New families of voltage-mode and current-mode filter circuits.

Tongfeng Qian
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NEW FAMILIES OF VOLTAGE-MODE AND CURRENT-MODE FILTER CIRCUITS

By

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University of Louisville
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ABSTRACT

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Tongfeng Qian

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In some previous papers, feedforward configurations of realizing second order allpass transfer functions with complex poles by adding some configurations to a first order circuit are discussed. In this dissertation, the above idea is extended to realize some other basic second order complex pole filter transfer functions.

A new corollary for circuit conversion is proposed and proved. This corollary is useful for converting op amp based voltage-mode circuits to their CCII based equivalent circuits, as are other existing theorems. But the new corollary is useful for converting circuits that cannot be converted by other theorems.

New voltage-mode feedforward filter realizations are proposed, as well as current-mode configurations. Current-mode circuits are constructed through the feedforward concept or converted from their voltage-mode counterparts. Simulations and laboratory work are done for some current-mode networks.
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CHAPTER I
INTRODUCTION

Active filters are widely used in communication, automatic control and instrumentation. The study of second order active networks is of importance because of their simplicity and usefulness [1, p.145]. Oftentimes a second order filter can achieve design objectives by itself. But if a higher order filter is needed, it can be implemented by cascading second order sections together with a single first order section if the higher order required is odd. Second order networks with complex poles have the capability of achieving steep magnitude characteristics which improves frequency selectivity. Thus, the emphasis in this work is primarily on new circuits that can realize complex poles.

The general form of a second order transfer function is

\[ T(s) = \frac{N(s)}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \]  

(1.1)

where \( \omega_0 \) is the undamped natural frequency [2], \( Q \) is the quality factor, and \( N(s) \) is a real polynomial whose order can be 2, 1 or zero. If \( 1/2 < Q < \infty \), the poles are complex and in the left half of the s-plane.

Second order transfer functions with complex poles can be achieved by passive RLC networks, but the size of the inductors and the poor performance of inductors at low frequency restrict their applications in passive networks. While passive resistor-capacitor
(RC) networks alone can not realize complex poles [3], active networks with RC and active building blocks are widely accepted as replacements for passive RLC networks, especially at lower frequencies.

Many novel second order circuits have been proposed in the literature from time to time, in voltage-mode, current-mode or mixed-mode realizations. Some of the circuits are capable of realizing complex poles, while others are not able to realize complex poles, or they may make poles barely complex ($Q$ may only be very little greater than 1/2), thus reducing the functionality of the circuits.

Whether or not a second order transfer function may have complex poles and the range of $Q$ are determined by the denominator polynomial of the transfer function. In some cases, it is easy to form a judgment by inspection, while in some other cases when the expressions for the coefficients of the polynomial are complicated, a more involved procedure may be needed to calculate the range of the $Q$ [4].

Active devices are common to all active filters. In the following sections of this chapter, the most widely used active devices are described and circuit analysis models are provided. These models are then utilized in the analysis and design of filter circuits presented in later chapters.

1.1 Operational Amplifiers

An operational amplifier [1, p. 190] (op amp) is a typical voltage-mode active element, and it is often represented as a voltage-controlled voltage source (VCVS). It is the most commonly used voltage-mode active building block in almost all analog applications in electronics.
Figure 1(a) shows a commonly accepted op amp schematic [5], and Figure 1(b) is its equivalent circuit [1, p. 203]. Ideally, the voltage gain $A$ is negatively infinite and is not dependent on frequency. Hence, the voltage between the negative (inverting) and positive (non-inverting) input terminals, $V_a$, is zero when the op amp is operating in the linear region. When non-ideal properties are considered, $A$ is no longer frequency independent and becomes $A(s)$, which can be represented as

$$A(s) = -\frac{A_0\omega_o}{s + \omega_o} = -\frac{GB}{s + \omega_o}$$  \hspace{1cm} (1.2)

in small signal operations. In (1.2), $A_0$ is the magnitude of $A(s)$ as $\omega \rightarrow 0$ (the magnitude of the open loop dc gain), $\omega_o$ is the open-loop 3-dB bandwidth and $GB$ is the open loop gain-bandwidth product of the op amp. Typical values for $A_0$ range from $10^5$ to $10^9$, and typical values for $\omega_o$ range from 10 to 20 rad/s.

Op amps can be employed to realize voltage-mode filters. Figure 2(a) is a KRC realization of a second order lowpass filter [1, p. 323]. The ideal transfer function (the transfer function found using the ideal op amp model, assuming the components are ideal, and assuming negligible parasitic elements) is

$$\frac{V_o}{V_i} = \frac{K}{R_1R_2C_1C_2s^2 + s \left[ \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} (1 - K) \right] + \frac{1}{R_1R_2C_1C_2}}$$  \hspace{1cm} (1.3)

where $K$ is the overall dc gain of the filter and the dc gain of the voltage amplifier. The voltage amplifier with gain $K$ (dashed block) is realized as shown in Figure 2(b).

Figure 3(a) shows another example of a second order lowpass filter which is realized by a $-KRC$ network [1, p. 339]. The voltage gain $-K$ (dashed block) is realized as shown
Figure 1. Operational amplifier model: 
(a) operational amplifier, (b) equivalent circuit
Figure 2. KRC lowpass circuit (a) and realization of positive gain $K$ (b)
Figure 3. $-KRC$ lowpass circuit (a) and realization of negative gain $-K$
in Figure 3(b). A third example, shown in Figure 4 [1, p. 350], uses an infinite gain voltage amplifier with RC components.

A second order lowpass network can also be realized by using two or more op amps with RC components. Such realizations, although more expensive because more components are used, often have advantages such as lower sensitivities to element values or they can be more easily adjusted to achieve design filter characteristics.

All the above realizations can achieve second order lowpass transfer functions with complex poles by proper assignment of the component values. Each realization usually has some advantage over the others, and so designers have many choices to make depending on the filter application.

Other types of second order filters, such as highpass, bandpass, bandstop (notch) and allpass filters, can also be realized by one or more op amps with RC components and their denominator poles can be complex. Figure 5 is a $-KRC$ realized second order bandpass circuit [1, p. 392]. From its transfer function

$$\frac{V_o}{V_i} = \frac{-K}{s} \frac{1}{RC_1} \frac{1}{s^2 + s \left( \frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_3C_3} \right) + \frac{(K+1)R_1 + R_2 + R_3}{R_1R_2R_3C_1C_2}}$$

(1.4)

one can see that the poles can be complex.

In Chapter 2, commonly used voltage-mode filters of second order are discussed in more detail.
Figure 4. Lowpass circuit with infinite gain amplifier

Figure 5. A $-KRC$ bandpass circuit
1.2 Current Conveyors

A current conveyor is an approximation to an ideal current-controlled current source (CCCS). Its invention resulted from seeking for precise voltage-to-current converters [6]. Current conveyors can operate at higher bandwidths [7], with greater linearity and larger dynamic range than their voltage-based circuit counterparts while keeping the same sensitivity properties [8]. Thus, applications for realizing voltage-mode and current-mode transfer functions using current conveyors have received much attention [9].

The first generation current conveyor (CCI) was introduced in 1966, and its schematic representation is shown in Figure 6 [6]. The ideal model representation is

\[
\begin{bmatrix}
i_y \\
v_x \\
i_z
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & \pm1 & 0
\end{bmatrix}
\begin{bmatrix}
v_y \\
i_x \\
v_z
\end{bmatrix},
\]

(1.5)

where the plus sign is for positive CCI (CCI+) and the minus sign is for negative CCI (CCI−). From (1.5) it can be seen that

(1) The current flowing into port \( Y \) equals the current going into port \( X \) and the voltage at port \( Y \) is independent of the current. Thus, the input at port \( Y \) appears as an ideal CCCS.

(2) The voltage at port \( X \) depends on the voltage at port \( Y \), and the current flowing into port \( X \) depends partly on this voltage. If \( V_Y \) is zero, then the input at port \( X \) would appear as a short circuit to ground (ideally).

(3) The current flowing into port \( Z \) equals the current going into port \( X \), independent of the voltage at port \( Z \). Thus, looking back into port \( Z \), we see a current source.
Figure 6. Schematic representation for CC

Figure 7. Model for non-ideal CCII and typical parasitic element values

\[ R_X = 50\Omega \quad C_X = 2\text{pF} \quad R_Y = 10\text{M}\Omega \]
\[ C_Y = 1.5\text{pF} \quad R_Z = 3\text{M}\Omega \quad C_Z = 4.5\text{pF} \]
\[ \beta = \pm 0.999 \text{ (} + \text{ for CCII+ and } - \text{ for CCII−)} \]
The second generation current conveyor (CCII) was introduced in 1968, soon after the invention of the CCI. The schematic diagram of a CCII is similar to that of the CCI, and its model representation is [6]

\[
\begin{bmatrix}
i_y \\
v_x \\
i_z
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
v_y \\
i_x \\
v_z
\end{bmatrix}
\]

(1.6)

where the plus sign is for positive CCII (CCII+) and the minus sign is for negative CCII (CCII−). The difference from CCI is that at port Y, the current is ideally zero; therefore the CCII ideally exhibits infinite input impedance at port Y. Although the CCI offers some advantages over CCII, such as in applications of external feedback without requiring a connection to port Z [10], it is far less used than CCIIIs because the latter have the property of construction simplicity. One fewer dependent source must be realized to construct a CCII in comparison with a CCI. Since the CCII was introduced, it has widely replaced the CCI in applications. For example, a synthesis procedure has been proposed to realize voltage transfer functions with CCIIIs [11], some CCII-based voltage-mode circuits are presented [12-14], and circuit conversions from some op amp based circuits to their CCII-based equivalent circuits have been introduced [15], [16].

Actual CCs behave differently from the matrix representations in (1.5) and (1.6) when parasitic elements are considered, and a commonly adopted non-ideal model for CCIIIs and the typical parasitic element values are shown in Figure 7 [17], [18]. Resistor \( R_x \) results in non-ideal behavior when it can not be compensated by additional circuitry. The error can be reduced by using an extra CCII. Gift [19] gives an example in which he uses two CCIIIs to design an instrumentation amplifier. The parasitic capacitor \( C_z \) introduces a lowpass property of the circuits, and \( R_z \) sometimes creates non-negligible errors when
other resistances in the circuits are not small enough compared to it and they can not be compensated. The $\beta$ would create error when feedback is applied. Generally speaking, other parasitic elements are not as significant as $R_x$, $C_z$, $R_z$ and $\beta$, and they will not be considered in the applications in this work in which CCII's are employed. A more detailed theoretical analysis of the performances of the CCII's with more complicated parasitic elements included is made by Fabre [18].

1.3 Adjoint Theorem and Its Application

Due to the superior performance of current-mode networks, voltage-mode networks can be improved in some instances by converting them to their current-mode counterparts. The advantages are generally wider bandwidth and greater dynamic range while achieving the same current transfer functions as the voltage transfer functions of their prototypes. A common method for converting many voltage-mode circuits is based on the adjoint theorem [8].

**Adjoint theorem:**

Given any network $N$, a corresponding network $N_a$, referred to as the adjoint network, can be created such that when the excitation and response of network $N$ are interchanged and network $N$ is replaced by $N_a$, the input-output transfer function remains the same.

In the conversion of a voltage-mode network to a current-mode network, the application of the adjoint theorem is shown in Figure 8. To construct $N_a$, the adjoint network of network $N$, one replaces each element in network $N$ by its adjoint element. Some useful electrical elements and their corresponding adjoint elements are shown in Figure 9.
\[
\frac{V_O}{V_i} = \frac{I_O}{I_i}
\]

Figure 8. Network \(N_a\) is the adjoint network of \(N\)

Figure 9. Electrical elements and their corresponding adjoint elements
Roberts and Sedra [20] employed the adjoint theorem to convert op amp-based voltage-mode filter circuits to their current amplifier-based current-mode counterparts. A current amplifier with gain other than unity was composed with two CCs due to the unity-gain property of the CCs. It is obvious that only a voltage-mode network with unity-gain voltage amplifiers can be converted to a CC-based current-mode network by using the adjoint theorem without the need of extra active components. The following example of frequency discriminators illustrates the application of the adjoint theorem to obtain a practical current-mode circuit.

Frequency discriminators are networks that can realize transfer functions which have a linear magnitude characteristic versus frequency over some frequency range of application. They convert a narrow band frequency modulated signal to an amplitude modulated signal, and thus they are useful in instrumentation applications for some types of sensors such as wind speed sensors.

Multiplying a Butterworth lowpass transfer function by \( s \) yields a frequency discriminator transfer function whose magnitude is proportional to frequency in the passband [1, pp. 527-529]. The higher the order of the Butterworth transfer function in use, the better the linearity of the discriminator. Frequency discriminators with order greater than two can be realized by cascading second order and/or first order transfer functions. Odd order discriminators can be realized by cascading one first order highpass function and one or more second order lowpass functions, depending on the order of the discriminator; even order discriminators are available by cascading one second order bandpass function and one or more second order lowpass functions, depending on the order of the discriminator. Figure 10 shows a voltage-mode sixth order discriminator
Figure 10. Voltage-mode sixth order frequency discriminator
realized by cascading one second order bandpass section (Figure 5) and two lowpass sections (Figure 2(a)). The required transfer functions can be obtained by proper assignment of the component values. And more, the required transfer functions are also achievable if all the $K$s (the gains of the voltage-mode amplifiers) in the network are set to unity. This means the voltage-mode transfer function can be converted to its counterpart in current-mode by using the adjoint theorem.

