	126	864	503	469	404	364	361	306	283	235	191	134	90	61
1	127	560	328	289	240	227	235	194	191	155	128	94	61	42
1	128	1687	1000	917	779	710	716	603	569	468	382	273	182	121
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1	130	603	349	311	264	249	249	208	202	164	135	99	65	45
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1	132	248	147	138	118	106	105	90	83	69	56	39	26	17

1	133	434	245	169	113	140	173	131	161	119	104	88	53	40
1	134	3017	1780	1596	1353	1266	1272	1062	1026	835	687	499	329	223
1	135	1855	1099	1056	909	787	789	678	606	510	411	282	192	126
1	136	3249	1881	1802	1548	1344	1350	1159	1040	874	705	484	330	224
1	137	2184	1285	1217	1040	914	922	787	716	598	484	336	228	151
1	138	791	466	444	378	329	334	286	258	217	175	121	82	54
1	139	3328	1946	1813	1541	1378	1395	1183	1096	909	739	520	349	236
1	140	4265	2483	2347	2005	1764	1781	1520	1386	1157	936	651	440	298
1	141	4661	2740	2637	2261	1949	1967	1692	1511	1272	1024	701	479	317
1	142	1167	681	646	549	480	488	417	379	317	256	178	120	81
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1	144	2377	1386	1349	1157	984	996	860	759	643	515	349	240	160
1	145	3449	2024	1953	1676	1441	1454	1251	1114	940	755	516	353	234
1	146	2744	1622	1525	1305	1156	1163	990	908	756	613	428	289	191
1	147	1650	977	918	788	702	700	596	547	455	369	258	174	115
1	148	1168	697	672	582	505	501	431	384	324	260	178	122	79
1	149	3653	2137	2061	1773	1528	1534	1321	1176	992	798	545	373	248
1	150	1614	938	909	780	666	674	581	515	435	349	237	163	109
1	151	1028	604	573	492	433	433	370	336	281	227	158	107	71
1	152	1791	1041	994	847	736	747	640	577	484	391	270	183	124
1	153	2140	1249	1178	1007	888	896	764	698	582	471	328	222	150
1	154	3278	1894	1772	1514	1350	1358	1154	1064	884	718	504	339	233
1	155	677	395	367	313	282	283	240	223	184	150	106	71	48
1	156	1932	1123	1071	917	799	806	690	623	522	421	291	198	133
1	157	1729	993	935	804	714	712	607	556	463	375	262	177	122
1	158	3011	1774	1694	1455	1269	1273	1091	983	825	666	459	312	206
1	159	4326	2540	2412	2072	1820	1822	1558	1413	1182	955	662	449	299
1	160	172	102	97	84	74	73	63	57	48	38	26	18	12
1	161	1924	1144	1078	930	827	821	699	640	533	432	301	203	134
1	162	2914	1706	1606	1371	1212	1223	1042	954	795	644	450	304	204
1	163	2141	1253	1227	1058	896	901	780	683	580	464	313	216	142
1	164	3730	2144	2024	1739	1538	1538	1312	1197	999	809	563	381	262
1	165	3736	2191	2114	1817	1565	1574	1355	1206	1017	818	559	382	253
1	166	242	145	140	121	104	104	90	80	67	54	37	25	16
1	167	99	59	57	50	42	42	37	32	27	22	15	10	7
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1	169	51	31	29	25	22	22	19	17	14	12	8	6	4
1	170	4034	2316	2210	1902	1662	1662	1424	1284	1077	869	600	408	280
2	1	2095	1201	1368	1105	875	904	749	654	557	445	299	207	138
2	2	544	313	328	264	223	227	188	174	144	117	83	55	40
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2	4	1498	657	948	767	617	625	531	473	399	321	219	150	100
2	5	4761	2828	3080	2544	1986	2059	1725	1539	1297	1043	713	487	322
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2	7	3759	2706	2372	1969	1565	1615	1342	1212	1016	820	567	385	261

2	8	2723	1628	1812	1508	1168	1245	982	842	722	574	380	265	180
2	9	4133	2287	2706	2222	1768	1782	1503	1305	1113	889	595	412	273
2	10	921	499	618	497	402	401	339	287	248	196	129	90	58
2	11	2592	1691	1592	1352	1102	1105	916	817	689	554	378	259	178
2	12	2854	1634	1736	1498	1234	1207	1024	911	768	618	421	288	196
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2	18	2402	1394	1549	1272	1050	1017	873	744	640	508	334	234	153
2	19	5301	2584	3366	2781	2283	2247	1905	1651	1409	1125	751	521	349
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2	22	2205	1160	1440	1192	962	949	806	684	589	468	307	215	139
2	23	5499	3157	3515	2908	2299	2346	1974	1747	1477	1186	806	553	367
2	24	4581	2453	2930	2395	1921	1930	1642	1455	1230	987	671	460	309
2	25	6068	3527	3704	3126	2525	2650	2066	1848	1555	1252	858	586	423
2	26	2519	1351	1506	1275	1035	1076	866	801	665	540	380	255	180
2	27	4500	2125	2599	2240	1874	1902	1524	1410	1169	950	669	449	322
2	28	2932	2035	1727	1443	1194	1253	984	912	756	615	433	291	210
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2	30	4935	2227	3204	2671	2079	2161	1781	1568	1329	1065	721	496	331
2	31	3417	1833	2123	1772	1404	1475	1187	1067	896	722	497	338	233
2	32	3423	2465	2007	1790	1470	1468	1209	1116	926	752	529	355	243
2	33	2364	1376	1388	1228	997	1001	825	758	630	511	358	241	165
2	34	2311	1569	1373	1176	981	986	794	709	597	480	328	224	160
2	35	2583	1803	1519	1283	1074	1125	845	757	637	513	352	240	184
2	36	7349	3987	4475	3814	3115	3098	2586	2297	1939	1558	1061	727	499
2	37	1025	661	637	548	445	437	370	326	277	222	150	103	70
2	38	277	158	163	146	120	122	97	88	74	60	42	28	20
2	39	3392	2135	2075	1814	1462	1460	1206	1055	897	718	483	333	226
2	40	583	469	355	303	249	252	207	187	157	127	88	60	41
2	41	3157	2237	1862	1637	1352	1339	1103	1000	837	676	468	317	222
2	42	403	273	240	220	175	175	144	131	110	89	61	42	28
2	43	1029	912	615	530	435	438	366	340	282	229	162	108	74
2	44	1676	815	964	862	715	700	587	540	449	364	255	172	118
2	45	766	441	465	405	327	332	270	242	204	164	113	77	53
2	46	612	495	355	313	263	262	217	206	169	138	99	66	45
2	47	1700	1260	962	861	725	726	574	531	441	358	252	169	122
2	48	313	270	191	169	137	134	113	101	85	69	47	32	22
2	49	1912	1529	1147	1055	836	852	680	603	510	409	279	191	130
2	50	1668	1353	997	864	722	739	569	511	429	346	237	162	121
2	51	767	549	456	408	331	349	260	233	196	158	108	74	54
2	52	2159	1079	1232	1080	904	936	713	662	548	446	315	211	156

2	53	2032	1285	1226	1068	877	934	676	587	501	400	267	185	142
2	54	1009	701	574	499	423	420	349	332	272	223	160	106	74
2	55	7083	4805	4124	3642	3056	2950	2482	2236	1876	1513	1043	709	496
2	56	1853	1525	1095	940	778	783	654	616	507	414	295	197	135
2	57	823	657	506	438	352	352	299	273	228	184	129	87	58
2	58	676	451	398	349	286	288	236	218	181	147	103	69	49
2	59	1764	1124	1044	950	758	752	632	575	480	388	270	183	121
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2	61	2362	343	1315	1194	963	950	800	734	611	495	346	234	156
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2	63	2337	1581	1357	1177	981	980	800	731	610	494	344	233	165
2	64	1918	1450	1121	990	819	827	668	623	515	419	297	199	139
2	65	607	421	361	313	259	259	214	197	164	133	93	63	43
2	66	1345	839	819	703	566	571	480	439	366	297	207	140	94
2	67	1276	466	734	642	531	524	442	408	339	275	194	130	89
2	68	1930	975	1121	947	794	808	653	610	504	411	291	195	141
2	69	1587	937	951	807	658	664	555	509	424	344	240	162	112
2	70	1930	1153	1137	977	805	813	679	638	526	429	306	204	139
2	71	1426	1374	893	837	628	620	539	461	396	314	207	145	90
2	72	3765	2703	2223	1929	1608	1610	1298	1170	982	792	546	371	266
2	73	2643	1759	1558	1360	1116	1128	916	844	701	569	399	269	190
2	74	1867	1171	1100	963	794	794	647	590	492	398	277	187	132
2	75	1326	1108	807	709	570	567	474	427	358	289	199	135	93
2	76	1599	1035	932	817	680	679	562	528	435	355	253	169	116
2	77	2128	1924	1283	1102	916	911	766	708	588	477	335	225	154
2	78	2460	1648	1429	1215	1023	1021	853	801	660	539	383	256	179
2	79	1476	1148	870	763	628	643	509	467	389	315	221	149	106
2	80	2288	1670	1363	1186	969	968	815	756	627	510	359	241	163
2	81	2742	2349	1588	1407	1182	1172	977	925	760	621	445	296	200
2	82	850	645	483	429	366	363	298	286	233	192	138	91	63
2	83	3982	2604	2318	2010	1676	1691	1378	1287	1063	866	614	410	288
2	84	2107	1559	1254	1119	899	898	749	679	568	459	318	216	144
2	85	1459	1243	877	781	632	634	528	484	403	327	228	154	103
2	86	3147	1757	1853	1647	1338	1316	1118	1019	851	688	479	324	217
2	87	2730	2227	1667	1475	1174	1162	990	890	747	603	415	282	187
2	88	2524	2057	1473	1267	1053	1078	861	801	663	540	381	255	183
2	89	1202	807	726	624	508	532	410	370	310	250	173	117	85
2	90	2219	1765	1286	1156	959	964	776	722	597	486	343	230	160
2	91	1525	1126	890	793	658	662	535	500	413	337	239	160	111
2	92	2995	1619	1745	1471	1229	1259	1015	946	782	637	451	302	216
2	93	2424	1776	1416	1248	1036	1060	833	768	638	518	364	245	174
2	94	521	469	326	277	228	232	188	167	141	113	77	53	37
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2	96	4881	2930	2976	2519	2046	2062	1708	1525	1285	1034	708	483	332
2	97	2205	1492	1381	1159	936	972	766	670	570	456	307	212	150

2	98	2077	1230	1325	1094	886	899	734	638	544	434	291	201	140
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2	100	1785	1145	1023	884	760	730	611	550	461	372	256	174	126
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2	103	581	419	347	302	250	262	200	182	152	123	86	58	42
2	104	331	293	197	173	142	145	117	108	90	73	51	34	24
2	105	2403	1808	1442	1252	1016	1055	825	746	625	504	348	237	169
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2	109	348	240	213	184	148	154	122	110	92	74	51	35	24
2	110	2385	1431	1457	1230	1002	1054	819	735	618	498	342	233	167
2	111	1083	757	675	563	454	471	383	347	290	235	162	110	76
2	112	312	173	193	165	131	135	110	99	83	67	46	31	21
2	113	1657	1092	979	864	706	712	583	534	445	360	251	170	116
2	114	574	398	344	295	243	247	205	192	158	129	91	61	41
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2	116	2931	1441	1788	1520	1225	1288	1012	900	760	611	417	285	198
2	117	2608	1712	1796	1463	1134	1196	987	831	719	569	371	261	165
2	118	89	28	53	48	38	39	31	28	24	19	13	9	6
2	119	3621	2571	2236	1858	1506	1518	1286	1169	977	790	548	371	252
2	120	3308	2210	1999	1700	1388	1414	1161	1062	885	717	500	338	233
2	121	734	346	411	358	305	306	247	236	193	158	114	75	54
2	122	1249	900	724	646	538	531	443	415	342	279	198	132	90
2	123	792	538	461	425	339	353	268	238	201	161	110	75	54
2	124	433	292	261	235	190	187	160	145	121	98	68	46	30
2	125	141	95	84	73	60	61	51	48	40	32	23	15	10
2	126	857	594	497	446	372	361	306	283	235	191	134	90	61
2	127	559	335	319	268	229	235	194	191	155	128	94	61	42
2	128	1699	1280	994	857	714	716	603	569	468	382	273	182	121
2	129	1384	1155	816	703	588	590	496	471	386	316	227	151	101
2	130	595	565	335	294	253	249	208	202	164	135	99	65	45
2	131	567	429	343	303	241	240	205	187	156	126	88	59	39
2	132	252	218	148	131	107	105	90	83	69	56	39	26	17
2	133	429	191	195	121	141	173	131	161	119	104	88	53	40
2	134	3024	2171	1721	1485	1274	1272	1062	1026	835	687	499	329	223
2	135	1862	1165	1138	1008	801	789	678	606	510	411	282	192	126
2	136	3242	2107	1950	1694	1391	1350	1159	1040	874	705	484	330	224
2	137	2191	1182	1322	1152	931	922	787	716	598	484	336	228	151
2	138	785	559	489	421	336	334	286	258	217	175	121	82	54
2	139	3311	2336	1976	1714	1403	1395	1183	1096	909	739	520	349	236
2	140	4262	3179	2556	2207	1810	1781	1520	1386	1157	936	651	440	298
2	141	4631	3452	2880	2495	1998	1967	1692	1511	1272	1024	701	479	317
2	142	1230	27	712	591	492	488	417	379	317	256	178	120	81

2	143	352	299	222	195	152	151	130	115	98	78	53	37	24
2	144	2390	1081	1486	1264	1021	996	860	759	643	515	349	240	160
2	145	3454	2226	2133	1841	1481	1454	1251	1114	940	755	516	353	234
2	146	2743	1946	1650	1446	1172	1163	990	908	756	613	428	289	191
2	147	1631	1407	983	887	710	700	596	547	455	369	258	174	115
2	148	1161	1004	711	650	510	501	431	384	324	260	178	122	79
2	149	3623	2649	2231	1939	1574	1534	1321	1176	992	798	545	373	248
2	150	1615	863	999	850	692	674	581	515	435	349	237	163	109
2	151	1069	350	616	544	441	433	370	336	281	227	158	107	71
2	152	1785	1125	1094	928	758	747	640	577	484	391	270	183	124
2	153	2130	1349	1280	1114	909	896	764	698	582	471	328	222	150
2	154	3254	2235	1916	1673	1387	1358	1154	1064	884	718	504	339	233
2	155	673	521	395	349	288	283	240	223	184	150	106	71	48
2	156	1925	1369	1165	1013	823	806	690	623	522	421	291	198	133
2	157	1708	1331	999	878	738	712	607	556	463	375	262	177	122
2	158	2990	2200	1830	1615	1294	1273	1091	983	825	666	459	312	206
2	159	4307	3505	2595	2294	1858	1822	1558	1413	1182	955	662	449	299
2	160	173	106	103	92	75	73	63	57	48	38	26	18	12
2	161	1917	1625	1145	1039	834	821	699	640	533	432	301	203	134
2	162	2926	1945	1745	1522	1237	1223	1042	954	795	644	450	304	204
2	163	2140	1579	1336	1151	931	901	780	683	580	464	313	216	142
2	164	3724	2187	2172	1907	1590	1538	1312	1197	999	809	563	381	262
2	165	3740	2367	2294	2005	1609	1574	1355	1206	1017	818	559	382	253
2	166	243	215	151	134	105	104	90	80	67	54	37	25	16
2	167	99	47	61	56	43	42	37	32	27	22	15	10	7
2	168	118	67	73	66	51	50	44	38	33	26	18	12	8
2	169	51	52	31	28	22	22	19	17	14	12	8	6	4
2	170	4082	1807	2372	2076	1728	1662	1424	1284	1077	869	600	408	280
3	1	2119	1177	1568	1206	897	928	749	654	557	445	299	207	138
3	2	536	313	370	287	231	229	188	174	144	117	83	55	40
3	3	2305	828	1750	1411	996	1067	832	716	613	488	324	225	149
3	4	1524	499	1080	835	633	629	531	473	399	321	219	150	100
3	5	4777	2828	3498	2825	2008	2088	1725	1539	1297	1043	713	487	322
3	6	1750	954	1263	1051	744	806	622	546	463	371	250	172	117
3	7	3752	3301	2678	2190	1593	1639	1342	1212	1016	820	567	385	261
3	8	2713	1628	2056	1667	1193	1326	982	842	722	574	380	265	180
3	9	4139	2150	3065	2420	1826	1814	1503	1305	1113	889	595	412	273
3	10	927	459	704	527	420	412	339	287	248	196	129	90	58
3	11	2587	1894	1752	1480	1138	1126	916	817	689	554	378	259	178
3	12	2841	1601	1877	1625	1277	1218	1024	911	768	618	421	288	196
3	13	4109	3274	2768	2408	1791	1798	1473	1330	1115	900	621	422	286
3	14	5001	2278	3499	2706	2182	2057	1743	1521	1294	1035	695	480	325
3	15	2745	1465	2008	1559	1237	1185	989	849	728	579	384	267	177
3	16	3412	1602	2466	1921	1537	1533	1214	1044	894	712	472	329	224
3	17	4888	2067	3380	2670	2188	2061	1739	1501	1284	1023	681	473	315

3	18	2429	1394	1723	1346	1109	1030	873	744	640	508	334	234	153
3	19	5330	2170	3744	2976	2389	2283	1905	1651	1409	1125	751	521	349
3	20	2646	1141	1874	1500	1200	1172	955	819	702	559	370	258	171
3	21	4711	2189	3280	2597	2127	2071	1667	1432	1227	977	647	451	305
3	22	2236	1044	1606	1273	1007	970	806	684	589	468	307	215	139
3	23	5553	3093	3971	3201	2348	2378	1974	1747	1477	1186	806	553	367
3	24	4598	2257	3327	2622	1985	1944	1642	1455	1230	987	671	460	309
3	25	6088	3527	4123	3471	2585	2776	2066	1848	1555	1252	858	586	423
3	26	2532	1243	1674	1431	1051	1100	866	801	665	540	380	255	180
3	27	4528	1742	2826	2474	1914	1950	1524	1410	1169	950	669	449	322
3	28	2960	2442	1919	1600	1213	1294	984	912	756	615	433	291	210
3	29	1339	927	854	738	568	585	468	444	364	298	214	142	98
3	30	4945	1692	3625	2965	2112	2220	1781	1568	1329	1065	721	496	331
3	31	3462	1687	2390	1975	1425	1521	1187	1067	896	722	497	338	233
3	32	3418	3007	2147	1990	1488	1488	1209	1116	926	752	529	355	243
3	33	2386	1376	1500	1375	1013	1015	825	758	630	511	358	241	165
3	34	2316	1851	1490	1275	1015	1020	794	709	597	480	328	224	160
3	35	2583	2200	1671	1404	1110	1195	845	757	637	513	352	240	184
3	36	7396	3747	4908	4166	3224	3152	2586	2297	1939	1558	1061	727	499
3	37	1016	728	696	598	463	441	370	326	277	222	150	103	70
3	38	276	152	173	163	121	126	97	88	74	60	42	28	20
3	39	3413	2306	2240	1986	1509	1501	1206	1055	897	718	483	333	226
3	40	579	638	388	330	253	258	207	187	157	127	88	60	41
3	41	3145	2729	2004	1805	1392	1363	1103	1000	837	676	468	317	222
3	42	403	312	254	248	176	178	144	131	110	89	61	42	28
3	43	1025	1368	671	589	441	441	366	340	282	229	162	108	74
3	44	1690	684	1024	952	730	704	587	540	449	364	255	172	118
3	45	766	433	505	448	334	341	270	242	204	164	113	77	53
3	46	605	673	382	348	265	263	217	206	169	138	99	66	45
3	47	1706	1613	1021	952	739	748	574	531	441	358	252	169	122
3	48	311	395	205	185	141	136	113	101	85	69	47	32	22
3	49	1911	2049	1207	1179	844	887	680	603	510	409	279	191	130
3	50	1627	1867	1079	950	746	777	569	511	429	346	237	162	121
3	51	764	658	488	455	333	372	260	233	196	158	108	74	54
3	52	2175	928	1330	1204	916	977	713	662	548	446	315	211	156
3	53	2011	1387	1327	1174	905	1022	676	587	501	400	267	185	142
3	54	1010	841	621	553	430	422	349	332	272	223	160	106	74
3	55	7054	5670	4387	3965	3178	2979	2482	2236	1876	1513	1043	709	496
3	56	1836	2135	1200	1049	791	786	654	616	507	414	295	197	135
3	57	814	880	554	489	358	353	299	273	228	184	129	87	58
3	58	673	514	433	390	292	293	236	218	181	147	103	69	49
3	59	1777	1213	1110	1065	766	759	632	575	480	388	270	183	121
3	60	501	327	324	288	211	210	176	164	136	110	78	52	36
3	61	2503	89	1399	1340	974	956	800	734	611	495	346	234	156
3	62	2758	1673	1760	1516	1170	1165	956	911	746	611	440	291	206