Current-mode frequency discriminators have been previously described [21-23]. Figure 11 depicts a current-mode sixth order frequency discriminator which is converted from the network in Figure 10 with all $K$s set to unity by employing the adjoint theorem.

Actually, frequency discriminators with odd orders and with even orders whose order is greater than four can all be realized using the above cascading method in both voltage-mode and current-mode realizations. But for discriminators with order of four, the situation is different. The fourth order Butterworth polynomial requires $Q$ of the bandpass section to be 0.54, while the maximum $Q$ is 0.53 if $K$ is set to unity. This means that voltage-mode networks are still realizable by setting $K$ greater than 1, but they can not be converted to CCII-based current-mode networks through the adjoint theorem without an extra CCII. Similarly, the network in Figure 5 can not be converted to a second order CCII-based current-mode frequency discriminator with a single CCII by the adjoint theorem, because the pole-$Q$ required is $1/\sqrt{2}$, greater than the maximally realizable $Q$ (0.53) unless an additional CCII is utilized.
Figure 11. Current-mode sixth order frequency discriminator
1.4 Other Operational Devices

Besides the op amps and CCs described above, there are some mixed-mode active building blocks available.

An operational transconductance amplifier (OTA) is a voltage-controlled current source (VCCS) [24]. Its schematic representation and ideal model are shown in Figure 12 where the transconductance \( g_m \) is proportional to \( I_{ABC} \). Figure 13 shows the non-ideal model of an OTA. The transconductance \( g_m \) is frequency dependent (in addition to being dependent on \( I_{ABC} \)), and the values of the parasitic elements are also dependent on \( I_{ABC} \). The dynamic ranges for OTAs are relatively narrow. One of the advantages of OTAs is that its \( g_m \) is controlled by means of the amplifier bias current \( I_{ABC} \). Thus, it is possible to electrically tune the circuit by changing \( I_{ABC} \). Another advantage of OTAs is that they are relatively simply devices that require only a few transistors to construct. Thus, in comparison with other active devices, they require less area on an integrated circuit chip.

A current feedback op amp (CFOA) is a current-controlled voltage source (CCVS). It can be considered to be a CCII+ followed by a voltage buffer (Figure 14). The advantage of the CFOA is that the buffered voltage output does not load the output of the CCII+, thus making feedback designs easy and flexible. Some applications of CFOA have been published in the literature [25], [26].

In the following chapters, new families of voltage-mode and current-mode second order active filters are presented as well as a new current-mode realization for frequency discriminators of order \( n \). These realizations can make use of well-known second order voltage-mode and current-mode realizations as building blocks. Thus, for the sake of
Figure 12. Operational transconductance amplifier (OTA):
(a) schematic representation, (b) ideal model

Figure 13. Model for non-ideal OTA

Figure 14. Schematic representation for CFOA
completeness, a brief summary of currently used well-known voltage-mode second order filters is also provided. These filters provided a benchmark against which the new filters can be compared for sensitivities, ease of tuning, spread of component values, and the number of passive and active components needed. The active devices used are op amps for voltage-mode circuits and CCIIIs for current-mode circuits.
CHAPTER II
CURRENTLY USED VOLTAGE-MODE FILTERS

In this chapter, some commonly used realizations of second order voltage-mode filters are presented with their properties provided for comparison purposes. They are the basis for the construction of new feedforward configurations that will be discussed in the following chapters.

A design for a network with a specified transfer function is not unique. There most often exist several circuit realizations with different topologies and different element values that result in the same transfer function. When the ideal op amp model is used and the passive element values are equal to their nominal values, networks based on different designs work equally well. In practice, however, one network may outperform another when non-ideal conditions are considered, such as the non-ideality of the op amp model (Figure 1 with frequency-dependent $A$), the departure of passive element values from their nominal values and their variations when time, temperature, pressure and humidity change.

Sensitivity functions are applied to compare network performances with regard to element variations. The root-sensitivity function is one such type of the sensitivity function. It is defined by [1, p167]
\[ S_q' = \lim_{{\Delta q \to 0}} \left( \frac{\Delta r}{|r|} \right) = q \frac{\delta r}{|r| \delta q} \quad (q \neq 0, q \neq \infty, r \neq 0), \quad (2.1a) \]

where \( r \) is a simple root, which is a pole or zero of a system function, and \( q \) is the element for which the sensitivity is being investigated. The root-sensitivity function provides a straightforward way to determine the effects of element variations on the poles and zeros of a network function. However, the root-sensitivity functions are generally complex.

Instead of root-sensitivity functions, the \( \omega_0 \)- and the \( Q \)-sensitivity functions can also be employed, giving an efficient way for the analysis for second order polynomials. Their definitions are as follows [1, p172]:

\[ S_{\omega_0}^q = \lim_{{\Delta \omega_0 \to 0}} \left( \frac{\Delta \omega_0}{\omega_0} \right) = q \frac{\delta \omega_0}{\omega_0 \delta q} \quad (q \neq 0, q \neq \infty, \omega_0 \neq 0), \quad (2.1b) \]

and

\[ S_Q^q = \lim_{{\Delta Q \to 0}} \left( \frac{\Delta Q}{Q} \right) = q \frac{\delta Q}{Q \delta q} \quad (q \neq 0, q \neq \infty, Q \neq 0), \quad (2.1c) \]

where \( \omega_0 \) and \( Q \) are associated with a pair of complex poles or a pair of complex zeros. Higher order polynomials can be studied using \( \omega_0 \)- and \( Q \)-sensitivities by examining these sensitivities for each second order factor.

In the following sections, various commonly used second order filters that can realize complex poles are presented and their properties are evaluated.
2.1 Commonly Used Lowpass Filters

A second order lowpass transfer function is characterized by two zeros at infinity. It attenuates high-frequency signals and passes low-frequency ones at steady state. The general transfer function is

\[ T_{lf}(s) = \frac{H}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \]  

(2.2)

where \( H \) is the magnitude scale factor.

The KRC realization of second order lowpass filters (Sallen-Key second order lowpass circuit [27]) is a widely used network which is shown in Figure 2(a) and its transfer function is given in (1.3). The \( \omega_0 \) and \( Q \) of the poles are given by

\[
\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad \text{and} \quad Q = \frac{1}{\sqrt{\frac{R_2 C_2}{R_1 C_1}} + \frac{R_1 C_2}{R_2 C_1} + (1 - K)\sqrt{\frac{R_1 C_1}{R_2 C_2}}} 
\]

and the \( \omega_0 \)-sensitivities and the \( Q \)-sensitivities are

\[
S_{\omega_0}^{\omega_0} = S_{R_1}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}, \quad S_{R_2}^{\omega_0} = 0, \quad S_{K_1}^{\omega_0} = -S_{R_2}^{\omega_0} = Q \left( \frac{R_2 C_2}{R_1 C_1} - \frac{1}{2} \right)
\]

\[
S_{C_1}^{Q} = -S_{C_2}^{Q} = Q \left( \sqrt{\frac{C_2}{C_1}} \left( \frac{R_1}{R_2} + \frac{R_2}{R_1} \right) - \frac{1}{2} \right), \quad S_{K_1}^{Q} = Q \left( \frac{R_1 C_1}{R_2 C_2} + \frac{R_2 C_2}{R_1 C_1} + \frac{R_1 C_1}{R_2 C_2} \right) - 1. \quad (2.3)
\]

All the \( \omega_0 \)-sensitivities are no greater than 1/2 in magnitude and independent of \( Q \), while the \( Q \)-sensitivities are all dependent on \( Q \). Moschytz [28] uses an impedance tapering procedure for a low-sensitivity circuit design when \( Q \) is moderate. In high-\( Q \) applications, \( S_{K_1}^{Q} \) is always high no matter how one selects component values, while other \( Q \)-sensitivities can be made low by making the \( R_s \) approximately equal and the ratio of \( C_1 \) to
$C_2$ approximately the order of $Q^2$. An excessive spread in capacitor values will make this circuit more susceptible to environmental conditions, and $Q$ will change as the environment conditions change.

If a unity-gain voltage amplifier is used, namely, $K = 1$, $S^Q_k$ becomes less important in the circuit construction because the op amp is used as a voltage buffer which has excellent gain accuracy and closed-loop bandwidth, but for other $Q$-sensitivities, the spread in capacitance values remains. By employing selectivity enhancement technique [29], the problem of the spread in capacitance values can be solved to some extent without severe sensitivity degradation. Figure 15 depicts a circuit in which another voltage amplifier is introduced in the Sallen-Key second order lowpass circuit to enhance selectivity. The transfer function is given by

$$
\frac{V_o}{V_i} = \frac{K_1K_2}{R_1R_2C_1C_2} \frac{1}{s^2 + s \left[ \frac{1}{R_1C_1} + \frac{1}{R_2C_2} (1 - K_1K_2) \right] + \frac{1}{R_1R_2C_1C_2}},
$$

(2.4)

where $K_1$ and $K_2$ are the gains of the two amplifiers, and therefore $K_1K_2$ is the $dc$ gain of the circuit. It is seen that the quantity $1/R_2C_1$ in the $s$-term of the denominator in (1.3) disappears in (2.4). For the same component values as used in Figure 2(a), there is an enhancement in $Q$. The use of $K_1$ and $K_2$ also yields a smaller spread in component values to realize higher $Q$. The $\omega_0$-sensitivities for (2.4) are the same as in (2.3), and the $Q$-sensitivities in (2.4) are given by

$$
S^{Q}_{C_1} = -S^{Q}_{C_2} = S^{Q}_{R_1} = -S^{Q}_{R_2} = Q \left( \frac{R_2C_2}{R_1C_1} - \frac{1}{2} \right), \quad S^{Q}_{K_1K_2} = Q \left( \sqrt{\frac{R_2C_1}{R_1C_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} \right) - 1.
$$

(2.5)
Figure 15. Lowpass $KRC$ with selectivity enhancement

Figure 16. Sallen-Key highpass circuit
The $Q$-sensitivities with respect to $R_s$ and $C_s$ can be made low by setting the ratios of $R_s$ and $C_s$ to be approximately equal to $Q$ in high-$Q$ applications; however, $S_{Q_i, k_2}$ is still high. If Bach’s circuit (circuit in Figure 15 with $K_1=K_2=1$) [30] is used, $S_{Q_i, k_2}$ is greatly reduced and its effect can be ignored.

If the non-ideal model of the op amp is used, the transfer function of the lowpass circuit in Figure 2(a) with $K = 1$ is

$$\frac{V_o}{V_I} = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2 + \frac{s}{GB}(s^2 + bs + \omega_0^2)} \quad (2.6)$$

where

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad \omega_0 = \frac{1}{Q} \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1}$$

and

$$b = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \quad (2.7)$$

while the transfer function of Bach’s circuit is

$$\frac{V_o}{V_I} = \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2 + \frac{s}{GB^2} + \frac{2}{GB}(s^2 + bs + \omega_0^2)} \quad (2.8)$$

where

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad \omega_0 = \frac{1}{Q} \frac{1}{R_1 C_1} \quad \text{and} \quad b = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2}.$$

Since $GB >> \omega_0$, $s^2/GB^2$ can be ignored in (2.8). The magnitude of the resulting real pole of the denominator in (2.8) is approximately one half of that in (2.6). Thus, there will be more attenuation in magnitude at high-frequency for Bach’s circuit. If $\omega_0 << GB$, the effect of the real axis pole is not significant.
The \(-KRC\) realization of a second order lowpass function (Figure 3(a)) uses four resistors and two capacitors. It has the advantage that all the \(\omega_\theta\)-sensitivities and \(Q\)-sensitivities are no greater than \(1/2\) in magnitude. The drawback of such a realization is that the non-ideal op amp model shows that the poles of the transfer function may move to the right half \(s\)-plane and result in oscillation in moderate-\(Q\) applications if \(GB\) is not very much greater than \(\omega_\theta\).

The realization with infinite gain voltage amplifier (also called a Rauch filter [31]) is shown in Figure 4 and its transfer function is the following:

\[
\frac{V_o}{V_i} = \frac{1}{R_2 R_3 C_1 C_2} \frac{1}{s^2 + \frac{1}{C_1} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{1}{R_1 R_2 C_1 C_2}}.
\]  

The \(dc\) gain is \(-R_1/R_3\). Compared with the \(KRC\) realization with unity-gain amplifier, this circuit can realize a \(dc\) gain other than unity by the addition of a resistor. The \(\omega_\theta\) and \(Q\) of the poles of (2.9) are

\[
\omega_\theta = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad Q = \sqrt{\frac{C_1}{C_2}} \sqrt{\frac{R_1}{R_2} + \frac{R_2}{R_1} + \frac{R_1 R_2}{R_3}},
\]

and the \(\omega_\theta\) and \(Q\)-sensitivity functions are given by

\[
S_{\omega_\theta}^{R_1} = S_{\omega_\theta}^{R_2} = S_{\omega_\theta}^{C_1} = S_{\omega_\theta}^{C_2} = -\frac{1}{2}, \quad S_{\omega_\theta}^{R_3} = 0, \quad S_{\omega_\theta}^{Q} = Q \frac{R_2 C_2}{R_1 C_1} - \frac{1}{2}.
\]

\[
S_{\omega_\theta}^{Q} = Q \frac{R_2 C_2}{R_2 C_1} - \frac{1}{2}, \quad S_{\omega_\theta}^{Q} = Q \frac{R_1 R_2}{R_3} \frac{C_2}{C_1}, \quad S_{C_1}^{Q} = -S_{C_2}^{Q} = \frac{1}{2}.
\]

The sensitivity functions have similar properties to those of the \(KRC\) realization with unity-gain amplifier. The \(\omega_\theta\)-sensitivities are all no greater than \(1/2\) in magnitude and the
\( Q \)-sensitivities can be made low in moderate-\( Q \) applications. The circuit requires a relatively large spread in capacitor values for higher-\( Q \) applications. The properties of the circuit when the non-ideal op amp model is used are almost the same as that for the KRC realization.