3	63	2352	1866	1470	1295	1009	1000	800	731	610	494	344	233	165
3	64	1904	1856	1209	1105	830	844	668	623	515	419	297	199	139
3	65	602	496	392	345	265	262	214	197	164	133	93	63	43
3	66	1346	890	901	783	575	575	480	439	366	297	207	140	94
3	67	1299	299	792	710	543	525	442	408	339	275	194	130	89
3	68	1925	858	1237	1049	815	824	653	610	504	411	291	195	141
3	69	1593	955	1053	897	674	670	555	509	424	344	240	162	112
3	70	1932	1176	1246	1089	816	816	679	638	526	429	306	204	139
3	71	1430	2198	931	938	633	623	539	461	396	314	207	145	90
3	72	3740	3351	2402	2112	1662	1658	1298	1170	982	792	546	371	266
3	73	2630	2005	1696	1521	1140	1151	916	844	701	569	399	269	190
3	74	1860	1265	1189	1068	816	812	647	590	492	398	277	187	132
3	75	1314	1573	877	792	585	575	474	427	358	289	199	135	93
3	76	1588	1138	1007	911	690	685	562	528	435	355	253	169	116
3	77	2088	2924	1395	1208	934	915	766	708	588	477	335	225	154
3	78	2453	1911	1565	1341	1048	1026	853	801	660	539	383	256	179
3	79	1471	1515	941	847	637	664	509	467	389	315	221	149	106
3	80	2279	2071	1484	1321	985	971	815	756	627	510	359	241	163
3	81	2724	3383	1696	1555	1191	1176	977	925	760	621	445	296	200
3	82	844	826	516	476	369	366	298	286	233	192	138	91	63
3	83	3981	2917	2515	2228	1703	1718	1378	1287	1063	866	614	410	288
3	84	2129	1964	1344	1246	909	910	749	679	568	459	318	216	144
3	85	1451	1764	939	868	635	640	528	484	403	327	228	154	103
3	86	3183	1687	1986	1826	1362	1319	1118	1019	851	688	479	324	217
3	87	2728	3073	1803	1644	1196	1167	990	890	747	603	415	282	187
3	88	2524	2879	1610	1408	1072	1104	861	801	663	540	381	255	183
3	89	1197	920	796	692	517	558	410	370	310	250	173	117	85
3	90	2210	2366	1367	1285	967	985	776	722	597	486	343	230	160
3	91	1512	1396	952	884	664	675	535	500	413	337	239	160	111
3	92	3008	1522	1927	1628	1256	1285	1015	946	782	637	451	302	216
3	93	2417	2202	1520	1381	1048	1095	833	768	638	518	364	245	174
3	94	509	704	356	301	234	240	188	167	141	113	77	53	37
3	95	500	142	309	270	207	235	157	146	121	99	70	47	37
3	96	4929	3047	3287	2763	2107	2102	1708	1525	1285	1034	708	483	332
3	97	2203	1731	1537	1270	968	1024	766	670	570	456	307	212	150
3	98	2073	1255	1493	1191	926	933	734	638	544	434	291	201	140
3	99	3662	2484	2555	2064	1593	1567	1294	1165	978	789	543	370	257
3	100	1782	1306	1095	949	799	739	611	550	461	372	256	174	126
3	101	1508	1759	924	847	651	660	526	503	411	337	243	161	112
3	102	4059	4335	2541	2350	1773	1873	1397	1279	1066	863	603	407	291
3	103	576	502	374	332	251	275	200	182	152	123	86	58	42
3	104	330	433	211	192	143	148	117	108	90	73	51	34	24
3	105	2402	2314	1573	1394	1033	1100	825	746	625	504	348	237	169
3	106	2214	1608	1360	1207	928	969	739	698	574	469	335	223	161
3	107	1541	1528	1008	918	670	712	546	501	417	338	236	159	111

3	108	755	451	493	440	326	332	272	248	207	167	117	79	53
3	109	348	278	232	204	150	159	122	110	92	74	51	35	24
3	110	2380	1460	1610	1354	1020	1104	819	735	618	498	342	233	167
3	111	1076	893	754	623	462	483	383	347	290	235	162	110	76
3	112	315	163	213	183	132	139	110	99	83	67	46	31	21
3	113	1667	1223	1052	957	713	725	583	534	445	360	251	170	116
3	114	573	461	375	325	244	250	205	192	158	129	91	61	41
3	115	1983	679	1190	1007	806	845	634	608	497	408	295	195	146
3	116	2978	1210	1963	1665	1238	1348	1012	900	760	611	417	285	198
3	117	2601	1849	2024	1556	1139	1253	987	831	719	569	371	261	165
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3	119	3623	3137	2507	2050	1545	1524	1286	1169	977	790	548	371	252
3	120	3306	2520	2202	1879	1414	1439	1161	1062	885	717	500	338	233
3	121	738	283	444	397	311	310	247	236	193	158	114	75	54
3	122	1242	1098	773	717	545	534	443	415	342	279	198	132	90
3	123	805	624	485	477	342	375	268	238	201	161	110	75	54
3	124	430	327	276	260	191	187	160	145	121	98	68	46	30
3	125	140	106	92	82	60	61	51	48	40	32	23	15	10
3	126	850	701	527	493	379	361	306	283	235	191	134	90	61
3	127	557	342	351	300	231	235	194	191	155	128	94	61	42
3	128	1711	1638	1077	943	719	716	603	569	468	382	273	182	121
3	129	1373	1617	886	775	593	590	496	471	386	316	227	151	101
3	130	588	916	360	328	257	249	208	202	164	135	99	65	45
3	131	570	549	372	338	244	240	205	187	156	126	88	59	39
3	132	256	322	158	145	108	105	90	83	69	56	39	26	17
3	133	423	149	224	129	141	173	131	161	119	104	88	53	40
3	134	3030	2649	1856	1631	1283	1272	1062	1026	835	687	499	329	223
3	135	1868	1235	1226	1117	815	789	678	606	510	411	282	192	126
3	136	3235	2360	2111	1853	1440	1350	1159	1040	874	705	484	330	224
3	137	2199	1088	1436	1276	949	922	787	716	598	484	336	228	151
3	138	779	671	538	467	343	334	286	258	217	175	121	82	54
3	139	3293	2803	2154	1907	1429	1395	1183	1096	909	739	520	349	236
3	140	4258	4069	2783	2430	1856	1781	1520	1386	1157	936	651	440	298
3	141	4602	4350	3145	2753	2048	1967	1692	1511	1272	1024	701	479	317
3	142	1296	1	786	636	505	488	417	379	317	256	178	120	81
3	143	350	424	242	217	155	151	130	115	98	78	53	37	24
3	144	2404	843	1638	1380	1060	996	860	759	643	515	349	240	160
3	145	3459	2449	2328	2022	1522	1454	1251	1114	940	755	516	353	234
3	146	2742	2335	1785	1603	1187	1163	990	908	756	613	428	289	191
3	147	1613	2026	1053	997	718	700	596	547	455	369	258	174	115
3	148	1154	1446	752	726	516	501	431	384	324	260	178	122	79
3	149	3593	3285	2415	2121	1623	1534	1321	1176	992	798	545	373	248
3	150	1617	794	1098	926	719	674	581	515	435	349	237	163	109
3	151	1112	203	662	602	450	433	370	336	281	227	158	107	71
3	152	1780	1215	1205	1016	781	747	640	577	484	391	270	183	124

3	153	2120	1457	1391	1232	930	896	764	698	582	471	328	222	150
3	154	3231	2638	2073	1850	1425	1358	1154	1064	884	718	504	339	233
3	155	670	688	425	389	293	283	240	223	184	150	106	71	48
3	156	1917	1671	1267	1119	849	806	690	623	522	421	291	198	133
3	157	1687	1783	1067	959	762	712	607	556	463	375	262	177	122
3	158	2969	2728	1976	1793	1321	1273	1091	983	825	666	459	312	206
3	159	4288	4837	2792	2540	1898	1822	1558	1413	1182	955	662	449	299
3	160	174	111	110	100	76	73	63	57	48	38	26	18	12
3	161	1911	2307	1216	1160	841	821	699	640	533	432	301	203	134
3	162	2939	2217	1897	1690	1262	1223	1042	954	795	644	450	304	204
3	163	2139	1990	1455	1253	967	901	780	683	580	464	313	216	142
3	164	3718	2231	2331	2090	1644	1538	1312	1197	999	809	563	381	262
3	165	3745	2556	2488	2211	1655	1574	1355	1206	1017	818	559	382	253
3	166	244	318	162	149	105	104	90	80	67	54	37	25	16
3	167	99	38	64	62	44	42	37	32	27	22	15	10	7
3	168	119	65	78	73	52	50	44	38	33	26	18	12	8
3	169	50	89	32	30	22	22	19	17	14	12	8	6	4
3	170	4130	1409	2545	2266	1796	1662	1424	1284	1077	869	600	408	280
4	1	2144	1153	1797	1315	919	953	749	654	557	445	299	207	138
4	2	528	313	417	312	239	231	188	174	144	117	83	55	40
4	3	2316	646	2000	1567	1017	1114	832	716	613	488	324	225	149
4	4	1549	379	1230	910	651	633	531	473	399	321	219	150	100
4	5	4794	2828	3972	3137	2030	2118	1725	1539	1297	1043	713	487	322
4	6	1757	915	1420	1170	755	839	622	546	463	371	250	172	117
4	7	3746	4027	3023	2436	1622	1664	1342	1212	1016	820	567	385	261
4	8	2704	1628	2333	1844	1218	1411	982	842	722	574	380	265	180
4	9	4144	2021	3473	2636	1886	1848	1503	1305	1113	889	595	412	273
4	10	934	422	802	559	439	423	339	287	248	196	129	90	58
4	11	2581	2121	1929	1620	1174	1149	916	817	689	554	378	259	178
4	12	2827	1569	2030	1762	1322	1228	1024	911	768	618	421	288	196
4	13	4094	3798	3032	2663	1826	1824	1473	1330	1115	900	621	422	286
4	14	5074	2050	3943	2901	2291	2075	1743	1521	1294	1035	695	480	325
4	15	2762	1406	2266	1673	1298	1206	989	849	728	579	384	267	177
4	16	3421	1442	2774	2059	1612	1592	1214	1044	894	712	472	329	224
4	17	4940	1778	3740	2830	2295	2086	1739	1501	1284	1023	681	473	315
4	18	2458	1394	1916	1424	1172	1044	873	744	640	508	334	234	153
4	19	5359	1823	4164	3184	2500	2319	1905	1651	1409	1125	751	521	349
4	20	2657	981	2075	1601	1253	1205	955	819	702	559	370	258	171
4	21	4747	1970	3635	2759	2237	2138	1667	1432	1227	977	647	451	305
4	22	2268	940	1792	1360	1054	992	806	684	589	468	307	215	139
4	23	5608	3032	4485	3523	2398	2412	1974	1747	1477	1186	806	553	367
4	24	4614	2076	3779	2871	2051	1959	1642	1455	1230	987	671	460	309
4	25	6109	3527	4589	3854	2646	2908	2066	1848	1555	1252	858	586	423
4	26	2545	1143	1861	1604	1066	1126	866	801	665	540	380	255	180
4	27	4557	1429	3074	2732	1955	1998	1524	1410	1169	950	669	449	322

4	28	2988	2930	2132	1774	1232	1336	984	912	756	615	433	291	210
4	29	1335	1001	930	814	572	593	468	444	364	298	214	142	98
4	30	4955	1286	4100	3293	2145	2280	1781	1568	1329	1065	721	496	331
4	31	3507	1552	2690	2201	1446	1569	1187	1067	896	722	497	338	233
4	32	3414	3669	2295	2212	1507	1509	1209	1116	926	752	529	355	243
4	33	2409	1376	1620	1538	1029	1030	825	758	630	511	358	241	165
4	34	2322	2184	1616	1383	1051	1055	794	709	597	480	328	224	160
4	35	2583	2684	1837	1538	1146	1269	845	757	637	513	352	240	184
4	36	7443	3523	5383	4550	3336	3207	2586	2297	1939	1558	1061	727	499
4	37	1007	800	759	654	481	446	370	326	277	222	150	103	70
4	38	275	146	185	181	123	130	97	88	74	60	42	28	20
4	39	3435	2490	2419	2174	1557	1543	1206	1055	897	718	483	333	226
4	40	576	867	424	361	257	263	207	187	157	127	88	60	41
4	41	3133	3329	2156	1989	1433	1388	1103	1000	837	676	468	317	222
4	42	403	355	269	279	178	181	144	131	110	89	61	42	28
4	43	1020	2052	733	654	447	444	366	340	282	229	162	108	74
4	44	1703	575	1089	1052	745	709	587	540	449	364	255	172	118
4	45	765	424	548	495	341	350	270	242	204	164	113	77	53
4	46	598	916	410	387	268	265	217	206	169	138	99	66	45
4	47	1711	2064	1085	1054	753	771	574	531	441	358	252	169	122
4	48	308	577	219	204	145	137	113	101	85	69	47	32	22
4	49	1910	2746	1271	1317	852	922	680	603	510	409	279	191	130
4	50	1588	2576	1168	1043	771	817	569	511	429	346	237	162	121
4	51	762	790	523	508	335	396	260	233	196	158	108	74	54
4	52	2191	798	1435	1341	928	1020	713	662	548	446	315	211	156
4	53	1991	1498	1435	1290	934	1118	676	587	501	400	267	185	142
4	54	1012	1010	672	613	438	423	349	332	272	223	160	106	74
4	55	7026	6691	4667	4316	3305	3007	2482	2236	1876	1513	1043	709	496
4	56	1820	2989	1315	1170	805	789	654	616	507	414	295	197	135
4	57	805	1179	607	546	364	354	299	273	228	184	129	87	58
4	58	670	586	470	436	298	299	236	218	181	147	103	69	49
4	59	1790	1311	1180	1195	773	765	632	575	480	388	270	183	121
4	60	502	347	356	323	214	211	176	164	136	110	78	52	36
4	61	2652	23	1488	1504	986	961	800	734	611	495	346	234	156
4	62	2741	1707	1935	1689	1194	1173	956	911	746	611	440	291	206
4	63	2366	2202	1592	1425	1037	1021	800	731	610	494	344	233	165
4	64	1889	2376	1303	1233	841	860	668	623	515	419	297	199	139
4	65	598	586	425	380	270	265	214	197	164	133	93	63	43
4	66	1346	943	992	872	585	579	480	439	366	297	207	140	94
4	67	1323	191	855	785	555	527	442	408	339	275	194	130	89
4	68	1921	755	1365	1162	837	839	653	610	504	411	291	195	141
4	69	1600	974	1166	996	690	675	555	509	424	344	240	162	112
4	70	1934	1200	1365	1214	827	819	679	638	526	429	306	204	139
4	71	1435	3517	969	1050	639	625	539	461	396	314	207	145	90
4	72	3714	4156	2596	2313	1718	1707	1298	1170	982	792	546	371	266

4	73	2616	2286	1847	1700	1165	1175	916	844	701	569	399	269	190
4	74	1853	1366	1285	1183	839	830	647	590	492	398	277	187	132
4	75	1302	2234	954	884	601	583	474	427	358	289	199	135	93
4	76	1576	1252	1088	1015	700	690	562	528	435	355	253	169	116
4	77	2049	4445	1517	1325	952	918	766	708	588	477	335	225	154
4	78	2446	2217	1715	1479	1073	1030	853	801	660	539	383	256	179
4	79	1466	2000	1017	940	646	685	509	467	389	315	221	149	106
4	80	2270	2568	1616	1472	1001	974	815	756	627	510	359	241	163
4	81	2706	4872	1810	1719	1199	1180	977	925	760	621	445	296	200
4	82	837	1057	550	528	372	369	298	286	233	192	138	91	63
4	83	3979	3267	2730	2469	1731	1745	1378	1287	1063	866	614	410	288
4	84	2151	2475	1440	1387	919	921	749	679	568	459	318	216	144
4	85	1443	2505	1005	965	639	647	528	484	403	327	228	154	103
4	86	3220	1620	2128	2025	1386	1322	1118	1019	851	688	479	324	217
4	87	2725	4240	1950	1833	1218	1171	990	890	747	603	415	282	187
4	88	2524	4031	1759	1564	1091	1131	861	801	663	540	381	255	183
4	89	1193	1048	872	768	526	586	410	370	310	250	173	117	85
4	90	2201	3170	1453	1428	975	1006	776	722	597	486	343	230	160
4	91	1500	1731	1018	986	670	687	535	500	413	337	239	160	111
4	92	3022	1431	2128	1802	1283	1311	1015	946	782	637	451	302	216
4	93	2411	2730	1632	1530	1059	1132	833	768	638	518	364	245	174
4	94	497	1056	389	326	239	248	188	167	141	113	77	53	37
4	95	502	100	338	301	210	250	157	146	121	99	70	47	37
4	96	4978	3169	3631	3031	2169	2143	1708	1525	1285	1034	708	483	332
4	97	2200	2007	1712	1392	1001	1078	766	670	570	456	307	212	150
4	98	2070	1280	1681	1296	968	968	734	638	544	434	291	201	140
4	99	3644	2682	2869	2265	1650	1587	1294	1165	978	789	543	370	257
4	100	1778	1489	1172	1019	840	749	611	550	461	372	256	174	126
4	101	1502	2463	989	940	655	670	526	503	411	337	243	161	112
4	102	4045	5809	2711	2603	1790	1949	1397	1279	1066	863	603	407	291
4	103	571	603	403	364	252	289	200	182	152	123	86	58	42
4	104	328	641	226	212	144	151	117	108	90	73	51	34	24
4	105	2402	2962	1717	1552	1049	1147	825	746	625	504	348	237	169
4	106	2226	1801	1474	1341	938	997	739	698	574	469	335	223	161
4	107	1530	1955	1088	1024	673	735	546	501	417	338	236	159	111
4	108	755	451	533	485	328	336	272	248	207	167	117	79	53
4	109	348	322	253	225	152	165	122	110	92	74	51	35	24
4	110	2376	1489	1780	1491	1039	1156	819	735	618	498	342	233	167
4	111	1069	1053	843	689	470	495	383	347	290	235	162	110	76
4	112	319	153	235	203	134	143	110	99	83	67	46	31	21
4	113	1677	1370	1131	1059	721	738	583	534	445	360	251	170	116
4	114	572	535	408	358	245	252	205	192	158	129	91	61	41
4	115	2006	530	1303	1110	820	870	634	608	497	408	295	195	146
4	116	3026	1017	2155	1824	1251	1410	1012	900	760	611	417	285	198
4	117	2595	1997	2282	1656	1144	1312	987	831	719	569	371	261	165

4	118	91	8	59	60	39	42	31	28	24	19	13	9	6
4	119	3624	3827	2811	2262	1586	1530	1286	1169	977	790	548	371	252
4	120	3303	2872	2426	2077	1441	1464	1161	1062	885	717	500	338	233
4	121	741	232	481	441	317	315	247	236	193	158	114	75	54
4	122	1235	1339	824	795	553	537	443	415	342	279	198	132	90
4	123	819	723	512	535	345	398	268	238	201	161	110	75	54
4	124	427	366	291	286	192	187	160	145	121	98	68	46	30
4	125	138	119	100	91	61	61	51	48	40	32	23	15	10
4	126	843	827	558	544	387	361	306	283	235	191	134	90	61
4	127	556	348	387	335	232	235	194	191	155	128	94	61	42
4	128	1723	2097	1168	1037	723	716	603	569	468	382	273	182	121
4	129	1361	2263	962	855	597	590	496	471	386	316	227	151	101
4	130	580	1484	387	366	261	249	208	202	164	135	99	65	45
4	131	572	703	403	378	247	240	205	187	156	126	88	59	39
4	132	259	477	169	160	109	105	90	83	69	56	39	26	17
4	133	418	116	258	138	142	173	131	161	119	104	88	53	40
4	134	3037	3232	2002	1791	1291	1272	1062	1026	835	687	499	329	223
4	135	1875	1309	1321	1238	830	789	678	606	510	411	282	192	126
4	136	3228	2643	2284	2028	1491	1350	1159	1040	874	705	484	330	224
4	137	2206	1001	1560	1414	966	922	787	716	598	484	336	228	151
4	138	774	805	593	519	350	334	286	258	217	175	121	82	54
4	139	3276	3363	2348	2121	1455	1395	1183	1096	909	739	520	349	236
4	140	4255	5208	3030	2674	1904	1781	1520	1386	1157	936	651	440	298
4	141	4572	5481	3434	3037	2100	1967	1692	1511	1272	1024	701	479	317
4	142	1366	0	866	684	518	488	417	379	317	256	178	120	81
4	143	349	602	263	241	158	151	130	115	98	78	53	37	24
4	144	2417	658	1805	1508	1101	996	860	759	643	515	349	240	160
4	145	3464	2694	2542	2221	1565	1454	1251	1114	940	755	516	353	234
4	146	2741	2802	1931	1777	1203	1163	990	908	756	613	428	289	191
4	147	1594	2917	1128	1122	726	700	596	547	455	369	258	174	115
4	148	1146	2083	795	810	522	501	431	384	324	260	178	122	79
4	149	3564	4074	2614	2320	1672	1534	1321	1176	992	798	545	373	248
4	150	1618	730	1206	1009	747	674	581	515	435	349	237	163	109
4	151	1156	118	712	666	459	433	370	336	281	227	158	107	71
4	152	1774	1312	1326	1113	805	747	640	577	484	391	270	183	124
4	153	2109	1573	1512	1363	951	896	764	698	582	471	328	222	150
4	154	3208	3113	2242	2044	1463	1358	1154	1064	884	718	504	339	233
4	155	666	908	457	433	299	283	240	223	184	150	106	71	48
4	156	1910	2038	1379	1237	875	806	690	623	522	421	291	198	133
4	157	1666	2390	1140	1048	787	712	607	556	463	375	262	177	122
4	158	2948	3383	2133	1990	1347	1273	1091	983	825	666	459	312	206
4	159	4269	6675	3003	2813	1938	1822	1558	1413	1182	955	662	449	299
4	160	175	115	116	109	77	73	63	57	48	38	26	18	12
4	161	1904	3277	1291	1296	848	821	699	640	533	432	301	203	134
4	162	2951	2527	2062	1876	1288	1223	1042	954	795	644	450	304	204

4	163	2138	2507	1584	1363	1005	901	780	683	580	464	313	216	142
4	164	3712	2275	2501	2292	1700	1538	1312	1197	999	809	563	381	262
4	165	3749	2760	2699	2439	1702	1574	1355	1206	1017	818	559	382	253
4	166	245	470	175	165	106	104	90	80	67	54	37	25	16
4	167	99	30	68	70	45	42	37	32	27	22	15	10	7
4	168	121	62	83	81	52	50	44	38	33	26	18	12	8
4	169	50	152	34	34	22	22	19	17	14	12	8	6	4
4	170	4178	1099	2732	2473	1867	1662	1424	1284	1077	869	600	408	280
5	1	2169	1130	2059	1435	942	979	749	654	557	445	299	207	138
5	2	519	313	471	340	248	234	188	174	144	117	83	55	40
5	3	2327	504	2286	1741	1038	1163	832	716	613	488	324	225	149
5	4	1576	288	1402	992	668	637	531	473	399	321	219	150	100
5	5	4810	2828	4510	3483	2053	2148	1725	1539	1297	1043	713	487	322
5	6	1764	879	1597	1302	765	874	622	546	463	371	250	172	117
5	7	3739	4913	3412	2710	1651	1689	1342	1212	1016	820	567	385	261
5	8	2694	1628	2647	2038	1244	1502	982	842	722	574	380	265	180
5	9	4150	1900	3935	2871	1948	1882	1503	1305	1113	889	595	412	273
5	10	941	388	913	593	459	435	339	287	248	196	129	90	58
5	11	2576	2375	2123	1774	1212	1171	916	817	689	554	378	259	178
5	12	2813	1538	2195	1912	1368	1239	1024	911	768	618	421	288	196
5	13	4080	4406	3320	2945	1862	1851	1473	1330	1115	900	621	422	286
5	14	5148	1845	4444	3110	2405	2093	1743	1521	1294	1035	695	480	325
5	15	2780	1350	2557	1795	1361	1228	989	849	728	579	384	267	177
5	16	3429	1298	3120	2207	1690	1652	1214	1044	894	712	472	329	224
5	17	4992	1529	4138	2999	2407	2112	1739	1501	1284	1023	681	473	315
5	18	2486	1394	2131	1506	1238	1059	873	744	640	508	334	234	153
5	19	5388	1531	4631	3407	2616	2356	1905	1651	1409	1125	751	521	349
5	20	2668	844	2298	1708	1310	1238	955	819	702	559	370	258	171
5	21	4784	1773	4027	2931	2353	2207	1667	1432	1227	977	647	451	305
5	22	2299	846	1999	1453	1104	1014	806	684	589	468	307	215	139
5	23	5663	2971	5067	3877	2449	2446	1974	1747	1477	1186	806	553	367
5	24	4631	1910	4292	3143	2119	1974	1642	1455	1230	987	671	460	309
5	25	6130	3527	5108	4280	2709	3047	2066	1848	1555	1252	858	586	423
5	26	2559	1052	2069	1800	1082	1152	866	801	665	540	380	255	180
5	27	4587	1172	3343	3017	1997	2048	1524	1410	1169	950	669	449	322
5	28	3017	3516	2369	1967	1251	1379	984	912	756	615	433	291	210
5	29	1332	1082	1013	896	575	601	468	444	364	298	214	142	98
5	30	4965	978	4638	3656	2178	2341	1781	1568	1329	1065	721	496	331
5	31	3554	1428	3028	2454	1467	1619	1187	1067	896	722	497	338	233
5	32	3409	4476	2455	2460	1525	1530	1209	1116	926	752	529	355	243
5	33	2431	1376	1751	1721	1045	1045	825	758	630	511	358	241	165
5	34	2327	2577	1754	1499	1088	1090	794	709	597	480	328	224	160
5	35	2583	3275	2020	1684	1184	1348	845	757	637	513	352	240	184
5	36	7491	3311	5905	4970	3453	3263	2586	2297	1939	1558	1061	727	499
5	37	998	880	829	714	499	451	370	326	277	222	150	103	70

5	38	274	140	196	201	124	134	97	88	74	60	42	28	20
5	39	3457	2690	2612	2380	1607	1586	1206	1055	897	718	483	333	226
5	40	573	1179	463	394	262	268	207	187	157	127	88	60	41
5	41	3121	4062	2320	2193	1475	1413	1103	1000	837	676	468	317	222
5	42	403	405	285	314	179	184	144	131	110	89	61	42	28
5	43	1016	3078	800	726	453	447	366	340	282	229	162	108	74
5	44	1717	483	1157	1163	761	714	587	540	449	364	255	172	118
5	45	765	415	595	548	348	360	270	242	204	164	113	77	53
5	46	591	1245	441	430	271	267	217	206	169	138	99	66	45
5	47	1717	2643	1152	1166	768	795	574	531	441	358	252	169	122
5	48	305	842	235	224	149	139	113	101	85	69	47	32	22
5	49	1908	3680	1338	1472	860	960	680	603	510	409	279	191	130
5	50	1549	3555	1264	1146	797	859	569	511	429	346	237	162	121
5	51	759	948	560	568	338	422	260	233	196	158	108	74	54
5	52	2207	686	1549	1494	939	1064	713	662	548	446	315	211	156
5	53	1971	1618	1553	1418	964	1224	676	587	501	400	267	185	142
5	54	1013	1212	727	679	446	425	349	332	272	223	160	106	74
5	55	6998	7896	4965	4698	3438	3036	2482	2236	1876	1513	1043	709	496
5	56	1804	4184	1441	1306	819	792	654	616	507	414	295	197	135
5	57	797	1580	665	608	369	355	299	273	228	184	129	87	58
5	58	667	668	510	488	304	304	236	218	181	147	103	69	49
5	59	1803	1415	1254	1340	781	772	632	575	480	388	270	183	121
5	60	504	368	390	364	217	211	176	164	136	110	78	52	36
5	61	2810	6	1583	1687	997	966	800	734	611	495	346	234	156
5	62	2725	1741	2127	1882	1219	1180	956	911	746	611	440	291	206
5	63	2381	2598	1725	1568	1066	1041	800	731	610	494	344	233	165
5	64	1875	3042	1405	1376	852	877	668	623	515	419	297	199	139
5	65	593	691	461	419	276	269	214	197	164	133	93	63	43
5	66	1346	999	1092	971	594	583	480	439	366	297	207	140	94
5	67	1347	122	923	868	567	529	442	408	339	275	194	130	89
5	68	1917	664	1506	1288	859	855	653	610	504	411	291	195	141
5	69	1607	994	1291	1107	707	681	555	509	424	344	240	162	112
5	70	1936	1224	1496	1354	838	822	679	638	526	429	306	204	139
5	71	1439	5626	1010	1177	644	628	539	461	396	314	207	145	90
5	72	3689	5153	2805	2533	1776	1758	1298	1170	982	792	546	371	266
5	73	2603	2606	2010	1901	1189	1199	916	844	701	569	399	269	190
5	74	1846	1475	1390	1311	862	848	647	590	492	398	277	187	132
5	75	1290	3172	1038	988	617	590	474	427	358	289	199	135	93
5	76	1565	1377	1176	1132	711	696	562	528	435	355	253	169	116
5	77	2011	6756	1649	1453	971	922	766	708	588	477	335	225	154
5	78	2439	2572	1878	1631	1099	1035	853	801	660	539	383	256	179
5	79	1461	2640	1099	1045	655	707	509	467	389	315	221	149	106
5	80	2260	3185	1760	1640	1017	977	815	756	627	510	359	241	163
5	81	2688	7015	1933	1900	1208	1185	977	925	760	621	445	296	200
5	82	831	1353	587	586	375	371	298	286	233	192	138	91	63

5	83	3978	3659	2962	2736	1760	1773	1378	1287	1063	866	614	410	288
5	84	2173	3118	1544	1544	930	933	749	679	568	459	318	216	144
5	85	1434	3558	1076	1073	642	654	528	484	403	327	228	154	103
5	86	3256	1555	2281	2245	1410	1326	1118	1019	851	688	479	324	217
5	87	2723	5852	2108	2043	1240	1176	990	890	747	603	415	282	187
5	88	2524	5644	1923	1738	1111	1159	861	801	663	540	381	255	183
5	89	1189	1195	957	853	535	615	410	370	310	250	173	117	85
5	90	2193	4248	1544	1588	983	1028	776	722	597	486	343	230	160
5	91	1487	2147	1089	1099	675	700	535	500	413	337	239	160	111
5	92	3035	1345	2350	1994	1311	1338	1015	946	782	637	451	302	216
5	93	2405	3385	1752	1694	1070	1170	833	768	638	518	364	245	174
5	94	485	1584	425	354	245	256	188	167	141	113	77	53	37
5	95	504	70	369	335	212	266	157	146	121	99	70	47	37
5	96	5026	3296	4011	3325	2234	2185	1708	1525	1285	1034	708	483	332
5	97	2197	2329	1906	1525	1035	1135	766	670	570	456	307	212	150
5	98	2067	1305	1893	1412	1011	1005	734	638	544	434	291	201	140
5	99	3627	2897	3222	2486	1709	1608	1294	1165	978	789	543	370	257
5	100	1775	1697	1254	1094	883	758	611	550	461	372	256	174	126
5	101	1496	3448	1058	1043	659	680	526	503	411	337	243	161	112
5	102	4032	7783	2893	2884	1808	2029	1397	1279	1066	863	603	407	291
5	103	566	723	435	400	253	303	200	182	152	123	86	58	42
5	104	327	949	243	235	145	155	117	108	90	73	51	34	24
5	105	2401	3791	1873	1728	1066	1195	825	746	625	504	348	237	169
5	106	2238	2017	1598	1489	948	1025	739	698	574	469	335	223	161
5	107	1518	2503	1175	1143	675	759	546	501	417	338	236	159	111
5	108	755	451	576	535	330	340	272	248	207	167	117	79	53
5	109	348	374	275	248	154	171	122	110	92	74	51	35	24
5	110	2371	1519	1967	1641	1057	1211	819	735	618	498	342	233	167
5	111	1062	1243	942	762	478	507	383	347	290	235	162	110	76
5	112	322	144	260	225	135	146	110	99	83	67	46	31	21
5	113	1687	1534	1215	1173	728	752	583	534	445	360	251	170	116
5	114	571	621	445	395	245	254	205	192	158	129	91	61	41
5	115	2029	413	1428	1223	835	895	634	608	497	408	295	195	146
5	116	3075	854	2366	1997	1264	1476	1012	900	760	611	417	285	198
5	117	2588	2157	2573	1761	1149	1375	987	831	719	569	371	261	165
5	118	92	4	62	67	39	43	31	28	24	19	13	9	6
5	119	3626	4669	3152	2496	1627	1536	1286	1169	977	790	548	371	252
5	120	3300	3275	2673	2296	1468	1490	1161	1062	885	717	500	338	233
5	121	745	191	520	489	322	320	247	236	193	158	114	75	54
5	122	1227	1634	879	882	560	540	443	415	342	279	198	132	90
5	123	834	839	539	601	348	423	268	238	201	161	110	75	54
5	124	424	410	307	316	194	187	160	145	121	98	68	46	30
5	125	137	133	110	102	61	61	51	48	40	32	23	15	10
5	126	837	975	592	601	396	361	306	283	235	191	134	90	61
5	127	554	355	426	375	234	235	194	191	155	128	94	61	42

5	128	1735	2684	1266	1141	728	716	603	569	468	382	273	182	121
5	129	1350	3168	1044	943	602	590	496	471	386	316	227	151	101
5	130	573	2404	416	409	265	249	208	202	164	135	99	65	45
5	131	575	900	436	422	250	240	205	187	156	126	88	59	39
5	132	263	706	182	177	110	105	90	83	69	56	39	26	17
5	133	413	91	296	148	143	173	131	161	119	104	88	53	40
5	134	3044	3942	2159	1966	1299	1272	1062	1026	835	687	499	329	223
5	135	1882	1387	1424	1373	845	789	678	606	510	411	282	192	126
5	136	3221	2960	2472	2219	1544	1350	1159	1040	874	705	484	330	224
5	137	2214	921	1695	1566	984	922	787	716	598	484	336	228	151
5	138	768	966	652	577	357	334	286	258	217	175	121	82	54
5	139	3259	4036	2559	2359	1481	1395	1183	1096	909	739	520	349	236
5	140	4251	6666	3299	2944	1953	1781	1520	1386	1157	936	651	440	298
5	141	4543	6906	3750	3351	2153	1967	1692	1511	1272	1024	701	479	317
5	142	1439	0	956	736	531	488	417	379	317	256	178	120	81
5	143	347	855	287	269	160	151	130	115	98	78	53	37	24
5	144	2431	513	1989	1647	1143	996	860	759	643	515	349	240	160
5	145	3468	2963	2776	2439	1609	1454	1251	1114	940	755	516	353	234
5	146	2740	3362	2089	1969	1219	1163	990	908	756	613	428	289	191
5	147	1576	4201	1208	1262	734	700	596	547	455	369	258	174	115
5	148	1139	2999	841	905	527	501	431	384	324	260	178	122	79
5	149	3535	5052	2829	2538	1723	1534	1321	1176	992	798	545	373	248
5	150	1619	672	1325	1100	776	674	581	515	435	349	237	163	109
5	151	1203	68	766	736	468	433	370	336	281	227	158	107	71
5	152	1768	1417	1460	1218	830	747	640	577	484	391	270	183	124
5	153	2099	1699	1643	1509	974	896	764	698	582	471	328	222	150
5	154	3185	3673	2425	2260	1503	1358	1154	1064	884	718	504	339	233
5	155	663	1198	492	482	305	283	240	223	184	150	106	71	48
5	156	1903	2487	1500	1367	901	806	690	623	522	421	291	198	133
5	157	1646	3202	1219	1145	813	712	607	556	463	375	262	177	122
5	158	2928	4194	2304	2209	1374	1273	1091	983	825	666	459	312	206
5	159	4250	9211	3231	3114	1979	1822	1558	1413	1182	955	662	449	299
5	160	175	120	123	119	78	73	63	57	48	38	26	18	12
5	161	1898	4653	1371	1448	856	821	699	640	533	432	301	203	134
5	162	2963	2881	2242	2083	1314	1223	1042	954	795	644	450	304	204
5	163	2138	3159	1725	1483	1044	901	780	683	580	464	313	216	142
5	164	3706	2321	2684	2513	1758	1538	1312	1197	999	809	563	381	262
5	165	3754	2981	2928	2690	1750	1574	1355	1206	1017	818	559	382	253
5	166	246	696	188	183	107	104	90	80	67	54	37	25	16
5	167	99	24	72	78	46	42	37	32	27	22	15	10	7
5	168	122	60	89	89	53	50	44	38	33	26	18	12	8
5	169	49	258	36	37	22	22	19	17	14	12	8	6	4
5	170	4228	857	2932	2699	1940	1662	1424	1284	1077	869	600	408	280
6	1	2194	1108	2360	1565	965	1005	749	654	557	445	299	207	138
6	2	511	313	531	370	257	236	188	174	144	117	83	55	40