Each realization above has advantages and drawbacks compared with other realizations. The situation provides flexibility for designers.

2.2 Commonly Used Highpass Filters

A second order highpass transfer function is characterized by two zeros at the origin. Therefore, at steady state, low-frequency signals are attenuated and high-frequency signals are passed. The general transfer function can be expressed as

\[
T_{hp}(s) = H \frac{s^2}{s^2 + s \omega_0 + \omega_0^2/Q}.
\]  

(2.10)

A second order highpass transfer function with complex poles can be realized by a single op amp with positive gain, negative gain or infinite gain, as is the situation for lowpass filters.

Figure 16 is the KRC realization of a second order highpass circuit (Sallen-Key second order highpass circuit) [1, p. 399]. It has the same topology as the Sallen-Key lowpass circuit, and it is obtained by exchanging \( R_s \) and \( C_s \) in the Sallen Key lowpass circuit in Figure 2(a). Its transfer function is given by

\[
\frac{V_o}{V_i} = \frac{Ks^2}{s^2 + \left[ \frac{1}{R_2} \left( \frac{1}{C_1} + \frac{1}{C_2} \right) + \frac{1-K}{R_1C_1} \right] + \frac{1}{R_1R_2C_1C_2}}.
\]  

(2.11)
The properties of $\omega_0$- and $Q$- sensitivity functions are almost identical to the lowpass KRC circuit. The $\omega_0$-sensitivities are all low and the $Q$-sensitivities depend on $Q$. When a unity-gain amplifier is employed the $Q$-sensitivities can be lowered if the $C$s can be selected to be equal, but a large spread of resistor values is required. If needed, selectivity enhancement techniques can also be used in the highpass network to decrease the spread of resistor values. If the non-ide al model of the op amp is applied, the transfer function of the highpass circuit in Figure 16 with unity-gain voltage amplifier is

$$\frac{V_O}{V_i} = \frac{s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2 + \frac{s}{GB}(s^2 + bs + \omega_0^2)}$$

where

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \quad \frac{\omega_0}{Q} = \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \quad \text{and} \quad b = \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_1 C_1}.$$  

The departures of $\omega_0$ and $Q$ from their nominal values are similar to those in the lowpass KRC realization.

Second order highpass functions can be obtained with a $-KRC$ realization [1, p. 404]. In the $-KRC$ realization, $K$ controls both $\omega_0$ and $Q$. The sensitivity properties are almost the same as for the $-KRC$ lowpass circuit. If $K = 1$, the maximum $Q$ that can be realized is $1/\sqrt{2}$.

By exchanging $R_s$ and $C_s$ in Figure 4, a second order highpass network with an infinite-gain voltage amplifier can be constructed [1, p. 406]. The $\omega_0$-sensitivities are all no greater than $1/2$ in magnitude and the $Q$-sensitivities for $R_s$ ($C_s$) are almost identical to the $Q$-sensitivities for $C_s$ ($R_s$) in Figure 4. High-$Q$ circuits of this variety require a large spread of resistor values if capacitance values are equal.
Generally, due to the non-ideal op amp and due to parastic elements in practice, the response of a highpass network will begin to fall off at some high frequency. The use of better (and more expensive) op amps with $GB$ of greater magnitude and the employment of high frequency construction techniques for the circuit will increase the frequency at which the response begins to fall off. Nevertheless, even with these remedies, a frequency will be reached at which the response begins to fall off thereby departing from the magnitude response of an ideal highpass network.

2.3 Commonly Used Bandpass Filters

The general form of a second order bandpass filter is

$$T_{BP}(s) = H \frac{\omega_0}{s} \frac{Q}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}.$$  \hspace{1cm} (2.13)

It has one zero at the origin and the other at infinity.

The $KRC$ realization of a second order bandpass circuit (Sallen-Key bandpass circuit) is shown in Figure 17 [32]. The $\omega_0$-sensitivities of the circuit are low, and the $Q$-sensitivities are all dependent on $Q$. It is similar to the lowpass $KRC$ realization in this respect. If a unity-gain amplifier is used, the maximum $Q$ that can be reached is 2.02.

Besides the bandpass circuit based on the circuit in Figure 3(a), another $-KRC$ bandpass realization is shown in Figure 18. The $\omega_0$ and $Q$ sensitivities are all low. However, quite large $K$ is required for high-$Q$ circuits, and $\omega_0$ and $Q$ are all dependent on $K$. The departure of the complex poles from their nominal positions is greater than in the
Figure 17. KRC bandpass circuit

Figure 18. A second –KRC bandpass circuit
A realization with an infinite-gain voltage amplifier is shown in Figure 19. The $\omega_o$-sensitivities are all low, and the $Q$-sensitivities with respect to $R_s$ are low; the $Q$-sensitivities to $C_s$ can be made equal to zero by making $C_s$ equal. The circuit has the same dependence of the poles on the $GB$ of the op amp as the KRC lowpass realization. In high-$Q$ applications, the magnitude of the gain at the peaking frequency is high which may result in distortion of the output signal. Thus the amplitude of the input signal should be restricted or attenuation is needed.

2.4 Commonly Used Bandstop Filters

A second order bandstop (notch) filter has the transfer function:

$$T_{rs}(s) = H \frac{s^2 + \omega_r^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$  \hspace{1cm} (2.14)

where $\omega_r$ is the rejection frequency. Most often, if the second order notch filter is used alone and not cascaded with other second order filters to make a higher order bandstop filter, $\omega_r = \omega_0$.

A notch filter can be constructed using a single op amp with $RC$ networks. Figure 20 shows two bandpass circuits with a twin-T network. It requires

$$\frac{R_1 R_2}{R_1 + R_2} \frac{C_1 + C_2}{C_3}$$

(2.15)

to realize imaginary zeros. The feedback from the output in (a) is added through the capacitor, and the feedback in (b) is added through the resistor. Both configurations of
Figure 19. Bandpass realization with infinite gain amplifier
Figure 20. Two twin-T circuits that can realize notch functions
feedback are employed to increase the pole-Q of the networks. Their transfer functions are

\[ \frac{V_o}{V_i} = \frac{s^2[(R_1 + R_2)R_3C_1C_2] + 1}{s^2[(R_1 + R_2)R_3C_1C_2] + s[R_1C_2 + R_2C_2 + (1 - K)R_3C_3] + 1} \]  

(2.16a)

and

\[ \frac{V_o}{V_i} = \frac{s^2[(R_1 + R_2)R_3C_1C_2] + 1}{s^2[(R_1 + R_2)R_3C_1C_2] + s[(1 - K)(R_1C_2 + R_2C_2) + R_3C_3] + 1} \]  

(2.16b)

respectively. While keeping (2.15) and tuning the circuits, both the zeros and poles of the circuits are affected. These two circuits are not easy to adjust.

Another single op amp realization of a notch circuit is shown in Figure 21, and its transfer function is given by

\[ \frac{V_o}{V_i} = k \frac{s^2 + s\left(\frac{1}{Q} - \frac{1 - k}{k} 2Q\right)\omega_0 + \omega_0^2}{s^2 + s\frac{\omega_b}{Q} + \omega_0^2} \]  

(2.17)

where

\[ \omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad \omega_b = \frac{1}{R_2C_1} + \frac{1}{R_2C_2}. \]

A notch function is realized with \( k = 2Q^2/(2Q^2 + 1) \). The \( \omega_0 \) and \( Q \) sensitivity functions are the same as the bandpass realization in Figure 19. If finite GB of the op amp is considered, the magnitude of this notch realization will attenuate at high frequencies, and in high-\( Q \) applications, there will be a pre-rejection peaking.

A third notch circuit with a single op amp can be constructed using Hilberman’s port interchange theorem [33]. It begins with a magnitude-normalized second order bandpass circuit (Figure 17), and by interchanging the input and ground leads of the network while
Figure 21. A notch/allpass realization with one op amp

\[ k = \frac{R_b}{R_a + R_b} \]

Figure 22. Hilberman notch filter realized by input-output exchange from the bandpass circuit in Figure 17
maintaining the normal connection of the power supply, a notch function can be realized. The resulting circuit is shown in Figure 22. The gain \( K \) of the bandpass network in Figure 17 must be greater than unity to realize a notch.

A notch filter is commonly realized by subtracting a proper bandpass function from unity or a fraction of unity. Figure 23 [1, p. 427, p. 430] shows two such realizations with \( KRC \) and \(-KRC\) bandpass circuits, respectively. In each realization, the denominator polynomial of the resulting notch function is inherited from the bandpass function. Compared with single op amp realizations, the circuits in Figure 23 are flexible for designers, but the high frequency attenuation is strengthened due to the cascading of op amps.

2.5 Commonly Used Allpass Filters

Allpass filters are used for phase corrections and for generating delay. They alter the phase characteristics of systems without affecting the magnitude characteristics. Oftentimes, only a second order allpass filter is needed for phase corrections (phase equalization). However, high order allpass filters can be constructed by cascading second order allpass filters in the same manner as higher order filters of other types are constructed (see Chapter 1). Thus, second order allpass filters are both useful by themselves and also are useful as building blocks for higher order filters.

A second order allpass function is given by

\[
T(s) = H \frac{s^2 - s \frac{\omega_0}{Q} + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}. \tag{2.18}
\]
Figure 23. Two realizations of notch/allpass circuits with two op amps
The zeros and poles of an allpass transfer function are located symmetrically with respect to the imaginary axis of the $s$-plane.

A second order allpass network with complex poles (and zeros) can be realized by using a single op amp. In Figure 21, the circuit performs an allpass function with $k = Q^2/(Q^2 + 1)$. The pole $\omega_0$ and $Q$ properties are the same as the notch functions realized from the same circuit. The finite $GB$ of the op amp will result in a pre-$\omega_0$ peaking and the subsequent dipping in the magnitude response, especially in high-$Q$ applications. This circuit is analyzed in more detail in the next chapter.

A second order allpass function can be obtained by subtracting a proper bandpass function from unity (or a fraction of unity), as for notch functions. The circuits in Figure 23 can be used to realize allpass functions as well, with different assignments of passive component values. The denominator polynomial is inherited from the bandpass component, as it is for the bandstop circuit.

In this chapter, currently used voltage-mode lowpass, highpass, bandpass, notch and allpass filters of second order realized with op amps are recalled. The $\omega_0$-sensitivities and $Q$-sensitivities of the presented circuits are discussed as well as the effects when the op amp model with finite $GB$ is applied.

Besides the realizations presented in this chapter, there are some other realizations with one or more op amps. Some circuits are able to realize different types of transfer functions simultaneously.
In Chapter 2, commonly used voltage-mode second order filters are presented, including the allpass circuits. In this chapter, a feedforward topology is introduced to realize second order voltage-mode allpass circuits, and then it is expanded to the realization of current-mode circuits as well. The properties and advantages of such allpass networks are also illustrated in this chapter.

3.1 Voltage-mode Feedforward Allpass Circuits

A second order allpass function with complex poles can be realized by adding an appropriate allpole function to a first order allpass function [34]. The diagram is shown in Figure 24(a). The allpass function in (2.18) with $H = 1$ can be obtained by adding an allpole function $T_F(s)$ to a first order allpass function

$$T_F(s) = \frac{s - \alpha}{s + \alpha}, \quad \alpha > 0. \quad (3.1)$$

Hence, the allpole function is expressed as

$$T_F(s) = \frac{2\alpha \omega_0^2}{(s^2 + s \frac{\omega_0}{Q} + \omega_0^2)(s + \alpha)} \quad (3.2)$$
Figure 24. Feedforward topology in block diagram form: original form (a) and modified version (b)
if $\alpha = \omega_0/Q$. Note that $T_F(s)$ is third order. However, $T_F(s)$ can be considered as the product of a second order lowpass transfer function and a first order lowpass transfer function as

$$T_F(s) = T_{P2}(s)T_{P1}(s) = \left( \frac{\omega_0^2/k}{s^2 + s\omega_0/Q + \omega_0^2} \right) \left( \frac{2k\alpha}{s + \alpha} \right) \quad (3.3)$$

where $T_{P2}(s)$ is the second order transfer function and $T_{P1}(s)$ is the first order transfer function. The constant $k$ is included for design flexibility.

In voltage-mode realizations, both $T_F(s)$ and $T_{P1}(s)$ can be achieved by a single op amp circuit. Therefore, the feedforward allpass transfer function can be obtained using two op amps and the block diagram in Figure 24(a) can be modified to that in Figure 24(b). Figure 25 gives an illustration of this kind of realization, and the transfer function is

$$\frac{V_o}{V_i} = \frac{s^3 + s^2\left(\frac{\omega_0}{Q} - a\alpha\right) + s(\omega_0^2 - a\alpha\frac{\omega_0}{Q}) + [K(1 + a) - a]\omega_0^2}{(s + \alpha)(s^2 + s\frac{\omega_0}{Q} + \omega_0^2)} \quad (3.4)$$

where

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}, \quad \frac{1}{Q} = \left( \frac{R_2C_2}{R_1C_1} \right)^{1/2} + \left( \frac{R_1C_2}{R_2C_1} \right)^{1/2} + (1 - K) \left( \frac{R_1C_1}{R_2C_2} \right)^{1/2},$$

$$\alpha = \frac{1}{R_AC}, \quad \text{and} \quad a = \frac{R_C}{R_B}.$$

If $K = 1$, $\alpha = \omega_0/Q$ and $R_B = R_C$, then the real axis zero and the real axis pole cancel resulting in the desired second order allpass transfer function with complex poles and complex zeros.