6	3	2338	393	2612	1933	1059	1214	832	716	613	488	324	225	149
6	4	1603	219	1597	1081	686	641	531	473	399	321	219	150	100
6	5	4826	2828	5122	3868	2076	2178	1725	1539	1297	1043	713	487	322
6	6	1770	844	1796	1449	775	910	622	546	463	371	250	172	117
6	7	3732	5994	3851	3014	1681	1714	1342	1212	1016	820	567	385	261
6	8	2684	1628	3003	2253	1270	1599	982	842	722	574	380	265	180
6	9	4156	1786	4458	3127	2013	1916	1503	1305	1113	889	595	412	273
6	10	947	357	1039	630	479	446	339	287	248	196	129	90	58
6	11	2571	2660	2337	1942	1251	1194	916	817	689	554	378	259	178
6	12	2800	1507	2373	2073	1416	1250	1024	911	768	618	421	288	196
6	13	4065	5110	3636	3257	1898	1879	1473	1330	1115	900	621	422	286
6	14	5223	1660	5008	3334	2525	2112	1743	1521	1294	1035	695	480	325
6	15	2798	1296	2885	1925	1427	1250	989	849	728	579	384	267	177
6	16	3438	1168	3510	2366	1772	1716	1214	1044	894	712	472	329	224
6	17	5045	1315	4578	3179	2524	2138	1739	1501	1284	1023	681	473	315
6	18	2515	1394	2370	1593	1307	1073	873	744	640	508	334	234	153
6	19	5417	1286	5150	3645	2737	2394	1905	1651	1409	1125	751	521	349
6	20	2680	726	2544	1822	1369	1273	955	819	702	559	370	258	171
6	21	4822	1596	4462	3114	2475	2278	1667	1432	1227	977	647	451	305
6	22	2332	761	2229	1553	1155	1036	806	684	589	468	307	215	139
6	23	5718	2911	5723	4267	2500	2480	1974	1747	1477	1186	806	553	367
6	24	4648	1757	4875	3441	2189	1989	1642	1455	1230	987	671	460	309
6	25	6151	3527	5685	4752	2774	3192	2066	1848	1555	1252	858	586	423
6	26	2572	968	2300	2018	1099	1178	866	801	665	540	380	255	180
6	27	4616	961	3636	3332	2040	2100	1524	1410	1169	950	669	449	322
6	28	3046	4220	2632	2180	1271	1424	984	912	756	615	433	291	210
6	29	1329	1168	1103	988	579	609	468	444	364	298	214	142	98
6	30	4974	743	5247	4060	2212	2404	1781	1568	1329	1065	721	496	331
6	31	3601	1313	3408	2735	1489	1670	1187	1067	896	722	497	338	233
6	32	3404	5461	2625	2735	1544	1552	1209	1116	926	752	529	355	243
6	33	2454	1376	1891	1926	1061	1060	825	758	630	511	358	241	165
6	34	2333	3041	1903	1626	1126	1127	794	709	597	480	328	224	160
6	35	2583	3995	2221	1844	1223	1431	845	757	637	513	352	240	184
6	36	7539	3113	6476	5428	3574	3319	2586	2297	1939	1558	1061	727	499
6	37	989	968	905	780	519	456	370	326	277	222	150	103	70
6	38	273	134	209	224	125	138	97	88	74	60	42	28	20
6	39	3479	2905	2821	2606	1658	1631	1206	1055	897	718	483	333	226
6	40	569	1604	506	430	266	274	207	187	157	127	88	60	41
6	41	3109	4955	2496	2418	1519	1439	1103	1000	837	676	468	317	222
6	42	403	462	302	353	181	187	144	131	110	89	61	42	28
6	43	1011	4617	874	806	459	450	366	340	282	229	162	108	74
6	44	1731	406	1230	1284	776	719	587	540	449	364	255	172	118
6	45	764	407	647	606	355	369	270	242	204	164	113	77	53
6	46	584	1694	474	478	273	268	217	206	169	138	99	66	45
6	47	1723	3382	1223	1290	782	819	574	531	441	358	252	169	122

6	48	303	1229	252	246	153	140	113	101	85	69	47	32	22
6	49	1907	4931	1408	1645	867	998	680	603	510	409	279	191	130
6	50	1512	4906	1369	1259	823	904	569	511	429	346	237	162	121
6	51	757	1138	600	634	340	449	260	233	196	158	108	74	54
6	52	2223	590	1672	1664	951	1110	713	662	548	446	315	211	156
6	53	1950	1748	1681	1558	995	1339	676	587	501	400	267	185	142
6	54	1014	1454	786	752	455	427	349	332	272	223	160	106	74
6	55	6970	9317	5282	5115	3575	3065	2482	2236	1876	1513	1043	709	496
6	56	1788	5858	1580	1457	832	795	654	616	507	414	295	197	135
6	57	788	2117	728	679	375	356	299	273	228	184	129	87	58
6	58	664	762	554	545	310	309	236	218	181	147	103	69	49
6	59	1817	1529	1333	1503	789	778	632	575	480	388	270	183	121
6	60	506	390	428	409	220	212	176	164	136	110	78	52	36
6	61	2977	2	1684	1893	1009	971	800	734	611	495	346	234	156
6	62	2708	1776	2339	2096	1244	1188	956	911	746	611	440	291	206
6	63	2396	3066	1868	1726	1096	1063	800	731	610	494	344	233	165
6	64	1862	3893	1515	1536	863	895	668	623	515	419	297	199	139
6	65	589	816	500	462	282	272	214	197	164	133	93	63	43
6	66	1346	1059	1202	1082	604	587	480	439	366	297	207	140	94
6	67	1371	78	997	959	580	530	442	408	339	275	194	130	89
6	68	1913	585	1661	1428	882	871	653	610	504	411	291	195	141
6	69	1613	1014	1429	1230	724	687	555	509	424	344	240	162	112
6	70	1938	1248	1639	1509	849	826	679	638	526	429	306	204	139
6	71	1444	9002	1052	1318	650	630	539	461	396	314	207	145	90
6	72	3664	6390	3031	2773	1836	1810	1298	1170	982	792	546	371	266
6	73	2590	2971	2188	2125	1215	1224	916	844	701	569	399	269	190
6	74	1839	1593	1503	1452	885	867	647	590	492	398	277	187	132
6	75	1279	4505	1129	1103	634	598	474	427	358	289	199	135	93
6	76	1554	1515	1271	1261	722	702	562	528	435	355	253	169	116
6	77	1974	10269	1793	1593	991	926	766	708	588	477	335	225	154
6	78	2433	2983	2057	1799	1126	1040	853	801	660	539	383	256	179
6	79	1456	3485	1188	1160	665	730	509	467	389	315	221	149	106
6	80	2251	3949	1916	1827	1033	980	815	756	627	510	359	241	163
6	81	2670	10102	2063	2101	1216	1189	977	925	760	621	445	296	200
6	82	824	1731	626	650	378	374	298	286	233	192	138	91	63
6	83	3976	4098	3215	3032	1789	1801	1378	1287	1063	866	614	410	288
6	84	2196	3929	1655	1719	941	945	749	679	568	459	318	216	144
6	85	1426	5052	1152	1193	646	660	528	484	403	327	228	154	103
6	86	3293	1493	2445	2489	1436	1329	1118	1019	851	688	479	324	217
6	87	2721	8076	2280	2277	1263	1180	990	890	747	603	415	282	187
6	88	2524	7901	2101	1931	1130	1188	861	801	663	540	381	255	183
6	89	1184	1362	1049	946	544	645	410	370	310	250	173	117	85
6	90	2184	5692	1640	1765	991	1051	776	722	597	486	343	230	160
6	91	1474	2662	1165	1225	681	713	535	500	413	337	239	160	111
6	92	3048	1264	2595	2207	1340	1366	1015	946	782	637	451	302	216

6	93	2399	4198	1881	1875	1082	1210	833	768	638	518	364	245	174
6	94	474	2376	464	383	251	265	188	167	141	113	77	53	37
6	95	507	49	403	374	214	283	157	146	121	99	70	47	37
6	96	5076	3427	4430	3648	2300	2228	1708	1525	1285	1034	708	483	332
6	97	2195	2701	2122	1672	1070	1195	766	670	570	456	307	212	150
6	98	2063	1332	2132	1537	1057	1042	734	638	544	434	291	201	140
6	99	3609	3129	3619	2729	1771	1628	1294	1165	978	789	543	370	257
6	100	1771	1935	1342	1175	929	768	611	550	461	372	256	174	126
6	101	1490	4827	1133	1158	663	689	526	503	411	337	243	161	112
6	102	4018	10430	3087	3195	1826	2112	1397	1279	1066	863	603	407	291
6	103	561	868	469	440	254	318	200	182	152	123	86	58	42
6	104	326	1404	260	259	146	158	117	108	90	73	51	34	24
6	105	2401	4852	2043	1923	1084	1246	825	746	625	504	348	237	169
6	106	2251	2259	1733	1654	959	1054	739	698	574	469	335	223	161
6	107	1507	3204	1270	1276	678	783	546	501	417	338	236	159	111
6	108	754	451	622	590	332	345	272	248	207	167	117	79	53
6	109	348	434	300	274	155	178	122	110	92	74	51	35	24
6	110	2366	1549	2174	1807	1077	1268	819	735	618	498	342	233	167
6	111	1055	1467	1053	842	486	519	383	347	290	235	162	110	76
6	112	325	135	287	250	136	150	110	99	83	67	46	31	21
6	113	1697	1718	1306	1299	736	765	583	534	445	360	251	170	116
6	114	570	720	485	435	246	256	205	192	158	129	91	61	41
6	115	2053	322	1564	1349	849	922	634	608	497	408	295	195	146
6	116	3125	717	2597	2187	1277	1544	1012	900	760	611	417	285	198
6	117	2581	2329	2901	1874	1154	1440	987	831	719	569	371	261	165
6	118	92	2	65	75	40	45	31	28	24	19	13	9	6
6	119	3627	5696	3534	2754	1670	1542	1286	1169	977	790	548	371	252
6	120	3298	3733	2944	2538	1496	1516	1161	1062	885	717	500	338	233
6	121	749	156	562	543	329	324	247	236	193	158	114	75	54
6	122	1220	1993	938	978	568	542	443	415	342	279	198	132	90
6	123	848	973	568	675	351	449	268	238	201	161	110	75	54
6	124	421	459	325	348	195	187	160	145	121	98	68	46	30
6	125	135	149	120	114	61	61	51	48	40	32	23	15	10
6	126	830	1151	627	663	404	361	306	283	235	191	134	90	61
6	127	553	363	469	418	236	235	194	191	155	128	94	61	42
6	128	1747	3436	1372	1255	733	716	603	569	468	382	273	182	121
6	129	1338	4436	1134	1040	606	590	496	471	386	316	227	151	101
6	130	565	3894	447	456	269	249	208	202	164	135	99	65	45
6	131	577	1152	473	472	252	240	205	187	156	126	88	59	39
6	132	267	1045	195	196	111	105	90	83	69	56	39	26	17
6	133	408	71	341	158	143	173	131	161	119	104	88	53	40
6	134	3050	4810	2328	2159	1307	1272	1062	1026	835	687	499	329	223
6	135	1889	1470	1534	1522	860	789	678	606	510	411	282	192	126
6	136	3213	3315	2675	2428	1598	1350	1159	1040	874	705	484	330	224
6	137	2221	847	1841	1734	1003	922	787	716	598	484	336	228	151

6	138	762	1159	718	641	364	334	286	258	217	175	121	82	54
6	139	3242	4843	2789	2623	1508	1395	1183	1096	909	739	520	349	236
6	140	4248	8533	3592	3241	2004	1781	1520	1386	1157	936	651	440	298
6	141	4514	8701	4095	3698	2207	1967	1692	1511	1272	1024	701	479	317
6	142	1517	0	1054	792	545	488	417	379	317	256	178	120	81
6	143	345	1214	313	299	163	151	130	115	98	78	53	37	24
6	144	2444	400	2192	1799	1187	996	860	759	643	515	349	240	160
6	145	3473	3260	3030	2679	1654	1454	1251	1114	940	755	516	353	234
6	146	2739	4035	2260	2183	1235	1163	990	908	756	613	428	289	191
6	147	1558	6050	1294	1419	743	700	596	547	455	369	258	174	115
6	148	1132	4319	890	1010	533	501	431	384	324	260	178	122	79
6	149	3506	6264	3062	2776	1776	1534	1321	1176	992	798	545	373	248
6	150	1620	618	1455	1199	806	674	581	515	435	349	237	163	109
6	151	1251	40	823	814	477	433	370	336	281	227	158	107	71
6	152	1763	1530	1608	1334	855	747	640	577	484	391	270	183	124
6	153	2089	1835	1785	1669	996	896	764	698	582	471	328	222	150
6	154	3162	4334	2623	2498	1544	1358	1154	1064	884	718	504	339	233
6	155	660	1582	529	537	310	283	240	223	184	150	106	71	48
6	156	1896	3034	1632	1510	929	806	690	623	522	421	291	198	133
6	157	1626	4291	1302	1251	840	712	607	556	463	375	262	177	122
6	158	2907	5201	2487	2452	1402	1273	1091	983	825	666	459	312	206
6	159	4232	12711	3476	3448	2021	1822	1558	1413	1182	955	662	449	299
6	160	176	124	131	129	79	73	63	57	48	38	26	18	12
6	161	1892	6607	1456	1618	863	821	699	640	533	432	301	203	134
6	162	2976	3285	2437	2312	1341	1223	1042	954	795	644	450	304	204
6	163	2137	3980	1878	1614	1084	901	780	683	580	464	313	216	142
6	164	3700	2367	2879	2755	1818	1538	1312	1197	999	809	563	381	262
6	165	3758	3220	3176	2967	1800	1574	1355	1206	1017	818	559	382	253
6	166	247	1030	202	203	107	104	90	80	67	54	37	25	16
6	167	99	19	77	87	46	42	37	32	27	22	15	10	7
6	168	123	57	94	99	54	50	44	38	33	26	18	12	8
6	169	49	438	38	41	22	22	19	17	14	12	8	6	4
6	170	4278	669	3146	2945	2017	1662	1424	1284	1077	869	600	408	280
7	1	2219	1086	2704	1707	989	1032	749	654	557	445	299	207	138
7	2	503	313	599	403	266	238	188	174	144	117	83	55	40
7	3	2350	307	2986	2147	1081	1268	832	716	613	488	324	225	149
7	4	1630	167	1819	1177	705	646	531	473	399	321	219	150	100
7	5	4843	2828	5817	4295	2099	2209	1725	1539	1297	1043	713	487	322
7	6	1777	810	2019	1613	786	948	622	546	463	371	250	172	117
7	7	3726	7313	4347	3353	1711	1740	1342	1212	1016	820	567	385	261
7	8	2674	1628	3408	2491	1297	1702	982	842	722	574	380	265	180
7	9	4162	1679	5051	3406	2079	1952	1503	1305	1113	889	595	412	273
7	10	954	329	1184	668	501	459	339	287	248	196	129	90	58
7	11	2566	2980	2573	2126	1291	1217	916	817	689	554	378	259	178
7	12	2786	1477	2565	2249	1466	1260	1024	911	768	618	421	288	196

7	13	4050	5928	3982	3603	1935	1906	1473	1330	1115	900	621	422	286
7	14	5299	1494	5644	3574	2651	2130	1743	1521	1294	1035	695	480	325
7	15	2816	1244	3256	2065	1496	1273	989	849	728	579	384	267	177
7	16	3447	1051	3948	2536	1858	1781	1214	1044	894	712	472	329	224
7	17	5099	1131	5065	3369	2647	2165	1739	1501	1284	1023	681	473	315
7	18	2544	1394	2636	1685	1381	1088	873	744	640	508	334	234	153
7	19	5446	1080	5728	3901	2864	2432	1905	1651	1409	1125	751	521	349
7	20	2691	624	2817	1944	1430	1308	955	819	702	559	370	258	171
7	21	4859	1436	4944	3308	2603	2351	1667	1432	1227	977	647	451	305
7	22	2364	685	2486	1659	1209	1060	806	684	589	468	307	215	139
7	23	5774	2853	6465	4697	2553	2515	1974	1747	1477	1186	806	553	367
7	24	4664	1617	5537	3767	2262	2004	1642	1455	1230	987	671	460	309
7	25	6171	3527	6327	5277	2840	3344	2066	1848	1555	1252	858	586	423
7	26	2585	890	2556	2264	1115	1205	866	801	665	540	380	255	180
7	27	4645	788	3954	3680	2084	2152	1524	1410	1169	950	669	449	322
7	28	3075	5064	2924	2417	1291	1470	984	912	756	615	433	291	210
7	29	1326	1261	1202	1088	583	617	468	444	364	298	214	142	98
7	30	4984	565	5935	4508	2246	2469	1781	1568	1329	1065	721	496	331
7	31	3648	1208	3836	3048	1511	1723	1187	1067	896	722	497	338	233
7	32	3399	6662	2807	3040	1563	1574	1209	1116	926	752	529	355	243
7	33	2477	1376	2043	2155	1078	1076	825	758	630	511	358	241	165
7	34	2338	3589	2065	1763	1166	1166	794	709	597	480	328	224	160
7	35	2583	4874	2443	2019	1264	1520	845	757	637	513	352	240	184
7	36	7587	2926	7103	5928	3699	3377	2586	2297	1939	1558	1061	727	499
7	37	981	1065	988	852	539	461	370	326	277	222	150	103	70
7	38	272	129	223	249	126	143	97	88	74	60	42	28	20
7	39	3501	3137	3046	2853	1711	1676	1206	1055	897	718	483	333	226
7	40	566	2182	553	469	271	279	207	187	157	127	88	60	41
7	41	3097	6046	2686	2665	1564	1465	1103	1000	837	676	468	317	222
7	42	403	526	319	398	182	191	144	131	110	89	61	42	28
7	43	1007	6925	954	895	466	453	366	340	282	229	162	108	74
7	44	1744	341	1308	1419	792	723	587	540	449	364	255	172	118
7	45	763	399	703	670	362	379	270	242	204	164	113	77	53
7	46	577	2303	510	531	276	270	217	206	169	138	99	66	45
7	47	1729	4330	1299	1427	797	844	574	531	441	358	252	169	122
7	48	300	1794	270	270	157	142	113	101	85	69	47	32	22
7	49	1906	6607	1482	1838	875	1039	680	603	510	409	279	191	130
7	50	1475	6770	1482	1383	851	950	569	511	429	346	237	162	121
7	51	754	1365	642	708	342	479	260	233	196	158	108	74	54
7	52	2240	507	1805	1854	964	1158	713	662	548	446	315	211	156
7	53	1931	1888	1818	1713	1027	1465	676	587	501	400	267	185	142
7	54	1015	1745	850	833	463	429	349	332	272	223	160	106	74
7	55	6942	10994	5619	5568	3718	3095	2482	2236	1876	1513	1043	709	496
7	56	1772	8201	1731	1625	847	798	654	616	507	414	295	197	135
7	57	780	2837	798	757	382	357	299	273	228	184	129	87	58