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Figure 25. Feedforward allpass circuit

Figure 26. Current-mode allpass/notch circuit
In Chapter 2, some second order allpass circuits are presented. The allpass networks can also be realized by more than two op amps. For realizations using two or more op amps, the feedforward topology mentioned above is capable of generating many different realizations and provides designers with more choices. Compared with the realizations using a single op amp, it provides better characteristics. The following is an illustration: Figure 21 is a network that can realize bandstop and allpass transfer functions. The transfer function is

$$\frac{V_o}{V_i} = k \frac{s^2 + s \left( \frac{1}{Q} - \frac{1 - k}{k} 2Q \right) \omega_0 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

(3.5)

where

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

and

$$Q = \frac{\sqrt{R_2 C_1 C_2}}{R_1 C_1 + C_2}$$

To obtain an allpass function, one sets $k = Q^2/(Q^2+1)$.

This circuit used only one op amp, but its gain $k$ is determined by $Q$ and is always less than unity. Table 1 provides the sensitivities of the upper half plane pole and upper half plane zero to passive elements. From the table, it is seen that for most passive elements, the zero sensitivity magnitudes are greater than pole sensitivity magnitudes.

For the feedforward circuit in Figure 25, the gain is unity and the zero sensitivity magnitudes are approximately equal to pole sensitivity magnitudes (Table 2)[34]. These are among the advantages of the feedforward circuits.

With the development of current-mode active building blocks, CCs replace op amps in some realizations of voltage-mode transfer functions, including allpass functions, and some voltage-mode allpass networks using CCs are proposed in the literature [35-40].
Table 1. Comparison of Pole and Zero Sensitivities to Passive Elements for the Circuit in Figure 21 in the Allpass Application

<table>
<thead>
<tr>
<th>Element</th>
<th>Pole Sensitivities</th>
<th>Zero Sensitivities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{R_1}$</td>
<td>$j\frac{Q}{\sqrt{4Q^2-1}}$</td>
<td>$\frac{1}{Q} - j\frac{Q}{\sqrt{4Q^2-1}}$</td>
</tr>
<tr>
<td>$\frac{1}{R_2}$</td>
<td>$-\frac{1}{2Q} + j\frac{Q-1}{2\sqrt{4Q^2-1}}$</td>
<td>$-\frac{1}{2Q} + j\frac{Q+1}{2\sqrt{4Q^2-1}}$</td>
</tr>
<tr>
<td>$\frac{1}{C_1}$</td>
<td>$-\frac{1}{2a} + j\frac{Q-1}{2\sqrt{4Q^2-1}}$</td>
<td>$-\frac{1}{2a} + j\frac{Q+1}{2\sqrt{4Q^2-1}}$</td>
</tr>
<tr>
<td>$\frac{1}{C_2}$</td>
<td>$-\frac{1}{2Q} + \frac{1}{2a} + j\frac{Q-1 + \frac{1}{2a}}{2\sqrt{4Q^2-1}}$</td>
<td>$\frac{1}{2Q} + \frac{1}{2a} + j\frac{Q-1 - \frac{1}{2a}}{2\sqrt{4Q^2-1}}$</td>
</tr>
<tr>
<td>$\frac{1}{R_a}$</td>
<td>-------------------------</td>
<td>$-\frac{1}{Q} + j\frac{Q}{\sqrt{4Q^2-1}}$</td>
</tr>
<tr>
<td>$\frac{1}{R_b}$</td>
<td>-------------------------</td>
<td>$\frac{1}{Q} - j\frac{Q}{\sqrt{4Q^2-1}}$</td>
</tr>
</tbody>
</table>

$$a = \left(\frac{R_2C_1}{R_1C_2}\right)^{\frac{1}{2}}$$
Table 2. Comparison of Pole and Zero Sensitivities to Passive Elements for the Circuit in Figure 25

<table>
<thead>
<tr>
<th></th>
<th>pole sensitivities</th>
<th>zero sensitivities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{R_1}$</td>
<td>$-\frac{1}{2d} + j\frac{Q - \frac{1}{2d}}{\sqrt{4Q^2 - 1}}$</td>
<td>$-\frac{q}{2d} + j\frac{Q - \frac{3q}{2d}}{\sqrt{4Q^2 - 1}}$</td>
</tr>
<tr>
<td>$\frac{1}{R_2}$</td>
<td>$-\frac{1}{2Q} + \frac{1}{2d} + j\frac{Q - \frac{1}{2Q} + \frac{1}{2d}}{\sqrt{4Q^2 - 1}}$</td>
<td>$\frac{q}{2d} - \frac{q}{2Q} + j\frac{Qq + \frac{q}{2Q} + \frac{3Qq}{2d}}{\sqrt{4Q^2 - 1}}$</td>
</tr>
<tr>
<td>$\frac{1}{C_1}$</td>
<td>$\frac{(1-K)d}{2} - \frac{1}{2Q} + j\frac{Q - \frac{1}{2Q} + \frac{(1-K)d}{2}}{\sqrt{4Q^2 - 1}}$</td>
<td>$\frac{(1-K)qd}{2} - \frac{q}{2Q} + j\frac{Qq - \frac{3qd(1-K)}{2}}{\sqrt{4Q^2 - 1}}$</td>
</tr>
<tr>
<td>$\frac{1}{C_2}$</td>
<td>$-\frac{(1-K)d}{2} + j\frac{Q - \frac{(1-K)d}{2}}{\sqrt{4Q^2 - 1}}$</td>
<td>$\frac{(K-1)qd}{2} + j\frac{Q - \frac{3qd(1-K)}{2}}{\sqrt{4Q^2 - 1}}$</td>
</tr>
<tr>
<td>$K$</td>
<td>$\frac{1}{2}Kd + j\frac{Kd}{2\sqrt{4Q^2 - 1}}$</td>
<td>$\left(\frac{1}{2}Kqd + \frac{q}{Q}\right)\left(1 + j\frac{3}{\sqrt{4Q^2 - 1}}\right)$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-------------------------------------------</td>
<td>$\frac{q}{Q} + \frac{q}{Q^3} + j\frac{Q - \frac{q}{Q^3}}{\sqrt{4Q^2 - 1}}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-------------------------------------------</td>
<td>$\frac{1}{2Q} - j\frac{1}{2Q\sqrt{4Q^2 - 1}}$</td>
</tr>
</tbody>
</table>

$$d = \left(\frac{R_1C_1}{R_2C_2}\right)^{\frac{1}{2}}, \quad q = \frac{Q^2}{Q^2 + 2}$$
the next section, current-mode allpass circuits and the feedforward topology are presented as well.

3.2 Current-mode Allpass Circuits

In recent years, several novel current-mode allpass networks have been proposed. Higashimura and Fukui [41] presented two circuits to realize first order transfer functions with high output impedance using a single CCII, one capacitor and three resistors. Aronhime et al. [10] realized a second order transfer function with high output impedance and complex poles using a single CCI, two grounded capacitors and four resistors. Fabre [42] used a single CCI, three capacitors and three resistors to realize complex pole second order functions in which the output does not have high impedance. Chang [43], [44] proposed second order filters with high output impedance and real axis poles using a single CCII, and $RC$ components. Then he presented complex-pole filters using a single CCII and $RC$ components in which the output impedance is not high [45]. Soliman [46], [47] obtained a second order network using a single CCII, two capacitors, one of which is grounded, and four resistors (Figure 26). The circuit can realize complex poles and the output impedance is high. It is converted from a voltage-mode realization in which the positive input terminal of the op amp is grounded [48] by using a nullor model for the op amp [49]. The same circuit can also be obtained by transforming a second order current-mode allpass circuit with real axis poles by using port interchange theorems [50].
Some current-mode circuits can be converted from voltage-mode circuits by applying the adjoint theorem, whereas other circuits do not meet the prerequisites of the adjoint theorem.

Celma proposed a theorem which extends the adjoint theorem to an extent. Celma’s theorem transforms a voltage-mode circuit with an op amp working as an infinite-gain voltage amplifier to a current-mode circuit using a CCII.

**Celma’s theorem:**

A general voltage-mode circuit, with a 4-terminal passive network $N$ in which there is no internal connection to the ground, and with an op amp whose non-inverting input terminal is grounded (Figure 27(a)), can be transformed into a current-mode circuit shown in Figure 27(b) which has the same current ratio transfer function as the voltage ratio transfer function of the original voltage-mode circuit [51]. The relation is shown as

$$\frac{I_o}{I_i} = \pm \frac{V_o}{V_i}$$  \hspace{1cm} (3.6)

where the plus sign is for CCII+ and the minus sign is for CCII−.

In Celma’s theorem, the output of the voltage-mode circuit can be taken at a node other than the output of the op amp. If a restriction is added that the output must be taken at the output of the op amp, the 4-terminal passive network is reduced to a 3-terminal network because the terminal of the passive network at which the output was taken can be erased. In this case, the theorem can be extended to applications in which the non-inverting input terminal of the op amp is not grounded. This novel extension is presented and proved below and is labeled as a corollary to Celma’s theorem.
Figure 27. Celma’ theorem: op amp based voltage-mode network (a) and its CCII based current-mode equivalent (b)
Corollary of Celma’s theorem:

Figure 28(a) shows a general voltage-mode circuit with two 3-terminal passive networks, \( N_1 \) and \( N_2 \), and an op amp. It is assumed that there is no internal connection to ground in networks \( N_1 \) and \( N_2 \). The circuit can be transformed into an equivalent current-mode circuit shown in Figure 28(b) and the relation is shown in (3.6).

The proof of the theorem employs the adjoint theorem and Celma’s theorem, and it is shown below:

**Proof:**

Let \( T_{21} \) be the voltage transfer function in the voltage-mode application (Figure 28(a)) with the non-inverting input terminal of the op amp grounded. Thus, the overall transfer function of the circuit shown in Figure 28(a) is

\[
\frac{V_o}{V_i} = T_{21}(1 - T_{s4}) + T_{s4}
\]  

(3.7)

where \( T_{s4} \) is the transfer ratio of \( V_i \) to the non-inverting terminal of the op amp and where we have employed Hilberman’s observation [33] that \( T_{31} + T_{32} = 1 \) for circuit \( N_1 \). Then we consider the current-mode application (Figure 28(b)). From Celma’s theorem, the following relation is obtained directly:

\[
\frac{I_1}{I_2} = -T_{21}.
\]  

(3.8)

Network \( N_2 \) has no influence on the ratio of \( I_1 \) to \( I_2 \) because \( V_x \) and \( V_y \) are equal at the CCII. Again, the adjoint theorem can be used for network \( N_2 \) because \( V_x = V_y \). The current relations of \( N_2 \) can be expressed as

\[
\frac{I_4}{I_i} = -T_{s4}
\]  

(3.9)
Figure 28. Corollary of Celma’s theorem: op amp based voltage-mode network (a) and its CCII based current-mode equivalent (b)
and
\[
\frac{I_6}{I_I} = \frac{-I_2}{I_I} = -1 + T_{54}. \tag{3.10}
\]

Combining (3.8), (3.9) and (3.10), we get
\[
\frac{I_X}{I_I} = \frac{-I_1 - I_4}{I_I} = T_{54} + T_{21}(1 - T_{54})
\]
and then
\[
\frac{I_O}{I_I} = \pm \frac{I_X}{I_I} = \pm \frac{V_O}{V_I} \tag{3.11}
\]

where the plus sign is for CCII+ and minus sign is for CCII-. This proves the corollary of Celma’s theorem.

The corollary of Celma’s theorem also can be applied to some circuit realizations in which \(N_2\) is an active network. It is seen that if \(I_I = -(I_I + I_6)\) in Figure 28(b) can be satisfied in the circuit construction, the circuit in Figure 28(a) can be converted to its current-mode counterpart in Figure 28(b) with \(N_2\) replaced by its adjoint network by using the relations shown in Figure 9.

By employing the corollary of Celma’s theorem, the allpass circuit in Figure 21 can be converted to the circuit in Figure 26 directly. They share the same input-output transfer function except that the transfer function for the circuit in Figure 21 is a voltage ratio, and the transfer function for the circuit in Figure 26 is a current ratio. The corollary of Celma’s theorem will also be employed in Chapter 5 for the realizations of current-mode feedforward circuits.

The realizations of current-mode second order allpass functions with complex poles can also be obtained using the feedforward method in section 3.1. Figure 29 shows a
Figure 29. Current-mode feedforward allpass circuit
circuit for realizing current-mode allpass transfer functions with complex poles [52]. Three CCIIIs are employed. Block 1 is a second order lowpass section which is cascaded with block 2 to convert current to voltage to apply to CCII 3. Block 3 is a first order current-mode allpass network. The overall transfer function of the circuit is

$$\frac{I_O}{I_1} = \frac{\left( s \frac{R_1 R_2 C_1}{R_1 + R_2} + 1 \right) \left( s^2 - s \frac{R_1 + R_2}{R_1 R_2 C_1} + \frac{1}{R_1 R_2 C_1 C_2} \right)}{(sRC + 1) \left( s^2 + s \frac{R_1 + R_2}{R_1 R_2 C_1} + \frac{1}{R_1 R_2 C_1 C_2} \right)}$$

(3.12)

if \(RC=R_1R_2C_1/(R_1+R_2)\), and the real axis pole cancels with the real axis zero, resulting in a second order allpass transfer function as desired. The complex pole and zero sensitivities of the circuit are the same as those of the allpass realization with the circuit in Figure 25.

The current-mode circuits inherit the properties of sensitivity from their voltage-mode counterparts. The advantage of the feedforward realization method presented is the same for voltage-mode and current-mode circuits from a sensitivity point of view. In the following chapters, the feedforward method will be expanded to the realizations of new types of filter circuits other than allpass filters. More detailed analysis and synthesis as well as simulations and laboratory results will be presented.
CHAPTER IV
VOLTAGE-MODE FEEDFORWARD FILTERS

Feedforward configurations to realize voltage-mode and current-mode second order allpass networks with complex poles are presented in Chapter 3. The observations in that chapter lead to the following question: Are there other useful filter sections that can be realized in a similar manner? We seek to devise new families of second order filter building blocks (other than allpass filters) by adding appropriate second order transfer functions to first order functions and then investigate the properties of the resulting circuits. In this chapter, the feedforward method for realizations of voltage-mode second order complex-pole filters of other types is presented. Current-mode realizations with CCIIs are presented and analyzed in the next chapter. To our knowledge, all of the feedforward realizations presented both in this chapter and the next are new and original. The approach of applying feedforward to convert a given first order network to a useful second order network with complex poles has not been utilized before, except for the case of allpass filters presented in [34] and [52].

4.1 Synthesis of Feedforward Filters

In Chapter 3, the feedforward diagram for allpass realization is shown in Figure 24. The scheme can be extended to use for all the voltage-mode realizations as shown in Figure 30. The block labeled \( T_f \) is a first order circuit with two inputs. This block realizes
Figure 30. Diagram of feedforward topology
two first order transfer functions $T_{IA}$ and $T_{IB}$. Block $T_2$ is a single input second order transfer function. The block shown in dotted outline labeled $T_p$ is a first order circuit which is sometimes needed to achieve proper pole-zero cancellation.

The general transfer function $T_{IA}$ from port 1 (terminal 1 to ground in Figure 30) to the output port of $T_I$ is

$$T_{IA} = \frac{\gamma s + \beta}{s + \alpha} \quad (4.1)$$

and $T_{IB}$ from port 2 (terminal 2 to ground in Figure 30) to the output port of $T_I$ is

$$T_{IB} = \frac{\delta s + \epsilon}{s + \alpha}, \quad (4.2)$$

In (4.1) and (4.2), all the coefficients are real and $\alpha$ is positive. The general expression for the transfer function product $T_2T_pT_{IB}$ from port 3 (terminal 3 to ground in Figure 30) to the output port of $T_I$ is

$$T_2T_pT_{IB} = \frac{(s + \alpha)D_m - (\gamma s + \beta)D}{(s + \alpha)D} \quad (4.3)$$

where

$$D = s^2 + s \frac{\omega_0}{Q} + \omega_0^2, \quad D_m = a_2s^2 + a_1 \frac{\omega_0}{Q}s + a_0 \omega_0^2,$$

and the constants $a_i$, $i = 0, 1, 2$ are real. The constants are selected to realize the desired filter type and specifications and, additionally, to insure that $T_2$ is indeed second order so that a realization of the filter can be accomplished with a reasonable parts count. The general expression for the resulting overall transfer function for the block diagram in Figure 30 is
Advantages of this approach include the generation of families of new filter circuits and the fact that the sensitivities of the poles depend on the circuit used for $T_2$.

Table 3 gives a list of some possible realizations for lowpass, highpass, bandpass and notch second order filters using the feedforward scheme. The first column from the left lists the first order functions to begin with, and the first row from the top are the resulting second order functions. Each third order function shown in the table is employed to add to the first order function at the left-most position in its row to realize the second order function at the top in its column. The detailed realizations of the transfer functions in Table 3 are presented in the following sections, and the properties are discussed.

\[
\frac{V_o}{V_i} = \frac{a_2s^2 + a_1\frac{\omega_0}{Q}s + a_0\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}. \tag{4.4}
\]

4.2 Feedforward Lowpass Filters

Usually second order lowpass networks with complex poles are used as basic building blocks by which, many other types of complex-pole networks can be constructed with flexibility. However, a second order lowpass circuit can be constructed from other types of filters.

The realization of a second order allpass functions with complex poles by adding a proper allpole function to a first order allpass function is presented in Chapter 3. The method can also be employed in the construction of second order lowpass functions, as illustrated in Table 3. A circuit realization is shown in Figure 31. It is constructed by adding a highpass section to a first order lowpass part. It is seen that if the non-inverting input terminal of the op amp is grounded, the circuit realizes a first order lowpass
Table 3. Synthesis of Second Order Feedforward Filters

<table>
<thead>
<tr>
<th></th>
<th>( \frac{\omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} )</th>
<th>( \frac{s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} )</th>
<th>( \frac{s \omega_0}{Q} )</th>
<th>( \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} )</th>
<th>( \frac{s^2 - s \frac{\omega_0}{Q} + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\alpha}{s + \alpha} )</td>
<td>(- \alpha \left[ s - \left( \frac{\omega_0}{\alpha} - \frac{\omega_0}{Q} \right) \right] ) ( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{s \omega_0}{Q} )</td>
<td>( \frac{\left( \omega_0 - \alpha \right) s^2 - \alpha \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\omega_0 - \alpha}{Q} \omega_0 s + s^3 )</td>
<td>( \frac{\omega_0 - \alpha}{Q} \omega_0 s + s^3 )</td>
</tr>
<tr>
<td>( \frac{s}{s + \alpha} )</td>
<td>(- \left[ \frac{\omega_0}{Q} - \alpha \right] s + \omega_0^2 ) ( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\alpha}{Q} - \frac{\omega_0}{Q} \omega_0 s - s^3 ) ( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\alpha - \omega_0}{Q} s^2 + \alpha \omega_0^2 ) ( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\alpha - \omega_0}{Q} s^2 + \alpha \omega_0^2 ) ( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\alpha - \omega_0}{Q} s^2 - 2 \alpha \omega_0^2 ) ( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
</tr>
<tr>
<td>( \frac{s - \alpha}{s + \alpha} )</td>
<td>( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{1}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\left( s - \alpha \right) s^2 + \left( \omega_0 - 2 \alpha \right) \omega_0 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{\alpha - 2 \omega_0}{Q} s^2 - (s - \omega_0) \omega_0 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
<td>( \frac{2 \left( \alpha - \omega_0 \right) s^2 - 2 \alpha \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) )</td>
</tr>
</tbody>
</table>

\[ A = \frac{\left( s - \omega_0 \right) s^2 + \left( \omega_0 - 2 \alpha \right) \omega_0 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) \]  

\[ B = \frac{\alpha - 2 \omega_0}{Q} s^2 - (s - \omega_0 Q) \omega_0 s}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) \]  

\[ C = \frac{2 \left( \alpha - \omega_0 \right) s^2 - 2 \alpha \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} (s + \alpha) \]
Figure 31. Feedforward lowpass filter with Sallen-Key highpass circuit as the second order building block

Figure 32. Feedforward highpass filter with Sallen-Key lowpass circuit as the second order building block
function. When the input signal is fed to the non-inverting input terminal through a second order highpass network and some passive network shown in Figure 31, and if \( C_3 = C_4 = C \), \( R_3R_4/(R_3 + R_4) = R_5R_6/(R_5 + R_6) \) and \( R_3 = KR_3 \) are satisfied, the overall ideal transfer function is given by

\[
\frac{V_o}{V_i} = \frac{-\frac{\omega_0}{Q} \left( s + \frac{\omega_0 Q}{Q + \omega_0^2} \right) \left( 1 + \frac{1}{R_6 C} \right)}{\left( s^2 + s \frac{\omega_0}{Q + \omega_0^2} + \frac{1}{R_6 C} \right)}. \tag{4.5}
\]

If \( \omega_0 Q = 1/R_6 C \), the real axis zero can cancel the real axis pole, resulting in a second order lowpass transfer function whose denominator polynomial is the same as that of the highpass function used to add to the first order lowpass network.

If the non-ideal model of the op amps is used, for the purpose of simplicity, (set \( K = 1, C_3 = C_4 = C, R_3 = R_5, \) and \( R_4 = R_6 \)), the transfer function becomes

\[
\frac{V_o}{V_i} = \left( -\frac{R_4}{R_3} \alpha \frac{\omega_0}{Q} \left[ s + \frac{\omega_0 Q + s}{GB} \frac{Q}{\omega_0} \left( s^2 + bs + \omega_0^2 \right) \right] \right) \left( s^2 + s \frac{\omega_0}{Q + \omega_0^2} + \frac{s}{GB} \left( s^2 + bs + \omega_0^2 \right) \right) \left[ s + \alpha + \frac{s}{GB} \left( s + \alpha + \frac{R_4}{R_3} \alpha \right) \right] \tag{4.6}
\]

where \( \alpha = 1/R_4 C \), and \( b = 1/(R_1 C_1) + 1/(R_2 C_1) + 1/(R_2 C_2) \). The terms in the square brackets can not be exactly cancelled with finite \( GB \) if \( \alpha = \omega_0 Q \) and it results in error.

### 4.3 Feedforward Highpass Filters

Second order complex-pole highpass functions can be realized by the feedforward method as illustrated in Table 3, and a realization is shown in Figure 32. The overall transfer function is
if \(1/(R_3C_3) = \omega_0/Q\), which causes the cancellation of the real axis zero with the real axis pole. The denominator polynomial of the resulting transfer function is the same as that of the second order lowpass section used in the circuit. Hence, the \(\omega_0\) and \(Q\) of the resulting function are the same as the lowpass section. This gives flexibility in the design. That is, for example, if a second order lowpass network is available and a higpass function with the same \(\omega_0\) and \(Q\) are to be constructed, a designer can add the lowpass network to a first order circuit with the proper \(\alpha\) to realize the highpass function, avoiding the construction of another second order network.

If a non-ideal model of the op amp is employed in the analysis and the circuit in Figure 2(a) is used to realize the lowpass function, the overall transfer function of the circuit in Figure 32 is

\[
\frac{V_O}{V_i} = \left( -\frac{R_4}{R_3} \right) \frac{s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \frac{s + \frac{\omega_0}{Q} + \frac{1}{GB}(s^2 + bs + \omega_0^2)}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2 + \frac{s}{GB}(s^2 + bs + \omega_0^2)} \left[ s + \alpha + \frac{s}{GB/(1 + R_4/R_3)} \right]
\]

where \(\alpha = 1/R_3C_3\), \(b\) is given in (2.7) for the lowpass circuit. The real axis zero and real axis pole that can be cancelled if the op amps are ideal may not be cancelled here, but the effect is not significant. Actually, (4.8) can be rewritten approximately as
if $GB >> \omega_0$. The transfer function in (4.9) is approximately equivalent to the ideal transfer function ((4.7) with $K = 1$) times a term $(GB/(1+R_4/R_3))/(s+GB/(1+R_4/R_3))$. The response falls off more at high frequencies than it does in some single op amp realizations.

4.4 Feedforward Bandpass Filters

The feedforward method can also be used to realize bandpass circuits as well as lowpass and highpass networks (Table 3), and with more flexible choices. A second order lowpass or a second order highpass network can be employed as the building block of a second order function. The circuit shown in Figure 33 is a realization of a second order bandpass transfer function which uses a second order lowpass building block together with a first order highpass network coupled by a passive $RC$ network. The ideal transfer function is

$$
\frac{V_o}{V_i} \approx \left( \frac{-R_4}{R_3} \right) s^2 \left[ \frac{1}{GB} \left( s + GB \left( s + \frac{\omega_0}{Q} \right) \right) \right]
$$

(4.9)

$$
\left[ \frac{1}{GB} \left( s + GB \left( s^2 + s \frac{\omega_0}{Q} + \omega_0^2 \right) \right) \right] \frac{1 + R_4 / R_3}{GB} \left( s + \alpha \left( s + \frac{GB}{1 + R_4 / R_3} \right) \right)
$$

where $\alpha = 1/R_4C_3$. The real axis zero can be cancelled with the real axis pole if $\alpha = \omega_0/Q$ and gives a second order bandpass transfer function.

If the non-ideal model of op amp is included, the transfer function obtained is
Figure 33. Feedforward bandpass filter with Sallen-Key lowpass circuit as the second order building block

Figure 34. Feedforward bandpass filter with Sallen-Key highpass circuit as the second order building block
\[
V_O = \frac{-\frac{R_4}{R_3} \alpha \left[ s \left( s + \frac{\omega_0}{Q} \right) + \frac{1}{GB} \left( s^2 + bs + \omega_0^2 \right) \right]}{s + \alpha + \frac{s}{GB} \left( s + \alpha + \frac{\alpha R_4}{R_3} \right) \left[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 + \frac{s}{GB} \left( s^2 + bs + \omega_0^2 \right) \right]}. \tag{4.11}
\]

When \( GB \gg \omega_0 \), the transfer function is approximately equivalent to

\[
\frac{V_O}{V_I} \approx \frac{-\frac{R_4}{R_3} \alpha \left[ \frac{1}{GB} s \left( s + \frac{\omega_0}{Q} \right) \left( s + GB \right) \right]}{s + \frac{1}{GB} \left( s + GB \right) \left[ \frac{1}{GB} \left( s^2 + s \frac{\omega_0}{Q} + \omega_0^2 \right) \left( s + GB \right) \right]}, \tag{4.12}
\]

which is an ideal bandpass transfer function times a lowpass term \( GB/(s + GB) \).