7	58	661	868	602	609	316	314	236	218	181	147	103	69	49
7	59	1830	1651	1417	1686	796	785	632	575	480	388	270	183	121
7	60	508	413	470	460	224	213	176	164	136	110	78	52	36
7	61	3155	0	1792	2124	1021	976	800	734	611	495	346	234	156
7	62	2692	1811	2571	2335	1269	1195	956	911	746	611	440	291	206
7	63	2411	3618	2024	1899	1127	1084	800	731	610	494	344	233	165
7	64	1848	4983	1633	1715	874	913	668	623	515	419	297	199	139
7	65	584	962	543	509	288	276	214	197	164	133	93	63	43
7	66	1347	1123	1324	1205	614	591	480	439	366	297	207	140	94
7	67	1396	50	1076	1061	593	532	442	408	339	275	194	130	89
7	68	1909	514	1833	1582	905	888	653	610	504	411	291	195	141
7	69	1620	1034	1582	1366	741	693	555	509	424	344	240	162	112
7	70	1940	1273	1796	1683	861	829	679	638	526	429	306	204	139
7	71	1449	14404	1096	1477	655	633	539	461	396	314	207	145	90
7	72	3639	7924	3275	3037	1897	1864	1298	1170	982	792	546	371	266
7	73	2577	3387	2382	2376	1241	1249	916	844	701	569	399	269	190
7	74	1832	1721	1625	1609	909	886	647	590	492	398	277	187	132
7	75	1267	6397	1227	1233	651	606	474	427	358	289	199	135	93
7	76	1544	1667	1374	1406	733	707	562	528	435	355	253	169	116
7	77	1937	15608	1949	1746	1010	930	766	708	588	477	335	225	154
7	78	2426	3461	2254	1984	1153	1044	853	801	660	539	383	256	179
7	79	1451	4601	1284	1288	674	753	509	467	389	315	221	149	106
7	80	2242	4897	2086	2035	1050	983	815	756	627	510	359	241	163
7	81	2652	14547	2203	2322	1225	1193	977	925	760	621	445	296	200
7	82	818	2216	667	721	381	377	298	286	233	192	138	91	63
7	83	3974	4590	3489	3360	1818	1829	1378	1287	1063	866	614	410	288
7	84	2219	4950	1773	1914	952	957	749	679	568	459	318	216	144
7	85	1418	7174	1233	1326	649	667	528	484	403	327	228	154	103
7	86	3331	1433	2620	2760	1461	1332	1118	1019	851	688	479	324	217
7	87	2719	11144	2466	2538	1286	1185	990	890	747	603	415	282	187
7	88	2524	11062	2296	2146	1151	1217	861	801	663	540	381	255	183
7	89	1180	1553	1150	1050	554	677	410	370	310	250	173	117	85
7	90	2175	7627	1743	1962	999	1074	776	722	597	486	343	230	160
7	91	1462	3301	1246	1366	687	727	535	500	413	337	239	160	111
7	92	3062	1189	2865	2443	1370	1394	1015	946	782	637	451	302	216
7	93	2392	5206	2019	2076	1094	1250	833	768	638	518	364	245	174
7	94	463	3564	506	416	258	273	188	167	141	113	77	53	37
7	95	509	34	441	417	216	301	157	146	121	99	70	47	37
7	96	5125	3565	4893	4001	2368	2272	1708	1525	1285	1034	708	483	332
7	97	2192	3133	2362	1832	1107	1258	766	670	570	456	307	212	150
7	98	2060	1358	2401	1673	1104	1082	734	638	544	434	291	201	140
7	99	3592	3379	4064	2995	1835	1649	1294	1165	978	789	543	370	257
7	100	1768	2206	1436	1262	977	777	611	550	461	372	256	174	126
7	101	1484	6758	1213	1286	667	699	526	503	411	337	243	161	112
7	102	4004	13976	3294	3539	1845	2198	1397	1279	1066	863	603	407	291

7	103	556	1042	506	484	255	334	200	182	152	123	86	58	42
7	104	325	2078	279	287	146	161	117	108	90	73	51	34	24
7	105	2400	6211	2229	2141	1101	1299	825	746	625	504	348	237	169
7	106	2264	2531	1879	1837	969	1083	739	698	574	469	335	223	161
7	107	1496	4101	1371	1424	681	809	546	501	417	338	236	159	111
7	108	754	451	671	650	335	350	272	248	207	167	117	79	53
7	109	348	503	327	303	157	184	122	110	92	74	51	35	24
7	110	2361	1580	2403	1989	1096	1328	819	735	618	498	342	233	167
7	111	1048	1731	1177	932	494	532	383	347	290	235	162	110	76
7	112	329	127	317	277	138	154	110	99	83	67	46	31	21
7	113	1707	1924	1403	1438	744	780	583	534	445	360	251	170	116
7	114	570	835	528	479	247	258	205	192	158	129	91	61	41
7	115	2077	251	1714	1487	864	949	634	608	497	408	295	195	146
7	116	3176	603	2851	2396	1291	1616	1012	900	760	611	417	285	198
7	117	2574	2515	3270	1993	1159	1509	987	831	719	569	371	261	165
7	118	93	1	69	84	40	46	31	28	24	19	13	9	6
7	119	3629	6950	3962	3038	1713	1548	1286	1169	977	790	548	371	252
7	120	3295	4256	3243	2805	1524	1543	1161	1062	885	717	500	338	233
7	121	753	128	607	603	335	329	247	236	193	158	114	75	54
7	122	1212	2431	1001	1085	576	545	443	415	342	279	198	132	90
7	123	863	1129	598	757	353	477	268	238	201	161	110	75	54
7	124	419	514	343	384	196	187	160	145	121	98	68	46	30
7	125	133	167	131	128	62	61	51	48	40	32	23	15	10
7	126	823	1358	665	732	412	361	306	283	235	191	134	90	61
7	127	551	370	516	468	237	235	194	191	155	128	94	61	42
7	128	1759	4398	1488	1381	737	716	603	569	468	382	273	182	121
7	129	1327	6210	1231	1147	610	590	496	471	386	316	227	151	101
7	130	558	6309	480	508	273	249	208	202	164	135	99	65	45
7	131	579	1474	512	527	255	240	205	187	156	126	88	59	39
7	132	271	1546	209	217	112	105	90	83	69	56	39	26	17
7	133	403	55	392	169	144	173	131	161	119	104	88	53	40
7	134	3057	5868	2510	2370	1316	1272	1062	1026	835	687	499	329	223
7	135	1895	1559	1653	1687	875	789	678	606	510	411	282	192	126
7	136	3206	3713	2895	2656	1654	1350	1159	1040	874	705	484	330	224
7	137	2229	779	2000	1921	1021	922	787	716	598	484	336	228	151
7	138	757	1391	791	713	371	334	286	258	217	175	121	82	54
7	139	3226	5812	3040	2917	1535	1395	1183	1096	909	739	520	349	236
7	140	4245	10922	3911	3567	2055	1781	1520	1386	1157	936	651	440	298
7	141	4485	10963	4472	4080	2263	1967	1692	1511	1272	1024	701	479	317
7	142	1598	0	1163	852	559	488	417	379	317	256	178	120	81
7	143	344	1724	341	333	166	151	130	115	98	78	53	37	24
7	144	2458	312	2415	1966	1232	996	860	759	643	515	349	240	160
7	145	3478	3586	3309	2943	1700	1454	1251	1114	940	755	516	353	234
7	146	2737	4842	2445	2419	1251	1163	990	908	756	613	428	289	191
7	147	1540	8711	1385	1597	751	700	596	547	455	369	258	174	115

7	148	1125	6219	941	1128	539	501	431	384	324	260	178	122	79
7	149	3477	7767	3315	3036	1831	1534	1321	1176	992	798	545	373	248
7	150	1622	569	1599	1306	837	674	581	515	435	349	237	163	109
7	151	1301	23	886	901	486	433	370	336	281	227	158	107	71
7	152	1757	1652	1770	1461	882	747	640	577	484	391	270	183	124
7	153	2079	1982	1939	1847	1019	896	764	698	582	471	328	222	150
7	154	3139	5114	2837	2761	1586	1358	1154	1064	884	718	504	339	233
7	155	656	2088	570	598	316	283	240	223	184	150	106	71	48
7	156	1888	3701	1776	1669	957	806	690	623	522	421	291	198	133
7	157	1606	5749	1392	1367	868	712	607	556	463	375	262	177	122
7	158	2887	6449	2686	2722	1431	1273	1091	983	825	666	459	312	206
7	159	4213	17542	3739	3818	2064	1822	1558	1413	1182	955	662	449	299
7	160	177	129	138	141	80	73	63	57	48	38	26	18	12
7	161	1885	9382	1547	1808	870	821	699	640	533	432	301	203	134
7	162	2988	3744	2649	2567	1369	1223	1042	954	795	644	450	304	204
7	163	2136	5015	2045	1756	1127	901	780	683	580	464	313	216	142
7	164	3694	2414	3090	3021	1880	1538	1312	1197	999	809	563	381	262
7	165	3763	3477	3445	3273	1851	1574	1355	1206	1017	818	559	382	253
7	166	248	1524	218	225	108	104	90	80	67	54	37	25	16
7	167	99	15	81	98	47	42	37	32	27	22	15	10	7
7	168	124	55	101	109	54	50	44	38	33	26	18	12	8
7	169	49	745	40	45	22	22	19	17	14	12	8	6	4
7	170	4328	522	3377	3214	2096	1662	1424	1284	1077	869	600	408	280
8	1	2245	1064	3099	1862	1014	1060	749	654	557	445	299	207	138
8	2	495	313	676	439	276	241	188	174	144	117	83	55	40
8	3	2361	239	3412	2385	1104	1324	832	716	613	488	324	225	149
8	4	1657	127	2072	1283	724	650	531	473	399	321	219	150	100
8	5	4859	2828	6605	4769	2123	2241	1725	1539	1297	1043	713	487	322
8	6	1784	777	2271	1795	796	987	622	546	463	371	250	172	117
8	7	3719	8922	4907	3730	1742	1766	1342	1212	1016	820	567	385	261
8	8	2665	1628	3866	2754	1325	1811	982	842	722	574	380	265	180
8	9	4168	1578	5723	3710	2148	1988	1503	1305	1113	889	595	412	273
8	10	961	302	1348	709	524	471	339	287	248	196	129	90	58
8	11	2561	3337	2832	2327	1333	1241	916	817	689	554	378	259	178
8	12	2773	1447	2774	2439	1517	1271	1024	911	768	618	421	288	196
8	13	4036	6877	4362	3984	1973	1935	1473	1330	1115	900	621	422	286
8	14	5377	1345	6361	3831	2783	2149	1743	1521	1294	1035	695	480	325
8	15	2834	1195	3674	2216	1569	1296	989	849	728	579	384	267	177
8	16	3456	946	4441	2719	1949	1849	1214	1044	894	712	472	329	224
8	17	5153	973	5604	3570	2777	2191	1739	1501	1284	1023	681	473	315
8	18	2574	1394	2932	1782	1459	1102	873	744	640	508	334	234	153
8	19	5476	908	6371	4174	2997	2471	1905	1651	1409	1125	751	521	349
8	20	2702	537	3119	2075	1495	1344	955	819	702	559	370	258	171
8	21	4897	1293	5478	3514	2738	2427	1667	1432	1227	977	647	451	305
8	22	2397	616	2773	1772	1266	1083	806	684	589	468	307	215	139

8	23	5831	2796	7303	5169	2608	2550	1974	1747	1477	1186	806	553	367
8	24	4681	1487	6289	4124	2337	2019	1642	1455	1230	987	671	460	309
8	25	6192	3527	7042	5859	2907	3503	2066	1848	1555	1252	858	586	423
8	26	2599	819	2842	2539	1132	1233	866	801	665	540	380	255	180
8	27	4675	646	4301	4065	2128	2206	1524	1410	1169	950	669	449	322
8	28	3105	6076	3248	2679	1311	1518	984	912	756	615	433	291	210
8	29	1323	1362	1308	1199	586	626	468	444	364	298	214	142	98
8	30	4994	429	6714	5006	2281	2536	1781	1568	1329	1065	721	496	331
8	31	3696	1112	4318	3397	1533	1777	1187	1067	896	722	497	338	233
8	32	3395	8128	3002	3380	1583	1596	1209	1116	926	752	529	355	243
8	33	2501	1376	2208	2412	1095	1091	825	758	630	511	358	241	165
8	34	2344	4235	2240	1912	1207	1205	794	709	597	480	328	224	160
8	35	2583	5946	2686	2211	1305	1614	845	757	637	513	352	240	184
8	36	7635	2750	7791	6475	3829	3436	2586	2297	1939	1558	1061	727	499
8	37	972	1172	1079	931	560	466	370	326	277	222	150	103	70
8	38	271	124	237	277	128	147	97	88	74	60	42	28	20
8	39	3524	3388	3289	3123	1766	1723	1206	1055	897	718	483	333	226
8	40	563	2967	604	513	276	285	207	187	157	127	88	60	41
8	41	3086	7376	2890	2938	1610	1492	1103	1000	837	676	468	317	222
8	42	403	600	338	448	184	194	144	131	110	89	61	42	28
8	43	1003	10387	1042	994	472	456	366	340	282	229	162	108	74
8	44	1758	286	1390	1568	809	728	587	540	449	364	255	172	118
8	45	763	391	763	741	370	389	270	242	204	164	113	77	53
8	46	570	3133	548	591	279	271	217	206	169	138	99	66	45
8	47	1735	5542	1380	1579	813	869	574	531	441	358	252	169	122
8	48	297	2620	290	297	161	143	113	101	85	69	47	32	22
8	49	1905	8854	1560	2054	883	1081	680	603	510	409	279	191	130
8	50	1440	9343	1604	1520	879	999	569	511	429	346	237	162	121
8	51	752	1638	687	790	345	510	260	233	196	158	108	74	54
8	52	2257	436	1948	2065	976	1208	713	662	548	446	315	211	156
8	53	1911	2039	1967	1882	1060	1603	676	587	501	400	267	185	142
8	54	1016	2094	920	923	472	431	349	332	272	223	160	106	74
8	55	6914	12973	5977	6061	3867	3124	2482	2236	1876	1513	1043	709	496
8	56	1757	11482	1897	1813	861	801	654	616	507	414	295	197	135
8	57	771	3802	874	844	388	358	299	273	228	184	129	87	58
8	58	658	990	654	681	322	320	236	218	181	147	103	69	49
8	59	1844	1783	1506	1891	804	791	632	575	480	388	270	183	121
8	60	510	438	516	517	227	213	176	164	136	110	78	52	36
8	61	3343	0	1906	2383	1033	982	800	734	611	495	346	234	156
8	62	2676	1847	2827	2601	1296	1203	956	911	746	611	440	291	206
8	63	2426	4269	2192	2090	1159	1106	800	731	610	494	344	233	165
8	64	1834	6379	1760	1914	886	931	668	623	515	419	297	199	139
8	65	579	1136	589	561	295	279	214	197	164	133	93	63	43
8	66	1347	1190	1457	1342	624	595	480	439	366	297	207	140	94
8	67	1422	32	1162	1173	606	533	442	408	339	275	194	130	89

8	68	1904	453	2023	1753	929	905	653	610	504	411	291	195	141
8	69	1627	1055	1751	1518	759	699	555	509	424	344	240	162	112
8	70	1942	1299	1968	1876	872	832	679	638	526	429	306	204	139
8	71	1453	23046	1142	1654	661	635	539	461	396	314	207	145	90
8	72	3614	9825	3539	3325	1961	1920	1298	1170	982	792	546	371	266
8	73	2564	3861	2593	2656	1267	1275	916	844	701	569	399	269	190
8	74	1825	1858	1757	1783	934	905	647	590	492	398	277	187	132
8	75	1256	9083	1335	1377	668	614	474	427	358	289	199	135	93
8	76	1533	1833	1485	1567	744	713	562	528	435	355	253	169	116
8	77	1901	23725	2119	1914	1030	934	766	708	588	477	335	225	154
8	78	2419	4015	2469	2189	1181	1049	853	801	660	539	383	256	179
8	79	1446	6073	1387	1431	684	778	509	467	389	315	221	149	106
8	80	2233	6072	2271	2267	1066	986	815	756	627	510	359	241	163
8	81	2635	20947	2352	2567	1234	1197	977	925	760	621	445	296	200
8	82	811	2836	712	800	385	380	298	286	233	192	138	91	63
8	83	3973	5140	3786	3723	1848	1858	1378	1287	1063	866	614	410	288
8	84	2242	6237	1901	2130	963	969	749	679	568	459	318	216	144
8	85	1409	10187	1320	1474	653	674	528	484	403	327	228	154	103
8	86	3369	1376	2809	3061	1487	1335	1118	1019	851	688	479	324	217
8	87	2717	15379	2666	2829	1310	1189	990	890	747	603	415	282	187
8	88	2524	15486	2509	2385	1171	1247	861	801	663	540	381	255	183
8	89	1176	1771	1261	1165	563	710	410	370	310	250	173	117	85
8	90	2166	10221	1852	2181	1008	1097	776	722	597	486	343	230	160
8	91	1450	4093	1332	1523	693	741	535	500	413	337	239	160	111
8	92	3075	1117	3164	2704	1400	1423	1015	946	782	637	451	302	216
8	93	2386	6455	2168	2299	1105	1292	833	768	638	518	364	245	174
8	94	452	5346	553	451	264	283	188	167	141	113	77	53	37
8	95	511	24	482	465	218	320	157	146	121	99	70	47	37
8	96	5176	3707	5405	4390	2438	2316	1708	1525	1285	1034	708	483	332
8	97	2190	3635	2630	2008	1144	1324	766	670	570	456	307	212	150
8	98	2057	1385	2704	1822	1154	1122	734	638	544	434	291	201	140
8	99	3575	3649	4564	3287	1901	1670	1294	1165	978	789	543	370	257
8	100	1764	2514	1537	1355	1027	787	611	550	461	372	256	174	126
8	101	1478	9461	1298	1428	671	709	526	503	411	337	243	161	112
8	102	3991	18728	3514	3921	1863	2288	1397	1279	1066	863	603	407	291
8	103	552	1250	545	532	256	350	200	182	152	123	86	58	42
8	104	324	3075	300	317	147	165	117	108	90	73	51	34	24
8	105	2400	7950	2432	2383	1119	1354	825	746	625	504	348	237	169
8	106	2276	2834	2037	2041	980	1114	739	698	574	469	335	223	161
8	107	1485	5249	1481	1589	683	835	546	501	417	338	236	159	111
8	108	754	451	725	717	337	354	272	248	207	167	117	79	53
8	109	348	584	356	334	159	191	122	110	92	74	51	35	24
8	110	2357	1612	2655	2190	1116	1391	819	735	618	498	342	233	167
8	111	1041	2042	1315	1030	503	545	383	347	290	235	162	110	76
8	112	332	119	350	307	139	158	110	99	83	67	46	31	21