Another feedforward construction of a second order bandpass filter by employing a second order highpass circuit as the second order building block is also available (also in Table 3). A configuration is shown in Figure 34, and its transfer function with ideal op amp model is

\[
\frac{V_O}{V_I} = \frac{-\frac{R_4}{R_3} \left( \frac{s}{Q} \right) \left( s + \frac{\omega_0}{Q} \right)}{s + \frac{1}{R_3 C_3} \left[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 \right]}. \tag{4.13}
\]

If \( \frac{1}{R_3 C_3} = \omega_0 Q \) is satisfied, the real axis zero can cancel the real axis pole. But when the non-ideal op amp model is considered, the transfer function expands to

\[
\frac{V_O}{V_I} = \frac{-\frac{R_4}{R_3} \left( \frac{s}{Q} \right) \left( s + \frac{\omega_0}{Q} \right) + \frac{s}{GB} \left( s^2 + bs + \omega_0^2 \right)}{s + \alpha + \frac{s}{GB \left( 1 + \frac{R_4}{R_3} \right)} \left[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 + \frac{s}{GB} \left( s^2 + bs + \omega_0^2 \right) \right]} \tag{4.14}
\].
It is seen that the factor of the numerator polynomial in square brackets is of third order and the terms in the square brackets not associated with $GB$ are of first order. When the effect of $GB$ is taken into account, the factor in the square brackets can be dissolved into a product of a first order factor and a second order factor with complex roots as

$$N_3 = \frac{1}{GB} \left( s^2 + b_1 s + \frac{\omega_0^2 GB}{\alpha_1} \right) \left( s + \alpha_1 \right)$$  \hspace{1cm} (4.15)$$

where $\alpha_1$ is close to $\alpha$ and $b_1$ is about the same order as $b$. Then (4.14) can be rewritten as

$$\frac{V_Q}{V_1} \approx \frac{- \frac{R_4}{R_3}}{1 + \frac{R_4}{R_3} \left( s + \alpha \right) \left( s + \frac{GB}{1 + \frac{R_4}{R_3}} \right) \left( 1 + \frac{\omega_0}{Q} \right)} \left( s + \frac{\omega_0^2}{Q} \right) \left( s + GB \right)$$  \hspace{1cm} (4.16)$$

The second order term in (4.15) may not be cancelled by the first order terms in the denominator in (4.16) and this will result in significant departure of the magnitude plot of the transfer function from its ideal appearance, especially near the frequency $\omega_0$. The error produced by this construction may not be ignored in practice. Therefore, the circuit in Figure 34 is inferior to that in Figure 33. Thus, op amps with high $GB$ are needed in the realization of these kinds of circuits, and it is concluded that the circuit in Figure 33 is the best of the bandpass filter derived here.

4.5 Feedforward Notch Filters

The feedforward method described above is suitable for notch filters (Table 3). Figure 35 is a notch filter obtained using the feedforward method. The notch network is realized by adding a second order allpole function to a first order highpass function. If $\alpha = 1/(RC) = \omega_0/Q$ is satisfied in the circuit, the resulting transfer function is
Figure 35. Feedforward notch filter with Sallen-Key lowpass circuit as a building block

Figure 36. Feedforward notch filter with Sallen-Key highpass circuit as a building block
\[
\frac{V_o}{V_i} = \frac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0^2}{Q} + \omega_0^2}
\]  (4.17)

where \(\omega_0\) and \(Q\) are the same as for the lowpass component. This circuit uses only one op amp and passive elements to realize a complex pole notch filter whose gain at very low frequency and at very high frequency is unity. This realization can be compared to the realization given in [1, p. 436] which requires four op amps and associated passive elements.

If a non-ideal op amp model is included, the transfer function becomes

\[
\frac{V_o}{V_i} = \frac{s\left[s^2 + \frac{\omega_0^2}{Q} + \omega_0^2 + \frac{s}{GB}\left(s^2 + bs + \omega_0^2\right)\right] + \alpha \omega_0^2}{s^2 + \frac{\omega_0^2}{Q} + \omega_0^2 + \frac{s}{GB}\left(s^2 + bs + \omega_0^2\right)}(s + \alpha)
\]  (4.18)

where \(b\) is defined in (2.7). The numerator of (4.18) is a fourth order polynomial which can be factored into

\[
N_4 = \frac{1}{GB}(s + \alpha_1)(s + GB_1)(s^2 + b_1s + \omega_1^2)
\]  (4.19)

in which \(\alpha_1\) is close to the nominal \(\alpha\), \(GB_1\) is close to \(GB\), \(\omega_1\) is close to \(\omega_0\), and \(b_1\) is very much smaller than the other coefficients in magnitude. The transfer function in (4.18) will have no zeros located on the imaginary axis of the \(s\)-plane, and therefore the notch depth of the magnitude plot of the transfer function will be decreased and the magnitude will never reach zero. If a zero magnitude of the transfer function is needed, it can be realized by tuning \(\alpha\) slightly away from its nominal value to make the \(s\) term of the second order factor in (4.19) disappear. The adjusted numerator is written as

\[
N_4 = \frac{1}{GB}(s + \alpha_2)(s + GB_2)(s^2 + \omega_2^2)
\]  (4.20)
where $a_z$ is close to the nominal $a$, $GB_z$ is close to $GB$ and $\omega_z$ is close and a little smaller than $\omega_0$. The real axis zeros may not cancel the real poles exactly, but their effect on the magnitude characteristic is small. The complex conjugate zeros are moved back on the imaginary axis of the $s$-plane. If a lowpass circuit with sensitivity enhancement is used, the effect is to replace $GB$ by $GB/2$ in the transfer function in (4.18), approximately. The attenuation at high frequency is thereby increased.

Another feedforward realization of notch filters, also in Table 3, is shown in Figure 36, in which a second order highpass network is used as a building block. If the ideal op amp model is employed and $\alpha = 1/(RC) = \omega_0 Q$ is satisfied in the circuit, the resulting transfer function is shown in (4.17). When the non-ideal op amp model is taken into consideration, the transfer function is

$$\frac{V_o}{V_i} = \frac{s^3 + \alpha s^2 + \alpha \frac{\omega_0}{Q} s + \alpha \omega_0^2 + \frac{s}{GB/\alpha} (s^2 + bs + \omega_0^2)}{s^2 + \frac{s}{Q} + \omega_0^2 + \frac{s}{GB} (s^2 + bs + \omega_0^2)} (s + \alpha).$$

Because $GB/\alpha$ is quite small compared to $GB$, the $GB$ related terms in the numerator polynomial become significant and even if a deep notch can be realized by adjusting $\alpha$, the notch may be far away from the desired position. This circuit is inferior to that in Figure 35.

In this chapter, new complex-pole voltage-mode lowpass, highpass, bandpass and notch filters of second order are discussed. The feedforward method for the realizations of allpass transfer functions is used to construct these filters. The effects of the non-ideal op amp model for circuits are presented and discussed. Actually, there are families of
realizations that can be obtained with the feedforward method. Here, we presented a few realizations for the purposes of demonstration.
CHAPTER V
CURRENT-MODE FILTERS AND FEEDFORWARD APPLICATIONS

In Chapter 4, new feedforward configurations to realize voltage-mode second order complex-pole circuits are presented. These circuits are lowpass, highpass, bandpass and notch filters. The feedforward concept can be used in current-mode networks as well. In this chapter, new feedforward configurations are employed for the realizations of current-mode second order complex-pole filters. Some of the feedforward current-mode circuits can be converted directly from their voltage-mode counterparts, and others are constructed using the feedforward concept.

The current-mode Sallen-Key lowpass and highpass circuits are mostly used in the feedforward applications, and their properties with the non-ideal CCII model are discussed. Then, the feedforward current-mode lowpass, highpass, bandpass and notch filters are presented. The advantages and drawbacks of different realizations are discussed. Simulation and laboratory results are provided for example networks which best illustrate the characteristics of the various new families of filter circuits presented.

5.1 Current-mode Second Order Filters

Besides current-mode allpass filters discussed in Chapter 3, there are other types of current-mode filters that can be converted from their commonly used voltage-mode
counterparts using the adjoint theorem, Celma’s theorem or the corollary of Celma’s
theorem.

Figure 37 is a current-mode realization of a second order lowpass complex-pole
circuit which is converted from the circuit with unity-gain amplifier in Figure 2(a) by
employing the adjoint theorem. This configuration is employed as the lowpass building
block for the realizations of the frequency discriminators (Figure 11) and for the
feedforward allpass networks (Figure 29). The lowpass current transfer function is the
same as the voltage transfer function of the circuit in Figure 2(a) and, for convenience, is
rewritten here as

\[
\frac{I_O}{I_1} = -\frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \tag{5.1}
\]

where

\[
\frac{\omega_0}{Q} = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} \quad \text{and} \quad \omega_0 = \left(\frac{1}{R_1R_2C_1C_2}\right)^{1/2}.
\]

In this circuit, the input impedance is low, and the circuit is suitable for cascading; the
capacitor across port Z of the CCII− can compensate the parasitic capacitance \(C_Z\). If the
parasitic elements are taken into account, the transfer function becomes

\[
\frac{I_O}{I_1} = -\frac{\frac{R_Y}{R_1} s^2 + s\left(\frac{1}{D_1} + \frac{R_Y}{R_Z}\right) - \frac{R_Y}{R_1C_2} + \beta \omega_0^2}{D_1\left[s^2 + s\left(\frac{1}{D_1} + \frac{1}{R_2C_2} + \frac{R_Y + (1 - \beta)R_1}{D_1R_1R_2C_2} + \frac{\omega_0^2}{D_1\left(1 + \frac{R_1 + R_2}{R_Z}\right)}\right]\right] \tag{5.2}
\]

where \(R_Y, R_Z\) and \(\beta\) are defined in Chapter 1, \(C_2\) includes the parallel parasitic capacitance
\(C_Z\) at port Z of the CCII−, \(\omega_0\) and \(Q\) are the same as defined in (5.1), and
Figure 37. Current-mode Sallen-Key lowpass circuit

Figure 38. Current-mode Sallen-Key highpass circuit
It is seen that nonzero \( R_X \) results in the \( s \) and \( s^2 \) terms in the numerator. These terms are error terms which become significant at high frequencies unless efforts are made to keep \( R_X \) low. Note that the constant term in the square brackets in the denominator can be fixed at its nominal value \( \omega_0^2 \) if \( R_X R_Z = R_1 R_2 \) is satisfied, and actual \( Q \) can be set very close to its nominal value in low-\( Q \) applications because nonzero \( R_X \) tends to increase \( Q \) and finite \( R_Z \) would decrease \( Q \). For moderate- and high-\( Q \) applications, actual \( Q \) tends to be somewhat smaller than the nominal value because both \( R_X \) and \( R_Z \) would decrease \( Q \). The \( \beta \) would increase (decrease) pole-\( Q \) if it is greater (smaller) than unity. The \( dc \) gain of this lowpass network is \( \beta R_Z/(R_1+R_2+R_Z) \).

The lowpass circuit in Figure 3(a) with unity-gain amplifier can also be converted to its current-mode counterpart, but the pole-\( Q \) can be no greater than \( 1/\sqrt{2} \) with a single CCII. Also, the lowpass circuit with infinite-gain amplifier (Figure 4) cannot be transformed to a current-mode circuit with a single CCII by using Celma’s theorem, but the current-mode realization can be obtained by using one more CCII.

The Sallen-Key highpass current-mode filter is shown in Figure 38, which is converted from the circuit in Figure 16 with unity-gain amplifier by using the adjoint theorem, and its ideal current transfer function is given by

\[
\frac{I_O}{I_I} = \frac{s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

where

\[
\frac{\omega_0}{Q} = \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \quad \text{and} \quad \omega_0 = \left( \frac{1}{R_1 R_2 C_1 C_2} \right)^{1/2}.
\]
The input impedance of the circuit is low, and so it is cascadable and lends itself to the construction of higher order filters.

If parasitic elements are included, the transfer function of the circuit in Figure 38 is

\[
\frac{I_Q}{I_t} = -\frac{s^2 \left( \frac{R_X}{R_1} + \frac{R_X C_Z}{R_1 R_2} + \beta \right) + s \frac{R_X}{R_1 R_2 C_2}}{s^2 \left( 1 + \frac{R_X}{R_1} \right) \left( 1 + \frac{C_Z}{C_1} + \frac{C_Z}{C_2} \right) + s \left[ \frac{\omega_0}{Q} \left( 1 + \frac{R_X}{R_1} \right) + \left( 1 - \beta + \frac{C_Z}{C_2} \right) \frac{1}{R_1 C_1} \right] + \omega_0^2} \tag{5.5}
\]

where \( R_X, C_Z \) and \( \beta \) are defined in Chapter 1, \( R_2 \) includes the parasitic \( R_Z \) at port Z of the CCII−, and \( \omega_0 \) and \( Q \) are the same as in (5.4). Nonzero \( R_X \) introduces an unwanted \( s \) term in the numerator, so \( R_X \) should be made as small as possible. The realized undamped critical frequency is smaller than \( \omega_0 \), and the realized \( Q \) may be greater or smaller than its nominal value depending on the parasitic elements and the \( Q \) value desired. Compared to the voltage-mode applications with op amps, this highpass transfer function does not have an \( s^3 \) term in its denominator. Thus at very high frequencies, the gain of the circuit rolls off only because of unmodeled parasitic elements in the circuit hardware configuration as opposed to the voltage-mode circuit for which the gain would roll off because of the \( s^3 \) term in the transfer function denominator as well as because of unmodeled parasitic elements.