8	113	1717	2155	1508	1592	752	794	583	534	445	360	251	170	116
8	114	569	969	575	528	248	260	205	192	158	129	91	61	41
8	115	2101	196	1878	1639	879	977	634	608	497	408	295	195	146
8	116	3227	506	3130	2624	1304	1691	1012	900	760	611	417	285	198
8	117	2568	2717	3687	2121	1164	1580	987	831	719	569	371	261	165
8	118	94	1	73	94	40	48	31	28	24	19	13	9	6
8	119	3630	8479	4442	3352	1758	1555	1286	1169	977	790	548	371	252
8	120	3293	4851	3573	3101	1552	1570	1161	1062	885	717	500	338	233
8	121	757	105	657	669	341	334	247	236	193	158	114	75	54
8	122	1205	2966	1068	1204	584	548	443	415	342	279	198	132	90
8	123	878	1310	630	850	356	506	268	238	201	161	110	75	54
8	124	416	576	362	423	197	187	160	145	121	98	68	46	30
8	125	132	187	143	143	62	61	51	48	40	32	23	15	10
8	126	817	1603	705	808	421	361	306	283	235	191	134	90	61
8	127	550	377	569	523	239	235	194	191	155	128	94	61	42
8	128	1771	5630	1613	1519	742	716	603	569	468	382	273	182	121
8	129	1316	8694	1336	1265	615	590	496	471	386	316	227	151	101
8	130	551	10221	516	567	277	249	208	202	164	135	99	65	45
8	131	582	1887	555	589	258	240	205	187	156	126	88	59	39
8	132	276	2288	224	240	113	105	90	83	69	56	39	26	17
8	133	398	43	451	180	145	173	131	161	119	104	88	53	40
8	134	3064	7159	2707	2603	1324	1272	1062	1026	835	687	499	329	223
8	135	1902	1652	1782	1870	891	789	678	606	510	411	282	192	126
8	136	3199	4159	3133	2907	1712	1350	1159	1040	874	705	484	330	224
8	137	2237	717	2173	2128	1040	922	787	716	598	484	336	228	151
8	138	751	1669	871	792	379	334	286	258	217	175	121	82	54
8	139	3209	6974	3314	3245	1563	1395	1183	1096	909	739	520	349	236
8	140	4241	13980	4258	3927	2108	1781	1520	1386	1157	936	651	440	298
8	141	4456	13814	4883	4502	2320	1967	1692	1511	1272	1024	701	479	317
8	142	1684	0	1282	917	573	488	417	379	317	256	178	120	81
8	143	342	2448	372	371	169	151	130	115	98	78	53	37	24
8	144	2472	243	2662	2147	1279	996	860	759	643	515	349	240	160
8	145	3483	3944	3612	3233	1748	1454	1251	1114	940	755	516	353	234
8	146	2736	5810	2645	2682	1267	1163	990	908	756	613	428	289	191
8	147	1523	12544	1484	1796	760	700	596	547	455	369	258	174	115
8	148	1118	8955	995	1259	545	501	431	384	324	260	178	122	79
8	149	3448	9631	3588	3321	1887	1534	1321	1176	992	798	545	373	248
8	150	1623	523	1756	1424	870	674	581	515	435	349	237	163	109
8	151	1353	13	952	997	496	433	370	336	281	227	158	107	71
8	152	1751	1785	1949	1600	909	747	640	577	484	391	270	183	124
8	153	2069	2141	2107	2043	1043	896	764	698	582	471	328	222	150
8	154	3116	6035	3068	3052	1629	1358	1154	1064	884	718	504	339	233
8	155	653	2756	613	666	322	283	240	223	184	150	106	71	48
8	156	1881	4516	1932	1844	987	806	690	623	522	421	291	198	133
8	157	1587	7704	1487	1494	897	712	607	556	463	375	262	177	122

8	158	2867	7997	2900	3021	1459	1273	1091	983	825	666	459	312	206
8	159	4195	24207	4023	4227	2107	1822	1558	1413	1182	955	662	449	299
8	160	178	135	147	154	81	73	63	57	48	38	26	18	12
8	161	1879	13322	1643	2019	878	821	699	640	533	432	301	203	134
8	162	3001	4269	2879	2850	1396	1223	1042	954	795	644	450	304	204
8	163	2135	6319	2226	1911	1171	901	780	683	580	464	313	216	142
8	164	3688	2463	3315	3312	1944	1538	1312	1197	999	809	563	381	262
8	165	3767	3755	3738	3610	1904	1574	1355	1206	1017	818	559	382	253
8	166	249	2255	235	250	109	104	90	80	67	54	37	25	16
8	167	99	12	86	109	48	42	37	32	27	22	15	10	7
8	168	125	53	107	121	55	50	44	38	33	26	18	12	8
8	169	48	1266	42	50	22	22	19	17	14	12	8	6	4
8	170	4379	407	3624	3508	2179	1662	1424	1284	1077	869	600	408	280
9	1	2271	1043	3552	2031	1039	1088	749	654	557	445	299	207	138
9	2	488	313	763	478	286	243	188	174	144	117	83	55	40
9	3	2372	187	3899	2648	1127	1382	832	716	613	488	324	225	149
9	4	1686	96	2361	1398	744	654	531	473	399	321	219	150	100
9	5	4876	2828	7501	5296	2147	2273	1725	1539	1297	1043	713	487	322
9	6	1791	746	2554	1998	807	1028	622	546	463	371	250	172	117
9	7	3712	10885	5539	4150	1773	1792	1342	1212	1016	820	567	385	261
9	8	2655	1628	4387	3045	1353	1928	982	842	722	574	380	265	180
9	9	4173	1483	6484	4041	2219	2024	1503	1305	1113	889	595	412	273
9	10	968	278	1534	753	547	484	339	287	248	196	129	90	58
9	11	2556	3738	3118	2548	1376	1266	916	817	689	554	378	259	178
9	12	2760	1418	2999	2645	1570	1282	1024	911	768	618	421	288	196
9	13	4021	7977	4777	4407	2012	1963	1473	1330	1115	900	621	422	286
9	14	5455	1210	7169	4107	2921	2168	1743	1521	1294	1035	695	480	325
9	15	2852	1147	4146	2377	1645	1319	989	849	728	579	384	267	177
9	16	3465	851	4995	2915	2043	1920	1214	1044	894	712	472	329	224
9	17	5207	836	6201	3784	2912	2219	1739	1501	1284	1023	681	473	315
9	18	2603	1394	3261	1885	1541	1117	873	744	640	508	334	234	153
9	19	5505	762	7086	4466	3136	2511	1905	1651	1409	1125	751	521	349
9	20	2713	461	3453	2214	1562	1382	955	819	702	559	370	258	171
9	21	4935	1163	6070	3734	2880	2505	1667	1432	1227	977	647	451	305
9	22	2431	555	3093	1894	1325	1108	806	684	589	468	307	215	139
9	23	5888	2740	8249	5689	2663	2585	1974	1747	1477	1186	806	553	367
9	24	4698	1368	7143	4515	2415	2035	1642	1455	1230	987	671	460	309
9	25	6213	3527	7838	6506	2976	3670	2066	1848	1555	1252	858	586	423
9	26	2612	754	3159	2848	1149	1261	866	801	665	540	380	255	180
9	27	4705	530	4677	4489	2174	2261	1524	1410	1169	950	669	449	322
9	28	3135	7291	3609	2970	1331	1567	984	912	756	615	433	291	210
9	29	1319	1471	1425	1321	590	635	468	444	364	298	214	142	98
9	30	5004	326	7595	5559	2317	2604	1781	1568	1329	1065	721	496	331
9	31	3745	1023	4860	3787	1556	1833	1187	1067	896	722	497	338	233
9	32	3390	9916	3211	3758	1602	1618	1209	1116	926	752	529	355	243

9	33	2524	1376	2385	2699	1112	1107	825	758	630	511	358	241	165
9	34	2350	4997	2430	2073	1250	1246	794	709	597	480	328	224	160
9	35	2583	7254	2953	2421	1348	1714	845	757	637	513	352	240	184
9	36	7684	2585	8545	7072	3963	3496	2586	2297	1939	1558	1061	727	499
9	37	963	1289	1178	1017	582	471	370	326	277	222	150	103	70
9	38	270	119	252	307	129	152	97	88	74	60	42	28	20
9	39	3546	3659	3551	3419	1822	1772	1206	1055	897	718	483	333	226
9	40	559	4035	660	560	281	291	207	187	157	127	88	60	41
9	41	3074	8998	3110	3239	1658	1519	1103	1000	837	676	468	317	222
9	42	403	684	358	504	185	197	144	131	110	89	61	42	28
9	43	998	15581	1138	1103	478	459	366	340	282	229	162	108	74
9	44	1772	240	1478	1732	826	733	587	540	449	364	255	172	118
9	45	762	383	829	820	378	400	270	242	204	164	113	77	53
9	46	564	4260	589	657	282	273	217	206	169	138	99	66	45
9	47	1741	7093	1465	1747	828	896	574	531	441	358	252	169	122
9	48	295	3825	311	326	166	145	113	101	85	69	47	32	22
9	49	1904	11864	1642	2295	892	1124	680	603	510	409	279	191	130
9	50	1405	12894	1737	1669	908	1051	569	511	429	346	237	162	121
9	51	750	1966	736	883	347	543	260	233	196	158	108	74	54
9	52	2273	375	2103	2300	988	1261	713	662	548	446	315	211	156
9	53	1891	2202	2129	2069	1094	1754	676	587	501	400	267	185	142
9	54	1018	2512	995	1022	480	432	349	332	272	223	160	106	74
9	55	6887	15308	6358	6598	4021	3154	2482	2236	1876	1513	1043	709	496
9	56	1741	16075	2079	2023	876	804	654	616	507	414	295	197	135
9	57	763	5095	957	942	394	359	299	273	228	184	129	87	58
9	58	655	1129	710	762	328	325	236	218	181	147	103	69	49
9	59	1857	1926	1601	2121	812	798	632	575	480	388	270	183	121
9	60	511	464	566	581	230	214	176	164	136	110	78	52	36
9	61	3542	0	2028	2674	1045	987	800	734	611	495	346	234	156
9	62	2660	1884	3108	2898	1322	1211	956	911	746	611	440	291	206
9	63	2441	5038	2375	2299	1192	1129	800	731	610	494	344	233	165
9	64	1821	8165	1897	2136	898	949	668	623	515	419	297	199	139
9	65	575	1340	639	618	301	283	214	197	164	133	93	63	43
9	66	1347	1262	1604	1495	634	599	480	439	366	297	207	140	94
9	67	1447	21	1254	1297	619	535	442	408	339	275	194	130	89
9	68	1900	398	2232	1943	954	922	653	610	504	411	291	195	141
9	69	1634	1076	1938	1687	777	705	555	509	424	344	240	162	112
9	70	1943	1325	2157	2092	884	836	679	638	526	429	306	204	139
9	71	1458	36873	1190	1853	667	638	539	461	396	314	207	145	90
9	72	3590	12183	3825	3641	2027	1977	1298	1170	982	792	546	371	266
9	73	2552	4402	2823	2970	1295	1301	916	844	701	569	399	269	190
9	74	1818	2007	1899	1975	960	925	647	590	492	398	277	187	132
9	75	1245	12898	1452	1538	686	623	474	427	358	289	199	135	93
9	76	1522	2017	1605	1747	755	719	562	528	435	355	253	169	116
9	77	1866	36061	2304	2099	1051	938	766	708	588	477	335	225	154

9	78	2412	4657	2704	2414	1209	1053	853	801	660	539	383	256	179
9	79	1441	8016	1499	1589	693	803	509	467	389	315	221	149	106
9	80	2225	7529	2473	2525	1083	989	815	756	627	510	359	241	163
9	81	2618	30164	2511	2837	1243	1202	977	925	760	621	445	296	200
9	82	805	3631	759	887	388	383	298	286	233	192	138	91	63
9	83	3971	5757	4108	4126	1878	1887	1378	1287	1063	866	614	410	288
9	84	2265	7859	2037	2371	974	981	749	679	568	459	318	216	144
9	85	1401	14465	1413	1638	656	681	528	484	403	327	228	154	103
9	86	3407	1321	3010	3394	1514	1338	1118	1019	851	688	479	324	217
9	87	2715	21223	2883	3153	1334	1194	990	890	747	603	415	282	187
9	88	2524	21681	2742	2650	1192	1277	861	801	663	540	381	255	183
9	89	1172	2019	1382	1293	573	745	410	370	310	250	173	117	85
9	90	2158	13696	1968	2424	1016	1121	776	722	597	486	343	230	160
9	91	1438	5076	1425	1698	699	754	535	500	413	337	239	160	111
9	92	3089	1050	3493	2992	1431	1452	1015	946	782	637	451	302	216
9	93	2380	8004	2327	2545	1117	1336	833	768	638	518	364	245	174
9	94	441	8019	603	489	271	292	188	167	141	113	77	53	37
9	95	513	17	527	519	220	341	157	146	121	99	70	47	37
9	96	5226	3855	5971	4815	2510	2362	1708	1525	1285	1034	708	483	332
9	97	2187	4216	2928	2201	1183	1394	766	670	570	456	307	212	150
9	98	2054	1413	3045	1984	1206	1164	734	638	544	434	291	201	140
9	99	3557	3941	5125	3608	1969	1691	1294	1165	978	789	543	370	257
9	100	1761	2866	1645	1455	1080	797	611	550	461	372	256	174	126
9	101	1472	13245	1389	1585	675	720	526	503	411	337	243	161	112
9	102	3977	25095	3750	4343	1882	2381	1397	1279	1066	863	603	407	291
9	103	547	1500	588	584	257	367	200	182	152	123	86	58	42
9	104	323	4551	322	351	148	169	117	108	90	73	51	34	24
9	105	2399	10176	2653	2653	1137	1411	825	746	625	504	348	237	169
9	106	2289	3174	2208	2267	991	1145	739	698	574	469	335	223	161
9	107	1474	6719	1599	1773	686	862	546	501	417	338	236	159	111
9	108	754	451	783	791	339	359	272	248	207	167	117	79	53
9	109	348	677	388	369	161	198	122	110	92	74	51	35	24
9	110	2352	1644	2935	2411	1136	1457	819	735	618	498	342	233	167
9	111	1034	2410	1470	1139	511	558	383	347	290	235	162	110	76
9	112	336	112	386	341	141	162	110	99	83	67	46	31	21
9	113	1728	2414	1620	1763	760	808	583	534	445	360	251	170	116
9	114	568	1124	627	581	249	262	205	192	158	129	91	61	41
9	115	2125	153	2057	1807	894	1005	634	608	497	408	295	195	146
9	116	3280	425	3436	2874	1318	1769	1012	900	760	611	417	285	198
9	117	2561	2934	4157	2256	1169	1656	987	831	719	569	371	261	165
9	118	95	0	77	105	41	50	31	28	24	19	13	9	6
9	119	3632	10344	4981	3699	1804	1561	1286	1169	977	790	548	371	252
9	120	3290	5531	3936	3428	1582	1598	1161	1062	885	717	500	338	233
9	121	761	86	710	743	347	339	247	236	193	158	114	75	54
9	122	1198	3619	1139	1336	591	551	443	415	342	279	198	132	90

9	123	893	1519	664	954	359	538	268	238	201	161	110	75	54
9	124	413	645	383	466	198	187	160	145	121	98	68	46	30
9	125	130	210	156	160	62	61	51	48	40	32	23	15	10
9	126	810	1891	747	893	430	361	306	283	235	191	134	90	61
9	127	548	385	627	584	241	235	194	191	155	128	94	61	42
9	128	1784	7206	1748	1672	747	716	603	569	468	382	273	182	121
9	129	1305	12172	1450	1395	620	590	496	471	386	316	227	151	101
9	130	544	16558	555	632	281	249	208	202	164	135	99	65	45
9	131	584	2415	602	658	261	240	205	187	156	126	88	59	39
9	132	280	3386	240	266	114	105	90	83	69	56	39	26	17
9	133	393	34	518	193	146	173	131	161	119	104	88	53	40
9	134	3071	8734	2920	2858	1333	1272	1062	1026	835	687	499	329	223
9	135	1909	1751	1920	2073	907	789	678	606	510	411	282	192	126
9	136	3192	4658	3390	3180	1773	1350	1159	1040	874	705	484	330	224
9	137	2244	659	2361	2357	1060	922	787	716	598	484	336	228	151
9	138	745	2003	959	880	387	334	286	258	217	175	121	82	54
9	139	3192	8369	3612	3609	1592	1395	1183	1096	909	739	520	349	236
9	140	4238	17894	4636	4323	2163	1781	1520	1386	1157	936	651	440	298
9	141	4428	17406	5333	4968	2378	1967	1692	1511	1272	1024	701	479	317
9	142	1775	0	1415	987	588	488	417	379	317	256	178	120	81
9	143	340	3476	405	412	172	151	130	115	98	78	53	37	24
9	144	2486	190	2933	2346	1328	996	860	759	643	515	349	240	160
9	145	3488	4339	3944	3551	1797	1454	1251	1114	940	755	516	353	234
9	146	2735	6972	2861	2972	1284	1163	990	908	756	613	428	289	191
9	147	1505	18064	1589	2020	768	700	596	547	455	369	258	174	115
9	148	1111	12895	1053	1405	551	501	431	384	324	260	178	122	79
9	149	3420	11943	3883	3632	1944	1534	1321	1176	992	798	545	373	248
9	150	1624	481	1930	1551	903	674	581	515	435	349	237	163	109
9	151	1407	8	1024	1102	506	433	370	336	281	227	158	107	71
9	152	1746	1927	2146	1752	936	747	640	577	484	391	270	183	124
9	153	2059	2312	2290	2260	1067	896	764	698	582	471	328	222	150
9	154	3094	7121	3319	3374	1673	1358	1154	1064	884	718	504	339	233
9	155	649	3638	660	742	329	283	240	223	184	150	106	71	48
9	156	1874	5509	2102	2038	1017	806	690	623	522	421	291	198	133
9	157	1567	10324	1589	1632	926	712	607	556	463	375	262	177	122
9	158	2846	9917	3132	3354	1489	1273	1091	983	825	666	459	312	206
9	159	4176	33406	4328	4680	2152	1822	1558	1413	1182	955	662	449	299
9	160	179	140	156	167	83	73	63	57	48	38	26	18	12
9	161	1872	18918	1745	2256	885	821	699	640	533	432	301	203	134
9	162	3013	4866	3130	3164	1425	1223	1042	954	795	644	450	304	204
9	163	2134	7962	2424	2080	1216	901	780	683	580	464	313	216	142
9	164	3683	2512	3557	3631	2010	1538	1312	1197	999	809	563	381	262
9	165	3772	4056	4055	3981	1958	1574	1355	1206	1017	818	559	382	253
9	166	250	3338	253	278	110	104	90	80	67	54	37	25	16
9	167	99	10	91	122	49	42	37	32	27	22	15	10	7