The −KRC voltage-mode second order highpass circuit [1, p. 404] with unity-gain amplifier can also be transformed to its current-mode counterpart, but the \( Q \) of the complex poles of the resulting circuit can be no greater than \( 1/\sqrt{2} \). The voltage-mode highpass circuit with infinite-gain amplifier [1, p. 406] cannot be transformed to a current-mode circuit with a single CCII by using Celma’s theorem.
For other types of filters, some current-mode circuits can also be obtained by conversion directly from voltage-mode circuits, and they are presented in the relevant sections to follow together with the feedforward realizations.

5.2 Feedforward Lowpass Filters

Besides the circuits presented in section 5.1, there are some other realizations of complex-pole lowpass current-mode circuits presented in the literature. Liu [53] presented a realization with one CCII and passive elements having input impedance that is not low and having high output impedance. This circuit can also realize complex-pole current-mode highpass and bandpass functions, as well as some notch and allpass functions. Chang [54] constructed a circuit with two CClIs that realizes lowpass and bandpass functions simultaneously and having input impedance that is not low and output impedance that is not high. Fabre [55] implemented lowpass as well as bandpass and highpass functions simultaneously with two CClIs. Fabre’s circuit has input impedance that is not low and output impedance that is not high. Both Chang’s and Fabre’s circuit require current buffers between stages to make higher order filters.

The feedforward concept can be used in the construction of current-mode lowpass filters. A realization of a feedforward circuit is shown in Figure 39 which is converted from the voltage-mode circuit in Figure 31 using the corollary of Celma’s theorem. In this realization, there is no capacitor across port Z of the CCII in the highpass section and, thus, the parasitic $C_Z$ cannot be compensated. This fact limits high frequency applications of this filter realization. Due to this uncompensated $C_Z$ of the CCs, the properties of this feedforward lowpass realization from highpass architecture is inferior to other lowpass
Figure 39. Feedforward lowpass circuit
circuits presented above. This circuit is employed as an illustration of the feedforward realization of current-mode circuits from their voltage-mode feedforward realizations by the corollary of Celma’s theorem.

5.3 Feedforward Highpass Filters

While the circuit in Figure 38 is the most commonly used second order complex-pole highpass network, Liu [53], Chang [54] and Fabre [55] also presented other circuits for the realization of highpass circuits. Chang’s and Fabre’s circuit use two CCIIIs and realize other functions simultaneously. The input impedance is not low and the output impedance is not high.

The feedforward method provides additional circuit realizations for the highpass functions. The circuit in Figure 40 is an example of the realization of a second order highpass circuit. It is converted from the voltage-mode circuit in Figure 32 using the corollary of Celma’s theorem.

Another feedforward realization is shown in Figure 41 which is constructed by using the feedforward concept directly. This circuit realizes lowpass, highpass and bandpass functions simultaneously. In the highpass application, the upper input current source and the dashed box construct a first order highpass network if the $Y$ terminal of the second CCII is grounded, while the second order lowpass section realizes the third order function with the dashed box if the upper current source is removed. The second CCII is also used to add the first order highpass function and the third order function together to realize a highpass function. In this circuit, CCII can be used for the second CCII as well because
Figure 40. Feedforward highpass filter obtained with the corollary of Celma’s theorem
Figure 41. Feedforward highpass (bandpass) circuit
no feedback network is connected from terminal \( Z \) of the second CCII. The overall ideal highpass transfer function of the circuit in Figure 41 is

\[
\frac{I_{HP}}{I_1} = \frac{s^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

(5.6)

if \( 1/(RC) = \omega_0/Q \) which causes the cancellation of the negative real axis zero with the real axis pole. The \( \omega_0 \) and \( Q \) are the same as for the lowpass building block. If CCII parasitic elements are taken into account, the transfer function can be expressed as

\[
\frac{I_{HP}}{I_1} = \frac{\beta_2 R_x (C - C_{Z2}) (R_x + R_z)}{R_z C (R_{X2} + R_x)} \frac{N}{s}
\]

(5.7)

where

\[
N = s^2 + s \left( \frac{R_z}{R_x + R_z} \frac{\omega_0}{Q} + \frac{1}{R_z C_2} + \frac{1 - \beta}{(R_x + R_z)C_2} \right) + \omega_0^2 \left( 1 - \beta + \frac{R_1 + R_2}{R_z} \right)
\]

(5.8)

and \( D_1 \) is defined in (5.3). In (5.7), the \( \beta \), \( R_x \) and \( R_z \) are the parasitic elements for the first CCII, and \( \beta_2 \), \( R_{X2} \) and \( C_{Z2} \) are for the second CCII. Also, \( C_2 \) includes its parallel parasitic capacitance \( C_{Z2} \), \( R \) includes the parasitic resistance and \( C \) includes \( C_{Z2} \). \( N \) can be rewritten as \( N = (s + a)(s + b) \) where \( a \) is close to \( \omega_0/Q \) in (5.6), and \( b \) is caused by finite \( R_z \) and non-unity \( \beta \) and is very close to zero. From (5.7), it is seen that: factor \( (s+a) \) of \( N \) cannot be exactly cancelled with \( (s+1/RC) \), but this fact is not significant to the magnitude characteristics of the transfer function; nonzero \( b \) introduces error at lower frequencies; and the parasitic elements of the second CCII only introduce a constant factor to the
transfer function and have no other influence; the error related to \( a, b \), and the \( \omega_0, Q \) are caused by the parasitic elements of the first CCII.

5.4 Feedforward Bandpass Filters

Current-mode second order bandpass circuits can be obtained from voltage-mode networks. The bandpass circuits in Figures 5, 17 and 18 with unity-gain amplifiers can be converted to their current-mode counterparts by using the adjoint theorem. The bandpass realization shown in Figure 19 can be transformed to a current-mode circuit with Celma’s theorem, and the resulting circuit has been presented by Liu [53]. The properties of the above circuits limit the high-Q applications as mentioned in section 2.3. The circuit in Figure 18 with non-unity \( K \) and output taken at node 1 can be converted to its current-mode counterpart with Celma’s theorem, and its gain at peak-frequency is less than unity. This circuit is also proposed by Fabre [32]. Some other bandpass realizations with two CCs were also presented by Chang, Fabre and Alami [54-57].

The feedforward concept can also be used to create new bandpass filters that can realize high \( Q \). The bandpass application shown in Figure 41 is constructed using the feedforward concept, and its ideal transfer function is

\[
\frac{I_{bp}}{I_1} = \frac{s \frac{\omega_0}{Q}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

(5.9)

if \( 1/(RC) = \omega_0/Q \), which causes the cancellation of the negative real axis zero and real axis pole. The \( \omega_0 \) and \( Q \) are the same as for the lowpass stage. If parasitic elements are taken into account, the transfer function is:
where $D_I$ is defined in (5.3), $N$ is defined in (5.8), and the definitions of parasitic elements are the same as for (5.7-5.8). Also, $R$ includes $R_{22}$ and $C$ includes $C_{22}$. $N$ can be rewritten as $N = (s + a)(s + b)$ where $a$ is close to $\omega_0/Q$ in (5.9), and $b$ is caused by finite $R_Z$ and non-unity $\beta$ and is very close to zero. The $(s+a)$ term of $N$ in the numerator can approximately cancel $(s+1/RC)$ in the denominator, and $b$ introduces magnitude error at lower frequencies.

The feedforward bandpass circuits in Figures 42 and 43 are converted from the voltage-mode realization in Figures 33 and 34, respectively, by means of the corollary of Celma’s theorem.

### 5.5 Feedforward Notch Filters

Current-mode notch filters can be realized in many ways: The twin-T circuits in Figure 20 with unity-gain amplifier can be converted to the current-mode circuits shown in Figure 44 by employing the adjoint theorem, the notch realization in Figure 21 can be transformed to a current-mode notch circuit shown in Figure 26 with the corollary of Celma’s theorem, and also, some other realizations of second order complex pole notch circuits are presented in the literature [42], [45]. Most of the allpass topologies can be used to realize notch functions if different component values are employed.

Notch filters, too, can be made with the feedforward concept. The circuit in Figure 45 is a feedforward realization of a current-mode second order notch filter with complex
Figure 42. Feedforward bandpass network converted from the circuit in Figure 33

Figure 43. Feedforward bandpass network converted from the circuit in Figure 34
Figure 44. Current-mode twin-T notch filters: 
(a) feedback through a capacitor; (b) feedback through a resistor
Figure 45. Feedforward notch filter constructed from Sallen-Key lowpass network

Figure 46. Feedforward notch filter built from Sallen-Key highpass network
poles. The circuit is converted directly from the voltage-mode circuit in Figure 35 by using the adjoint theorem. Its ideal current transfer function is

\[
\frac{I_O}{I_i} = \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]  (5.11)

where

\[
\omega_0 = \frac{1}{R_1C_1} + \frac{1}{R_2C_2}
\]

and

\[
\omega_0 = \left(\frac{1}{R_1R_2C_1C_2}\right)^{1/2}
\]

if \( \alpha = 1/(RC) \) is set equal to \( \omega_0/Q \) of the lowpass section. If the non-ideal CCII model is considered, the transfer function of the circuit in Figure 45 is

\[
\frac{I_O}{I_i} = \frac{\frac{D + \frac{R_x}{R}}{s^2 + s \left[ \frac{1}{R_2 + \frac{1}{R_z}} \right] + \frac{\omega_0}{Q} + \omega_0^2 \left(1 + \frac{R_1 + R_2}{R_z}\right)} + N}{D\left(\frac{1 + \frac{R_x}{R}}{R} + \frac{R_x}{R} s^2 + s \left[ \frac{1}{R_2 + \frac{1}{R_z}} \right] + \frac{\omega_0}{Q} + \omega_0^2 \left(1 + \frac{R_1 + R_2}{R_z}\right)\right)}
\]  (5.12)

where

\[
D = \left[1 + \frac{R_x}{R_1} + \frac{R_x}{R_2}\right] s^2 + s \left[ \frac{1}{D_1} \frac{\omega_0}{Q} + \frac{1}{R_2C_2} + \frac{R_x + (1 - \beta)R_1}{D_1R_1R_2C_2} + \frac{\omega_0^2}{D_1} \left(1 + \frac{R_1 + R_2}{R_z}\right)\right],
\]

\[
N = \frac{1}{RC} \left[ \frac{R_x}{R_1} s^2 + s \left( \frac{1}{R_2} + \frac{1}{R_z} \right) \frac{R_x}{R_1C_2} + \beta \omega_0^2 \right],
\]

and the \( \omega_0 \) and \( Q \) are the same as in (5.11). With \( \alpha = 1/(RC) = \omega_0/Q \), the transfer function in (5.12) can be written as

\[
\frac{I_O}{I_i} = -\frac{(s^2 + \varepsilon s + \omega_0^2)(s + \alpha)}{(s^2 + s \frac{\omega_0}{Q_p} + \omega_0^2)(s + \alpha_p)}
\]  (5.13)

where parameters with subscripts \( Z \) or \( P \) are the actual values but not the nominal values, but they are close to their nominal values, and the quantity \( \varepsilon \) is close to zero. The nonzero
\( \varepsilon \) prevents the notch from reaching zero. By tuning \( RC \), the complex zeros can be moved back to the imaginary axis and \( \varepsilon \) reaches zero. The first order terms in the numerator and the denominator in the ideal expression cannot cancel each other exactly here, but the effect is small. The notch frequency, \( \omega_{0Z} \), and \( Q_p \) are close to their nominal values. At very low frequencies, the gain is \( \beta R_Z/(R_1+R_2+R_Z) \) which is close to unity for large \( R_Z \) and \( \beta \) close to 1, and at very high frequencies, the gain is unity.

Another feedforward realization is shown in Figure 46 in which a second order highpass network is used as a building block. This circuit is converted from the voltage-mode circuit in Figure 36 by the adjoint theorem, and its ideal current transfer function is the same as the voltage transfer function in (4.17) if \( a = 1/RC = \omega_0 Q \). If the parasitic elements are considered, the complex zeros may move away from the imaginary axis, and by tuning \( RC \), the zeros can be moved back to the imaginary axis and the actual zeros are close to the nominal values. The non-ideal transfer function can be written as

\[
\frac{I_o}{I_i} = -\frac{K(s^2 + \omega_{0Z}^2) \left( s + \frac{\alpha_p \omega_{0P}^2}{K \omega_{0Z}^2} \right)}{\left( s^2 + s \frac{\omega_{0P}}{Q_p} + \omega_{0P}^2 \right) \left( s + \alpha_p \right)}
\]

(5.14)

where the parameters with subscripts \( Z \) or \( P \) are the actual values which are close to the nominal values, and \( K \) is expressed as

\[
K = 1 + \frac{R_Z R_1}{R_X + R_1} \left( 1 - \beta + \frac{C_Z}{C_1} + \frac{C_Z}{C_2} + \frac{R_X C_Z}{R_1 C_1} \right)
\]

\[
\left( R + \frac{R_X R_1}{R_X + R_1} \right) \left( \frac{R_Y}{R_1} + \frac{R_X C_Z}{R_1 C_2} + \beta \right)
\]
which is close to unity. The real axis zero in (5.14) can approximately cancel the real axis pole. At very low frequencies, the gain is unity, and at very high frequencies, the gain is $K$.