9	168	127	51	114	134	55	50	44	38	33	26	18	12	8
9	169	48	2152	45	55	22	22	19	17	14	12	8	6	4
9	170	4431	317	3889	3829	2265	1662	1424	1284	1077	869	600	408	280
10	1	2298	1022	4070	2216	1064	1117	749	654	557	445	299	207	138
10	2	480	313	861	520	297	246	188	174	144	117	83	55	40
10	3	2384	146	4456	2941	1150	1443	832	716	613	488	324	225	149
10	4	1714	73	2689	1523	764	658	531	473	399	321	219	150	100
10	5	4892	2828	8518	5880	2171	2305	1725	1539	1297	1043	713	487	322
10	6	1797	717	2872	2224	818	1070	622	546	463	371	250	172	117
10	7	3705	13280	6253	4616	1805	1819	1342	1212	1016	820	567	385	261
10	8	2646	1628	4977	3367	1382	2052	982	842	722	574	380	265	180
10	9	4179	1394	7347	4401	2292	2061	1503	1305	1113	889	595	412	273
10	10	975	256	1747	799	572	497	339	287	248	196	129	90	58
10	11	2551	4186	3432	2790	1420	1290	916	817	689	554	378	259	178
10	12	2746	1390	3243	2869	1625	1293	1024	911	768	618	421	288	196
10	13	4007	9253	5232	4874	2051	1992	1473	1330	1115	900	621	422	286
10	14	5535	1089	8080	4402	3067	2187	1743	1521	1294	1035	695	480	325
10	15	2870	1101	4678	2550	1725	1343	989	849	728	579	384	267	177
10	16	3474	766	5618	3125	2143	1993	1214	1044	894	712	472	329	224
10	17	5262	719	6860	4010	3054	2246	1739	1501	1284	1023	681	473	315
10	18	2634	1394	3627	1994	1628	1133	873	744	640	508	334	234	153
10	19	5535	640	7881	4778	3281	2551	1905	1651	1409	1125	751	521	349
10	20	2725	397	3823	2362	1632	1420	955	819	702	559	370	258	171
10	21	4974	1047	6725	3967	3029	2586	1667	1432	1227	977	647	451	305
10	22	2465	499	3450	2023	1387	1132	806	684	589	468	307	215	139
10	23	5946	2685	9318	6262	2719	2622	1974	1747	1477	1186	806	553	367
10	24	4715	1259	8114	4943	2495	2050	1642	1455	1230	987	671	460	309
10	25	6235	3527	8723	7224	3047	3844	2066	1848	1555	1252	858	586	423
10	26	2626	693	3511	3194	1166	1290	866	801	665	540	380	255	180
10	27	4735	434	5087	4958	2220	2318	1524	1410	1169	950	669	449	322
10	28	3165	8750	4009	3293	1352	1618	984	912	756	615	433	291	210
10	29	1316	1589	1552	1455	594	643	468	444	364	298	214	142	98
10	30	5014	248	8591	6172	2353	2675	1781	1568	1329	1065	721	496	331
10	31	3794	941	5471	4221	1579	1891	1187	1067	896	722	497	338	233
10	32	3385	12097	3433	4178	1622	1641	1209	1116	926	752	529	355	243
10	33	2548	1376	2577	3020	1130	1124	825	758	630	511	358	241	165
10	34	2355	5897	2637	2248	1294	1289	794	709	597	480	328	224	160
10	35	2583	8850	3248	2651	1393	1820	845	757	637	513	352	240	184
10	36	7734	2430	9372	7724	4101	3557	2586	2297	1939	1558	1061	727	499
10	37	955	1418	1286	1111	605	476	370	326	277	222	150	103	70
10	38	269	114	268	342	130	157	97	88	74	60	42	28	20
10	39	3569	3952	3835	3743	1881	1821	1206	1055	897	718	483	333	226
10	40	556	5487	722	611	286	297	207	187	157	127	88	60	41
10	41	3062	10978	3346	3571	1707	1547	1103	1000	837	676	468	317	222
10	42	403	780	380	567	186	201	144	131	110	89	61	42	28

10	43	994	23372	1243	1225	485	462	366	340	282	229	162	108	74
10	44	1787	202	1571	1914	843	738	587	540	449	364	255	172	118
10	45	761	376	900	907	385	411	270	242	204	164	113	77	53
10	46	557	5794	633	730	284	274	217	206	169	138	99	66	45
10	47	1747	9080	1556	1933	844	923	574	531	441	358	252	169	122
10	48	292	5584	333	358	171	147	113	101	85	69	47	32	22
10	49	1903	15898	1729	2564	900	1170	680	603	510	409	279	191	130
10	50	1371	17793	1880	1834	939	1105	569	511	429	346	237	162	121
10	51	747	2359	788	985	350	578	260	233	196	158	108	74	54
10	52	2290	323	2270	2563	1001	1316	713	662	548	446	315	211	156
10	53	1872	2378	2303	2273	1130	1919	676	587	501	400	267	185	142
10	54	1019	3015	1076	1133	489	434	349	332	272	223	160	106	74
10	55	6859	18063	6764	7183	4182	3185	2482	2236	1876	1513	1043	709	496
10	56	1726	22505	2279	2256	891	807	654	616	507	414	295	197	135
10	57	755	6827	1049	1050	400	360	299	273	228	184	129	87	58
10	58	653	1287	771	852	335	331	236	218	181	147	103	69	49
10	59	1871	2080	1702	2379	820	804	632	575	480	388	270	183	121
10	60	513	492	621	654	234	215	176	164	136	110	78	52	36
10	61	3753	0	2157	3000	1057	992	800	734	611	495	346	234	156
10	62	2644	1922	3417	3228	1349	1218	956	911	746	611	440	291	206
10	63	2456	5945	2572	2530	1225	1152	800	731	610	494	344	233	165
10	64	1807	10451	2045	2384	909	968	668	623	515	419	297	199	139
10	65	570	1581	694	681	308	286	214	197	164	133	93	63	43
10	66	1347	1337	1765	1665	644	603	480	439	366	297	207	140	94
10	67	1474	13	1354	1433	633	537	442	408	339	275	194	130	89
10	68	1896	351	2463	2153	979	939	653	610	504	411	291	195	141
10	69	1641	1097	2146	1874	796	711	555	509	424	344	240	162	112
10	70	1945	1351	2363	2333	896	839	679	638	526	429	306	204	139
10	71	1462	58997	1239	2076	673	640	539	461	396	314	207	145	90
10	72	3565	15107	4133	3987	2095	2036	1298	1170	982	792	546	371	266
10	73	2539	5018	3073	3320	1322	1328	916	844	701	569	399	269	190
10	74	1811	2168	2053	2189	986	945	647	590	492	398	277	187	132
10	75	1233	18315	1579	1718	704	631	474	427	358	289	199	135	93
10	76	1511	2218	1735	1947	767	725	562	528	435	355	253	169	116
10	77	1831	54813	2505	2301	1071	942	766	708	588	477	335	225	154
10	78	2406	5402	2962	2663	1239	1058	853	801	660	539	383	256	179
10	79	1436	10581	1620	1765	703	829	509	467	389	315	221	149	106
10	80	2216	9336	2693	2813	1101	992	815	756	627	510	359	241	163
10	81	2600	43436	2680	3137	1252	1206	977	925	760	621	445	296	200
10	82	799	4647	810	984	391	386	298	286	233	192	138	91	63
10	83	3970	6448	4459	4572	1909	1917	1378	1287	1063	866	614	410	288
10	84	2288	9902	2183	2640	985	993	749	679	568	459	318	216	144
10	85	1393	20540	1513	1821	660	688	528	484	403	327	228	154	103
10	86	3446	1268	3226	3763	1541	1342	1118	1019	851	688	479	324	217
10	87	2712	29288	3118	3514	1359	1198	990	890	747	603	415	282	187

10	88	2524	30353	2996	2945	1213	1309	861	801	663	540	381	255	183
10	89	1167	2301	1516	1435	583	781	410	370	310	250	173	117	85
10	90	2149	18352	2091	2695	1024	1145	776	722	597	486	343	230	160
10	91	1426	6294	1524	1892	706	769	535	500	413	337	239	160	111
10	92	3102	987	3857	3312	1462	1482	1015	946	782	637	451	302	216
10	93	2374	9925	2499	2818	1129	1381	833	768	638	518	364	245	174
10	94	431	12028	659	530	278	302	188	167	141	113	77	53	37
10	95	515	12	576	578	223	362	157	146	121	99	70	47	37
10	96	5277	4010	6595	5282	2584	2408	1708	1525	1285	1034	708	483	332
10	97	2184	4891	3261	2412	1223	1467	766	670	570	456	307	212	150
10	98	2050	1441	3430	2160	1260	1208	734	638	544	434	291	201	140
10	99	3540	4257	5756	3961	2040	1713	1294	1165	978	789	543	370	257
10	100	1757	3268	1760	1562	1136	807	611	550	461	372	256	174	126
10	101	1466	18543	1487	1760	679	730	526	503	411	337	243	161	112
10	102	3964	33628	4001	4811	1901	2478	1397	1279	1066	863	603	407	291
10	103	542	1800	634	642	258	386	200	182	152	123	86	58	42
10	104	321	6736	345	388	149	172	117	108	90	73	51	34	24
10	105	2399	13025	2895	2954	1156	1471	825	746	625	504	348	237	169
10	106	2302	3555	2394	2518	1002	1178	739	698	574	469	335	223	161
10	107	1463	8600	1727	1979	689	890	546	501	417	338	236	159	111
10	108	753	451	846	872	341	364	272	248	207	167	117	79	53
10	109	348	786	423	408	162	205	122	110	92	74	51	35	24
10	110	2347	1677	3243	2655	1156	1526	819	735	618	498	342	233	167
10	111	1027	2844	1643	1260	520	572	383	347	290	235	162	110	76
10	112	339	105	426	378	142	167	110	99	83	67	46	31	21
10	113	1738	2704	1741	1952	768	823	583	534	445	360	251	170	116
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10	127	547	392	690	652	243	235	194	191	155	128	94	61	42
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10	132	284	5012	257	294	115	105	90	83	69	56	39	26	17

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10	137	2252	607	2565	2610	1079	922	787	716	598	484	336	228	151
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10	170	4483	248	4174	4179	2354	1662	1424	1284	1077	869	600	408	280

Table 50 Competitors with highest attraction with respect to demand point

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Table 51 Attraction value for Dortmunder Volksbank's facilities with respect to demand points

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36	104	2
36	105	5
36	106	3
36	107	3
36	108	4
36	109	3
36	110	5
36	111	4
36	112	4
36	113	3
36	114	4
36	115	4
36	116	3
36	117	4
36	118	2
36	119	4
36	120	3
36	121	3
36	122	3

36	123	2
36	124	2
36	125	2
36	126	3
36	127	3
36	128	3
36	129	3
36	130	2
36	131	2
36	132	2
36	133	3
36	134	3
36	135	3
36	136	2
36	137	2
36	138	2
36	139	2
36	140	2
36	141	2
36	142	2
36	143	2
36	144	3
36	145	3
36	146	3
36	147	2
36	148	1
36	149	1
36	150	1
36	151	4
36	152	3
36	153	4
36	154	2
36	155	2
36	156	2
36	157	2
36	158	2
36	159	1
36	160	2
36	161	2
36	162	2
36	163	2
36	164	2
36	165	2
36	166	2
36	167	2

36	168	2
36	169	2
36	170	1

Appendix I LINGO file for case study model

Model:

```
!Variables

assignment: Assignment variable
expanded: Expand
shrunk: Shrink
uPlus: Units of expansion
uMinus: Units of shrinkage
totalCapacity: totalCapacity of facility
open: Have facility open
opened: Open facility at end of period
closed: Close down facility at end of period
usedCapacity: The used Capacity at the facility in certain period ;

!Parameters
initalcapacity: Initial Capacity of faciltiy in period 1
initialOpen: Initial setting for having open facility in period 1
price: Price for selling
variableCost: Unit cost for selling
openingCost: Cost for opening
closingCost: Cost for closing
fixCost: cost for an open facility in a period
attraction: Attraction index of own facility
competitorsAttraction: Attraction index of competitors
expansionCost: Fix cost for expansion
shrinkageCost: Fix cost for shrinkage
unitExpansionCost: unit (variable) cost for expanding
unitShrinkageCost: unit (variable) cost for shrinkage
demand: demand at demand location ;

Sets:
    !Set of facilities;
    locations /1..36/:initialOpen, initialCapacity, fixCost,
expansionCost, shrinkageCost, openingCost, closingCost;

    !Set of demand points;
    demands /1..170/;

    !Set of time periods;
    periods /1..10/;

    !Set of products;
    products /1..13/;
```

```

indexip (locations, products);
indexjt (demands, periods): competitorsAttraction;
indexijt (locations, demands, periods): assignment, attraction;
indexitp (locations, periods, products): usedCapacity, price,
variableCost, unitExpansionCost, unitShrinkageCost;
indexjtp (demands, periods, products): demand;
indexit (locations, periods): expanded, shrunk, open, opened,
closed, totalFixCost, totalCapacity, uPlus, uMinus, threshold1,
threshold2, threshold3, threshold4, threshold5, threshold6, threshold7,
threshold8, threshold9, threshold10, thresholdSwitch1, thresholdSwitch2,
thresholdSwitch3, thresholdSwitch4, thresholdSwitch5, thresholdSwitch6,
thresholdSwitch7, thresholdSwitch8, thresholdSwitch9, thresholdSwitch10,
step1, step2, step3, step4, step5, step6, step7, step8, step9, step10,
stepCost1, stepCost2, stepCost3, stepCost4, stepCost5, stepCost6,
stepCost7, stepCost8, stepCost9, stepCost10;

```

Endsets

```

@for(locations(i):
    @for(demands(j):
        @for(periods(t):
            @for(products(p):
                @bin(assignment(i,j,t));
                @bin(expanded(i,t));
                @bin(shrunk(i,t));
                @bin(open(i,t));
                @bin(opened(i,t));
                @bin(closed(i,t));
                @gin(uPlus(i,t));
                @gin(uMinus(i,t));
                @gin(totalCapacity(i,t));
                @gin(usedCapacity(i,t,p));
                @bin(threshold1(i,t));
                @bin(threshold2(i,t));
                @bin(threshold3(i,t));
                @bin(threshold4(i,t));
                @bin(threshold5(i,t));
                @bin(threshold6(i,t));
                @bin(threshold7(i,t));
                @bin(threshold8(i,t));
                @bin(threshold9(i,t));
                @bin(threshold10(i,t));
                @bin(thresholdSwitch1(i,t));
                @bin(thresholdSwitch2(i,t));
                @bin(thresholdSwitch3(i,t));
                @bin(thresholdSwitch4(i,t));
                @bin(thresholdSwitch5(i,t));
                @bin(thresholdSwitch6(i,t));
                @bin(thresholdSwitch7(i,t));
                @bin(thresholdSwitch8(i,t));
                @bin(thresholdSwitch9(i,t));
                @bin(thresholdSwitch10(i,t));
            )
        )
    )
)

```

```

    )
);

Data:

    competitorsAttraction =
@odbc('dissertation','cs_competitors_view','COMPETITORSATTRACTION');
    attraction =
@odbc('dissertation','cs_attractivity_view','ATTRACTIVITY');
    initialCapacity =
@odbc('dissertation','CASE_STUDY_DATA_INITIALS_CAP','INITIALCAP');
    initialOpen =
@odbc('dissertation','CASE_STUDY_INITIALS','INITIALOPEN');
    variableCost =
@odbc('dissertation','CS_PRODUCT_VIEW','VARIABLECOST');
    fixCost =
@odbc('dissertation','CASE_STUDY_DATA_FACILITIES_NPR','FIXCOST');
    openingCost =
@odbc('dissertation','CASE_STUDY_DATA_FACILITIES_NPR','OPENINGCOST');
    closingCost =
@odbc('dissertation','CASE_STUDY_DATA_FACILITIES_NPR','CLOSINGCOST');
    unitExpansionCost =
@odbc('dissertation','CS_PRODUCT_VIEW','UNITEXPANSIONCOST');
    expansionCost =
@odbc('dissertation','CASE_STUDY_DATA_FACILITIES_NPR','EXPANSIONCOST');
    unitShrinkageCost =
@odbc('dissertation','CS_PRODUCT_VIEW','UNITSHRINKAGECOST');
    shrinkageCost =
@odbc('dissertation','CASE_STUDY_DATA_FACILITIES_NPR','SHRINKAGECOST');
    price = @odbc('dissertation','CS_PRODUCT_VIEW','PRICE');
    demand = @odbc('dissertation','CS_DEMAND_VIEW','DEMAND');

enddata

    max = @sum(indexitp(i,t,p): usedCapacity(i,t,p)*(price(i,t,p) -
variableCost(i,t,p))) - @sum(indexit(i,t): totalFixCost(i,t) +
closingcost(i)*closed(i,t) + openingcost(i)*opened(i,t)) -
@sum(indexit(i,t): expansionCost(i)*expanded(i,t)) -
@sum(indexitp(i,t,p): unitExpansionCost(i,t,p)*uPlus(i,t)) -
@sum(indexit(i,t): shrinkageCost(i)*shrunk(i,t)) - @sum(indexitp(i,t,p):
unitShrinkageCost(i,t,p)*uMinus(i,t));

    !Initializing open and totalCapacity data;
@for(locations(i):
    open(i,1)= initialOpen;
);

@for( periods(t) |(t) #GT#1 :
    open(1,t)= 1;
);

@for(locations(i):
    totalCapacity(i,1) = initialCapacity(i);

```



```

);

@for(locations(i):
    @for(periods(t):
        @for(products(p):
            step1(i,t) = 600;
            step2(i,t) = 1200;
            step3(i,t) = 1800;
            step4(i,t) = 2400;
            step5(i,t) = 3000;
            step6(i,t) = 3600;
            step7(i,t) = 4200;
            step8(i,t) = 4800;
            step9(i,t) = 5400;
            step10(i,t) = 6000;

            stepCost1(i,t) = 100000;
            stepCost2(i,t) = 100000;
            stepCost3(i,t) = 100000;
            stepCost4(i,t) = 100000;
            stepCost5(i,t) = 100000;
            stepCost6(i,t) = 100000;
            stepCost7(i,t) = 100000;
            stepCost8(i,t) = 100000;
            stepCost9(i,t) = 100000;
            stepCost10(i,t) = 100000;

        )
    )
);

!Modification of fix cost in terms of change in facility size;
@for(locations(i):
    @for(periods(t):

        9999999*threshold1(i,t) >= (totalCapacity(i,t) /
step1(i,t))-1;
        9999999*threshold2(i,t) >= (totalCapacity(i,t) /
step2(i,t))-1;
        9999999*threshold3(i,t) >= (totalCapacity(i,t) /
step3(i,t))-1;
        9999999*threshold4(i,t) >= (totalCapacity(i,t) /
step4(i,t))-1;
        9999999*threshold5(i,t) >= (totalCapacity(i,t) /
step5(i,t))-1;
        9999999*threshold6(i,t) >= (totalCapacity(i,t) /
step6(i,t))-1;
        9999999*threshold7(i,t) >= (totalCapacity(i,t) /
step7(i,t))-1;
        9999999*threshold8(i,t) >= (totalCapacity(i,t) /
step8(i,t))-1;
        9999999*threshold9(i,t) >= (totalCapacity(i,t) /
step9(i,t))-1;