In the above two feedforward notch circuits, only one CCII is employed in each, and each circuit is easy to adjust and is superior to the twin-T circuits in Figure 44.

5.6 Simulation and Laboratory Results

To verify the feedforward method for the realization of second order complex-pole current-mode filters, simulations of the performances for three new circuits presented above are done with PSPICE, and two of the circuits are constructed and tested in the laboratory as well.

The bandpass circuit in Figure 43 is an example of the feedforward realizations that the current-mode network is converted from a voltage-mode circuit with Sallen-Key highpass circuit as the second order building block by employing the corollary of Celma's theorem. This circuit is simulated with parasitic elements, $R_X$, $R_Z$, $C_Z$ and $\beta$, of the non-ideal CCII in Figure 7 included. The peak frequency of the bandpass circuit, $f_0$, is set to be $1000\text{Hz}$ and $Q$ is $4.80$, for comparison purposes with other circuits. The component values selected are shown in Table 4, and Figure 47 gives the magnitude plot of the ideal transfer function and the simulated result (magnitude-normalized) in linear-log scale. The ideal and simulated results are summarized in Table 5. The simulated curve basically tracks the ideal one, the peak frequency is close to the designed value, and the pole-$Q$ is decreased somewhat with the effect of the parasitic element of the
Table 4. Component Values in Simulation for the circuit in Figure 43

<table>
<thead>
<tr>
<th>symbol</th>
<th>$R_1$ (kΩ)</th>
<th>$R_2$ (kΩ)</th>
<th>$R_3$ (kΩ)</th>
<th>$R_4$ (kΩ)</th>
<th>$C_1$ (nF)</th>
<th>$C_2$ (nF)</th>
<th>$C_3$ (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>1.658</td>
<td>152.8</td>
<td>3.316</td>
<td>3.316</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 5. The $f_0$ and $Q$ for Bandpass Circuit in Figure 43

<table>
<thead>
<tr>
<th></th>
<th>Peak frequency</th>
<th>-3dB bandwidth</th>
<th>pole-Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>ideal</td>
<td>1000</td>
<td>208</td>
<td>4.80</td>
</tr>
<tr>
<td>simulated</td>
<td>996</td>
<td>241</td>
<td>4.13</td>
</tr>
</tbody>
</table>

Figure 47. Magnitude plot for the bandpass circuit in Figure 43: ideal (dashed) and simulated (solid)
CCIIs. The simulation verified the correctness of the feedforward realization shown in Figure 43, and we can infer that the circuits of the same topology should work as well.

To further verify the feedforward realizations, a bandpass filter and a notch filter are simulated with the non-ideal CCII model, and they are constructed and tested in the laboratory as well.

The circuit in Figure 41 is a realization with the feedforward concept. In its bandpass application, the peak frequency of the bandpass circuit, \( f_0 \), is set to be 1000Hz and \( Q \) is 4.80 for convenience and availability of passive component values. In the laboratory, AD844As [58] are used as CCIIIs, and two AD844As are employed to realize a CCII-. The passive component values used in the simulation and laboratory are listed in Table 6. The \( C_2 \) is reduced by 4.5pF from its designed value to compensate the \( C_2 \) of CCII 1.

Figure 48 depicts the magnitude characteristic of the frequency response for the bandpass filter from the PSPICE simulation and laboratory measurement, as well as from the theoretical calculation (ideal curve). The simulation curve and laboratory data are magnitude-normalized for comparison purposes. The results are summarized in Table 7. The uncompensated parasitic elements of the first CCII, nonzero \( R_X \), finite \( R_Z \), and less-than-unity \( \beta \), all contribute to the decrease of \( Q \). The actual peak-frequency is close to 1000Hz because the products \( R_1R_2 \) and \( R_XR_Z \) are close in value. The \( Q \) value from the simulation is smaller than that from the laboratory work. It results from the fact that the typical parasitic values of CCIIIs may be farther away from the ideal values than the actual values are. The results from the simulation and the laboratory measurement agree with each other well within the passband, but the laboratory data shows a tendency to have less attenuation at frequencies far away from the passband than the simulation curve.
Table 6. Component Values in Simulation and Laboratory Work for the Bandpass Circuit in Figure 41

<table>
<thead>
<tr>
<th>symbol</th>
<th>R₁ (kΩ)</th>
<th>R₂ (kΩ)</th>
<th>R (kΩ)</th>
<th>C₁ (nF)</th>
<th>C₂ (nF)</th>
<th>C (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>11.91</td>
<td>19.96</td>
<td>13.70</td>
<td>102.5</td>
<td>1.036</td>
<td>55.78</td>
</tr>
</tbody>
</table>

Table 7. The \( f₀ \) and \( Q \) for Bandpass Application in Figure 41

<table>
<thead>
<tr>
<th></th>
<th>ideal</th>
<th>simulation</th>
<th>lab</th>
</tr>
</thead>
<tbody>
<tr>
<td>peak frequency (Hz)</td>
<td>1000</td>
<td>1005</td>
<td>1003</td>
</tr>
<tr>
<td>-3db bandwidth (Hz)</td>
<td>208</td>
<td>275</td>
<td>238</td>
</tr>
<tr>
<td>pole-Q</td>
<td>4.80</td>
<td>3.65</td>
<td>4.21</td>
</tr>
</tbody>
</table>

Figure 48. Magnitude plot for the bandpass circuit in Figure 41: ideal (dashed), simulated (solid), and measured (+)
does. This effect results from the tolerances of passive element values, the extra parasitic elements not included in the CCII model, and parasitic elements arising from the circuit built on a breadboard.

The notch circuit in Figure 45 is simulated with PSPICE, built and tested in the laboratory, too. Its notch frequency $f_2$ is set to 1000Hz and the pole $Q$ and $f_0$ are 4.80 and 1000Hz, respectively. The same lowpass network and component values as the above bandpass circuit (Table 7) are employed here for the circuit design. Due to the fact that the parasitic elements of the CCII would prevent the notch to reach zero, $R$ and/or $C$ are tuned to realize a deep notch. In the laboratory, $R$ is reduced to 11.9kΩ to realize a deepest notch. The simulated curve and laboratory data and the corresponding simulated curve, as well as the ideal magnitude plots are shown in Figure 49, and the results are shown in Table 8. The parasitic elements and the tuning of $R$ to reach a deep notch result in the difference of the simulated and measured data from the designed values. The parasitic elements contribute more to the error than the $R$. The actual $Q$ is closer to the ideal value than the simulated because of the same reason as the bandpass circuit presented above.

In this chapter, current-mode Sallen-Key lowpass and highpass filters with non-ideal CCII are analyzed first, and then they are employed as building blocks for the feedforward realizations of other types of current-mode second order complex pole filters. The resulting filters inherit the pole $Q$ and $\omega_0$ properties from the second order building block and give designers the flexibility of choosing different schemes under different situations. The characteristics of the filters are considered with non-ideal CCII as well,
Table 8. The Results of $f_0$ and $Q$ for the Feedforward Notch Circuit in Figure 45

<table>
<thead>
<tr>
<th></th>
<th>ideal</th>
<th>simulation</th>
<th>lab</th>
</tr>
</thead>
<tbody>
<tr>
<td>notch frequency (Hz)</td>
<td>1000</td>
<td>1001</td>
<td>1007</td>
</tr>
<tr>
<td>-3db bandwidth (Hz)</td>
<td>208</td>
<td>270</td>
<td>230</td>
</tr>
<tr>
<td>pole-Q</td>
<td>4.80</td>
<td>3.70</td>
<td>4.35</td>
</tr>
<tr>
<td>notch depth</td>
<td>0</td>
<td>0.13</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Figure 49. Magnitude plot for the notch filter in Figure 45: ideal (dashed), simulated (solid), and measured (+)
and compensation methods are introduced to approach desired properties. A bandpass feedforward circuit is simulated with PSPICE to prove the correction of the proposed topology, another bandpass circuit and a notch circuit are constructed and tested in the laboratory as well as simulated. The results prove that the feedforward realizations proposed are correct theoretically and easy to use in practice. Some new circuits presented have advantages over others in some aspects.

In this chapter and in the previous one, several new feedforward realizations of voltage-mode and current-mode continuous-time active filters are presented. These new circuits are derived using the feedforward concept as well as the adjoint theorem and the corollary to Celma’s theorem (also newly proposed here). It should be emphasized again that the new circuit circuits proposed in these chapters are merely examples of members of new families of circuits. Many other new circuits can be derived by employing other building blocks in the feedforward topology.
CHAPTER VI
CONCLUSIONS AND RECOMMENDATIONS

The existing complex pole active filters in voltage-mode and current-mode are recalled, as well as the adjoint theorem and Celma’s theorem that can be employed to transform voltage-mode active networks to their current-mode counterparts. The current transfer functions of the resulting current-mode networks are the same as the voltage transfer functions of their voltage-mode prototypes. The popularly used second generation current-conveyors (CCIIIs) are employed to construct new current-mode circuits while the traditional operational amplifiers (op amps) are used as active building blocks for voltage-mode networks.

A new corollary of conversion of voltage-mode circuits to current-mode circuits is presented as the corollary of Celma’s theorem that can be applied in some situations in which the adjoint theorem and Celma’s theorem do not work. It deals with the transition between voltage-mode op amp based circuits with the non-inverting terminal of the op amp not grounded and current-mode CCII based circuits whose output impedance is high. The detailed proof is given, in which the adjoint theorem and Celma’s theorem are employed as presumptions. With the new corollary, some current-mode circuits otherwise realized by complex procedures can be converted from their voltage-mode counterparts directly, and more new circuits may be obtained. The corollary of Celma’s theorem, as
well as the adjoint theorem and Celma's theorem, are widely used in the current-mode filter constructions presented in this dissertation.

The voltage-mode second order complex pole allpass filters realized by adding a third order allpole function to a first order allpass function was proposed in the literature in 1975, and the advantages of the realizations of these feedforward filters were discussed. The same feedforward concept can be used for current-mode allpass circuits. The current-mode realizations inherit the properties of the voltage-mode networks with respect to the transfer functions, the sensitivities of passive elements.

The feedforward concept is then extended for the construction of voltage-mode filters other than allpass filters. This use of the feedforward concept is new. A voltage-mode second order complex pole filter can be realized by adding an appropriate third order function, not necessarily an allpole function, to a first order proper function. New families of lowpass, highpass, bandpass and notch filters are proposed based on the feedforward topology. Examples of the new circuits are analyzed with the non-ideal op amp model, and the results are compared with other circuits proposed in the literature.

The feedforward topology is applicable for the realizations of families of current-mode filters as well as voltage-mode ones. Some new current-mode circuits are presented as examples of these new families obtained from the feedforward topology. The circuits are converted by using the adjoint theorem, Celma's theorem, the corollary of Celma's theorem, or constructed with the feedforward concept. In the feedforward circuit realizations presented, the Sallen-Key lowpass and highpass filters are used as the second order building blocks. The properties of the resulting circuits are analyzed, and the methods of reducing errors introduced by non-ideal CCIIIs to obtain design characteristics
are given. The feedforward realization of a bandpass filter with Sallen-Key highpass circuit as the second order building block is simulated. This circuit is converted from its voltage-mode prototype with the corollary of Celma’s theorem. The result agrees with what expected. Another feedforward bandpass circuit with a different topology and a notch filter, all constructed with Sallen-Key lowpass circuit as the second order building block, are simulated with PSPICE, built in the laboratory, and tested. All the simulation curves and laboratory data are analyzed. From the results, it is seen that, usually, the actual $\omega_0$ is close to its nominal value, and the actual $Q$ tends to be reduced in moderate to high $Q$ applications. In order for $Q$ to reach back to its nominal value, a $Q$ value greater than designed should be set to begin with, and with the parasitic elements taken into account, the $Q$ goes to its desired value.

With the feedforward topology, many voltage-mode and current-mode networks can be constructed, and only a few of the networks possible are presented here as illustrations and verifications of the theories and concepts.

In this dissertation, a corollary of Celma’s theorem for the conversions of voltage-mode op amp based circuits to their current-mode CCII based counterparts is proposed and proved. It can be employed to convert some voltage-mode circuits to their current-mode counterparts directly, and vice versa. These circuits may not be converted with the adjoint theorem and Celma’s theorem. The new corollary significantly improved the circuit conversion theories and it is proved to be correct by circuit simulations and laboratory data. However, this corollary is still restricted to some circumstances. It applies to passive $N_2$ and only some active $N_2$ (see Figure 28 and the proof for the corollary of Celma’s theorem). The current input and output relation for the network $N_2$
needs to be satisfied for the applications with active $N_2$. Also, in this corollary, $N_I$ is to be passive and has no internal connection to the ground, no feedback is allowed from the output to $N_2$, and, in the voltage-mode network, the output must be taken at the output of the op amp. All the above restrict the general applications of the corollary. More theoretical research is needed in the future to find general theorems that can be employed globally in circuit conversions. Strict proof is also required for new theorems.

In the feedforward circuits proposed, only Sallen-Key lowpass and highpass circuits are used as second order building blocks, in voltage-mode and current-mode realizations. It is because the realizations are straightforward. Some other second order complex pole circuits, commonly used or new, may also be applicable for the feedforward constructions with the proposed and/or new topologies and further research is suggested on the constructions of more new circuits. There will be more useful and practical circuits available, and some of them may have advantages over the existing ones.
REFERENCES


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