```

```

step10(i,t))-1;
        9999999*threshold10(i,t) >= (totalCapacity(i,t) /
threshold1(i,t))+(1-open(i,t));
        threshold1(i,t) + open(i,t) + thresholdSwitch1(i,t)
<= 2;

        thresholdSwitch2(i,t)*9999999 >= (1-
threshold2(i,t))+(1-open(i,t));
        threshold2(i,t) + open(i,t) + thresholdSwitch2(i,t)
<= 2;

        thresholdSwitch3(i,t)*9999999 >= (1-
threshold3(i,t))+(1-open(i,t));
        threshold3(i,t) + open(i,t) + thresholdSwitch3(i,t)
<= 2;

        thresholdSwitch4(i,t)*9999999 >= (1-
threshold4(i,t))+(1-open(i,t));
        threshold4(i,t) + open(i,t) + thresholdSwitch4(i,t)
<= 2;

        thresholdSwitch5(i,t)*9999999 >= (1-
threshold5(i,t))+(1-open(i,t));
        threshold5(i,t) + open(i,t) + thresholdSwitch5(i,t)
<= 2;

        thresholdSwitch6(i,t)*9999999 >= (1-
threshold6(i,t))+(1-open(i,t));
        threshold6(i,t) + open(i,t) + thresholdSwitch6(i,t)
<= 2;

        thresholdSwitch7(i,t)*9999999 >= (1-
threshold7(i,t))+(1-open(i,t));
        threshold7(i,t) + open(i,t) + thresholdSwitch7(i,t)
<= 2;

        thresholdSwitch8(i,t)*9999999 >= (1-
threshold8(i,t))+(1-open(i,t));
        threshold8(i,t) + open(i,t) + thresholdSwitch8(i,t)
<= 2;

        thresholdSwitch9(i,t)*9999999 >= (1-
threshold9(i,t))+(1-open(i,t));
        threshold9(i,t) + open(i,t) + thresholdSwitch9(i,t)
<= 2;

        thresholdSwitch10(i,t)*9999999 >= (1-
threshold10(i,t))+(1-open(i,t));
        threshold10(i,t) + open(i,t) + thresholdSwitch10(i,t)
<= 2;

)

```

```

);

    @for(locations(i):
        @for( periods(t):
            totalFixCost(i,t) = open(i,t)*fixCost(i) +
stepCost1(i,t)*(threshold1-thresholdSwitch1(i,t)) +
stepCost2(i,t)*(threshold2(i,t)-thresholdSwitch2(i,t)) +
stepCost3(i,t)*(threshold3(i,t)-thresholdSwitch3(i,t)) +
stepCost4(i,t)*(threshold4(i,t)-thresholdSwitch4(i,t))+
stepCost5(i,t)*(threshold4(i,t)-thresholdSwitch5(i,t)) +
stepCost6(i,t)*(threshold6(i,t)-thresholdSwitch6(i,t)) +
stepCost7(i,t)*(threshold7(i,t)-thresholdSwitch7(i,t))+
stepCost8(i,t)*(threshold8(i,t)-thresholdSwitch8(i,t))+
stepCost9(i,t)*(threshold9(i,t)-thresholdSwitch9(i,t))+
stepCost10(i,t)*(threshold10(i,t)-thresholdSwitch10(i,t));
        )
    );

    !Only assign if higher attraction level than competitor;
    @for(indexijt(i,j,t):
        (assignment(i,j,t))*(attraction(i,j,t)-
competitorsAttraction(j,t))>= 0);

    !Usage does not exceed the totalCapacity at i in t;
    @for(locations(i):
        @for( periods(t):
            @sum(products(p): usedCapacity(i,t,p)) <=
totalCapacity(i,t);
        )
    );

    !Usage will never be greater than demand;
    @for(locations(i):
        @for( periods(t):
            @for( products(p):
                usedCapacity(i,t,p) <= @sum(demands(j):
                    (assignment(i,j,t)*demand(j,t,p))
                )
            )
        )
    );

    !totalCapacity if the facility is determined by expansion or
shrinking in the previous period;
    @for(locations(i):
        @for( periods(t)|(t) #NE#10:
            totalCapacity(i,t+1) = totalCapacity(i,t) +
uPlus(i,t) - uMinus(i,t)
        )
    );

    !Central setting of open, opening or closing down;
    @for(locations(i):

```

```

        @for( periods(t) | (t) #NE#10: open(i,t+1) = open(i,t) +
opened(i,t) - closed(i,t) ));

!If is expanded, expanded must be set to 1;
@for(locations(i):
    @for( periods(t):
        9999999*expanded(i,t) >= uPlus(i,t)
    )
);

!If is shrunk, shrunk must be set to 1;
@for(locations(i):
    @for( periods(t):
        9999999*shrunk(i,t) >= uMinus(i,t)
    )
);

!totalCapacity must be bigger than shrinkage;
@for(locations(i):
    @for( periods(t):
        totalCapacity(i,t) >= uMinus(i,t)
    )
);

!Only open facilities can be expanded;
@for(locations(i):
    @for( periods(t):
        expanded(i,t) <= open(i,t)
    )
);

!Only open facilities can be shrunk;
@for(locations(i):
    @for( periods(t):
        shrunk(i,t) <= open(i,t)
    )
);

!Only have used Capacity if open ;
@for(locations(i):
    @for( periods(t):
        @for( products(p):
            9999999*open(i,t) >= usedCapacity(i,t,p)
        )
    )
);

!Only have a maximum of one assignment ;
@for(demands(j):
    @for( periods(t):
        @sum(locations(i): assignment(i,j,t)) <= 1));

```

end

Appendix J JAVA Source Code of annealing algorithm for case study

```
//package simulatedannealing;  
  
import com.lindo.*;  
  
import java.io.*;  
  
public class SimulatedAnnealingCaseStudy extends Lindo {  
  
    // Set initial temp  
    public static double startTemp = 100    ;  
    public static double temp = startTemp;  
    // Cooling rate  
    public static double coolingRate = 0.01  ;  
  
    public static long startTime = 0;  
  
    public static String energyTracker;  
    public static String bestEnergyTracker;  
  
    public static int counter =0;  
    public static int fileCounter = 0;  
    //Retrieving variable indexes  
  
    public static boolean initialized = false;  
  
    public static int[][] startConfiguration;  
    public static int[][] currentConfiguration;  
    public static int[][] bestConfiguration;  
    public static int[][] neighbourConfiguration;  
  
    public static double startEnergy;  
    public static double currentEnergy = 0;  
    public static double bestEnergy;  
    public static double neighbourEnergy;  
  
    public static int facilities = 36;  
    public static int periods = 10;  
  
    public static int[][] variables = new int[facilities][periods];
```

```

        public static String license = "D:/Dropbox/PhD/Dissertation Sven
        Brose/09_Development/workspace/lib/lindoapi/license/lndapi100.lic";

        private static int nErrorCode[] = new int[1];
        private static StringBuffer cErrorMessage = new
        StringBuffer(LS_MAX_ERROR_MESSAGE_LENGTH);
        private static StringBuffer cLicenseKey = new
        StringBuffer(LS_MAX_ERROR_MESSAGE_LENGTH);
        double[][] array = null;
        double[] adC = null;
        double[] adB = null;

        /* Number of constraints, Number of variables */static int nM = 0, nN = 0;
        /* The length of each column. Since we aren't leaving any blanks in our
        matrix, we can set this to null */
        static int pnLenCol[] = null;

        /* The nonzero coefficients */static double adA[] = null;

        /* The row indices of the nonzero coefficients */static int anRowX[] =
        null;

        /* Simple upper and lower bounds on the variables.
        By default, all variables have a lower bound of zero
        and an upper bound of infinity. Therefore pass null
        pointers in order to use these default values. */
        static double pdLower[] = null, pdUpper[] = null;

        /* The number of nonzeros in the constraint matrix */static int nNZ = 0;
        /* The indices of the first nonzero in each column */static int anBegCol[]
        = null;
        /* The length of each column. Since there are no blanks can be null
        */static int[]
        Alencol = null;
        /* LB and UB of the variable, null means 0 LB and infinity UB */static
        double[]
        lb = null, ub = null;

        static String varnames[] = null, connames[] = null;

        public static void main(String[] args) {

            startTime = System.currentTimeMillis();
            System.out.println("Runtime in MilliSeconds: " +
            getElapsedTime());
            System.loadLibrary("lindojni");
            nErrorCode[0] = LSloadLicenseString(license, cLicenseKey);

            // Create startConfiguration (all Facilities open)

```

```

startConfiguration = new int[facilities][periods];

for(int t = 0; t < periods; t++)
{
    startConfiguration[0][t] = 1;
}

for(int i = 1; i < 33; i++)
{
    startConfiguration[i][0] = 1;
    for(int t = 1; t < periods; t++)
    {
        startConfiguration[i][t] = 0;
    }
}

for(int i = 33; i < facilities; i++)
{
    startConfiguration[i][0] = 0;
    for(int t = 1; t < periods; t++)
    {
        startConfiguration[i][t] = 0;
    }
}

//((int)Math.round(Math.random()));
}

System.out.println("Starting with temperature " + temp + " and
coolingRate " + coolingRate);

// Initialize solution
bestEnergy = getEnergy(startConfiguration);

System.out.println("Initial solution: " + bestEnergy);

// Set start as best and as current
bestConfiguration = startConfiguration;
currentConfiguration = startConfiguration;
currentEnergy = bestEnergy;

energyTracker = Double.toString(currentEnergy)+"\n";
bestEnergyTracker = Double.toString(bestEnergy)+"\n";

// Loop until system has cooled
while (temp > 1) {

```



```

        // Create new configuration

        neighbourConfiguration =
createNewConfiguration(currentConfiguration);

        // Get energy of solutions
        //double currentEnergy = getEnergy(currentConfiguration);
        double neighbourEnergy = getEnergy(neighbourConfiguration);

        // Decide if we should accept the neighbour
        if (acceptanceProbability(currentEnergy, neighbourEnergy, temp) >
Math.random()) {
            currentConfiguration = neighbourConfiguration;
            currentEnergy = neighbourEnergy;
            System.out.println("accepted neighbour"+ neighbourEnergy);
        }

        // Keep track of the best solution found
        if (currentEnergy < bestEnergy)
        {
            bestConfiguration = currentConfiguration;
            //writeFile(input, fileName);
            //System.out.println(configurationToString(bestConfiguration));
            try{
                writeFile(configurationToString(bestConfiguration)+"\r\n",
"D:/Dropbox/PhD/Dissertation Sven Brose/11_Case_Study/Volksbank/Szenario
3/Best2.txt");
            }catch(Exception e)
            {
                e.toString();
            }
            bestEnergy = currentEnergy;
            System.out.println("New Best "+ bestEnergy);
            //printConfiguration(bestConfiguration);
        }

        energyTracker = getElapsedTime() + ";" +
Double.toString(currentEnergy)+ ";" + temp;
        bestEnergyTracker = getElapsedTime() + ";" +
Double.toString(bestEnergy)+ ";" + temp;

        // Cool system
        temp *= 1-coolingRate;
        //temp = temp-coolingRate;
        System.out.println("Getting colder... ");
        System.out.println("New temperature is: "+temp);

        try{
            writeFile(bestEnergyTracker, "D:/Dropbox/PhD/Dissertation Sven
Brose/11_Case_Study/Volksbank/Simulated annealing/"+facilities+"x"+periods+"-
"+"bestEnergies-T"+startTemp+"-C"+coolingRate+".csv");
        }
    }
}

```

```

        writeFile(energyTracker, "D:/Dropbox/PhD/Dissertation Sven
Brose/11_Case_Study/Volksbank/Simulated annealing/" + facilities + "x" + periods + "-"
+"allEnergies-T"+startTemp+"-C"+coolingRate+".csv");

        }catch(Exception e)
        {
            e.toString();
        }

    }

    System.out.println("Final solution energy: " + bestEnergy);

    System.out.println("Configurations created: " + counter);

} //main

// Generalized error Reporting function
private static void APIErrorCheck(Object pEnv) {
    if (0 != nErrorCode[0]) {
        LSgetErrorMessage(pEnv, nErrorCode[0], cErrorMessage);
        System.out.println("\nError " + nErrorCode[0] + ": " + cErrorMessage);
        System.out.println();
        System.exit(1);
    }
}

// Calculate the acceptance probability
public static double acceptanceProbability(double energy, double newEnergy,
double temperature) {
    // If the new solution is better, accept it

    if (newEnergy < energy) {
        return 1.0;
    }
    // If the new solution is worse, calculate an acceptance probability
    double acceptanceProb = Math.exp(-((energy - newEnergy) /
temperature));

    System.out.println("Acceptance probability:" + acceptanceProb);

    return acceptanceProb;
}

```

```

    public static void writeFile (String input, String fileName) throws
IOException
    {

        BufferedWriter out = new BufferedWriter(
            new OutputStreamWriter(
                new FileOutputStream(fileName, true)));

        out.write(input);
        out.newLine();
        out.close();

    }

    public static long getElapsedTime() {
        long elapsed;

        elapsed = (System.currentTimeMillis() - startTime);

        return elapsed;
    }

    public static double getEnergy(int[][] configuration){

        Object pEnv = null;
        Object pModel = null;

        pEnv = LScreeEnv(nErrorCode, cLicenseKey.toString());
        APIErrorCheck(pEnv);

        pModel = LScreeModel(pEnv, nErrorCode);
        APIErrorCheck(pEnv);

        String fileName = "D:/Dropbox/PhD/Dissertation Sven
Brose/11_Case_Study/Volksbank/Szenario 3/model_voba_S3.mps";
        //System.out.println("Loading file ... ");
        //System.out.println(fileName);

        nErrorCode[0] = LSreadMPSFile(pModel, fileName, LS_UNFORMATTED_MPS);
        APIErrorCheck(pEnv);

        int[] iVar = new int[1];
        iVar[0] = 1;

```

```

        for(int i = 0; i < facilities; i++)
        {
            for(int t = 0; t < periods; t++)
            {
                if(!initialized){

                    System.out.println("Initializing variable

index");

                    String varName = "OPEN(" + (i+1) + "," + (t+1)

+");";

                    nErrorCode[0] = LSgetVariableIndex(pModel,

varName, iVar);

                    variables[i][t] = iVar[0];

                    System.out.println(varName);
                    System.out.println("index is: " + iVar[0]);

                    System.out.println("Runtime in MilliSeconds:

" + getElapsedTime());

                }

                int nCon = 1;
                String achCtype = "E";
                String apszCname[] = {"New constraint for "+i +", " +

t};

                int ia[] = new int [] {variables[i][t]};
                int ka[] = new int [] {0,1};
                double a[] = new double [] {1};
                double rhs[] = new double [] {configuration[i][t]} ;

                if(variables[i][t] != -1)
                {
                    nErrorCode[0] = LSaddConstraints(pModel,

nCon,achCtype,apszCname,ka,a,ia,rhs);
                }

            }

            initialized = true;

            //b "nErrorCode[0] =
            LSwriteMPSFile(pModel,"createdModels/mod"+fileCounter+".mps", LS_BASFILE_MPS);

            //time to call Lindo with configuration

            nErrorCode[0] = LSoptimize( pModel, LS_METHOD_PSIMPLEX, null);

            APIErrorCheck(pEnv);

            //nErrorCode[0] = LSwriteSolution(pModel, "solution.txt");

```

```

        double dObj[] = new double[1];

        nErrorCode[0] = LSgetInfo(pModel, LS_DINFO_POBJ, dObj);

        APIErrorCheck(pEnv);
        System.out.println("Current Configuration");
        //printConfiguration(configuration);
        System.out.println("Runtime in MilliSeconds: " +
getElapsedTime());

        System.out.println( "Objective Value for configuration = "+ " "+ -
dObj[0]);

        nErrorCode[0] = LSdeleteModel(pModel);
        nErrorCode[0] = LSdeleteEnv(pEnv);

        //System.out.print(nErrorCode[0]);

        APIErrorCheck(pEnv);

        fileCounter++;

        return -dObj[0];

    }

```

```

    public static int[][] createNewConfiguration(int[][]
currentConfiguration)
    {
        counter++;
        int[][] newConfiguration = new int[facilities][periods];
        newConfiguration = currentConfiguration;
        int indexi = 0;
        int indext = 0;

        while(indext == 0)
        {
            indext = (int)(Math.random()*(periods));
        }

        while(indexi == 0)
        {
            indexi = (int)(Math.random()*(facilities));
        }

        for(int t1 = indext; t1 < periods; t1++)
        {
            newConfiguration[indexi][t1] = 0;
        }
    }

```

```

        for(int t2 = 0; t2 <=indext; t2++)
        {
            newConfiguration[indexi][t2] = 1;
        }

        return newConfiguration;
    }

    public static void printConfiguration(int[][] configuration)
    {
        for(int i = 0; i < facilities ; i++)
        {
            System.out.println("");
            for(int t = 0;t < periods; t++)
            {
                System.out.print(configuration[i][t]);
            }
            System.out.println("");
        }

        public static String configurationToString(int[][] configuration)
        {
            System.out.println("drin");
            String config = "";
            for(int i = 0; i < facilities ; i++)
            {
                for(int t = 0;t < periods; t++)
                {
                    config = config + (configuration[i][t]);
                }
                config = config + "\r\n";
            }
            return config;
        }
    }
}

```

CURRICULUM VITA

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Research Interests

Applied Operations Research, Supply Chain Management, Logistics, , Management Information Systems, Decision Support Systems, , Software Engineering, Software Development, Design of Experiments

Education

Ph.D in Industrial Engineering at University of Louisville, USA (since September, 2009)

Diploma in Industrial Engineering at Technische Universitat Dortmund, Germany
(October, 2003 - September, 2008)

Vocational Training in Software Development at adesso AG, Dortmund, Germany
(August, 2000 - February, 2003)

Working Experience

Manager in Business Consultancy at Sopra Steria GmbH, Hamburg, Germany (since February 2014)

Senior Consultant in Requirements Engineering at adesso AG, Dortmund, Germany
(October 2010 – January 2014)

Financial Controller at Eurowings Luftverkehrs AG, Dortmund, Germany (June 2007 – June 2010)

Professional Skills

Analysis Software: Lingo (mathematical programming), MiniTab (statistics analysis), Arena (simulation), Excel, Gephi BIC Design

Common Software: Microsoft Word, PowerPoint, Project, Visio, Access, etc.

Programming Languages: Java, Visual Basic, UML, SQL, LATEX, PHP

OS Platforms: Microsoft Windows, Linux

Database Platforms: Oracle, MySQL

Languages: Fluent in German and English, basics in Spanish

Awards and Honors

Golden Key International Honour Society (2012)

Intra-corporate award for paperless business models (2013)

ISTQB Certified Tester (2011)

Lean Six Sigma Green Belt (2012)

Lean Six Sigma Black Belt (2013)

Lean Six Sigma Master Black Belt (2015)

Affiliations

Verein Deutscher Ingenieure (VDI)